LOW FREQUENCY GUIDED PI. ATE WAVES PROPAGATION IN FIBER REINFORCED COMPOSITES

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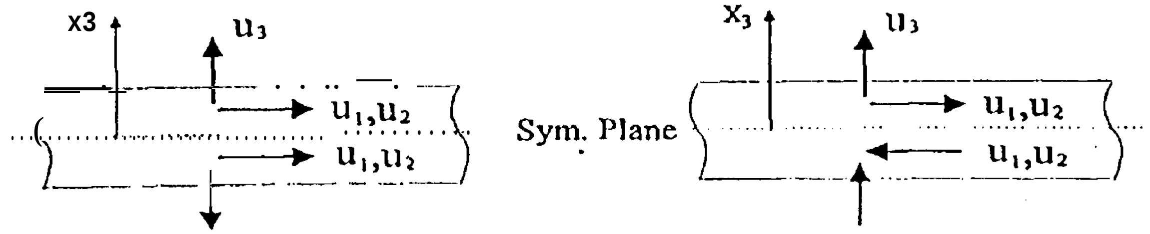
INTRODUCTION

The use of composite materials has increased steadily during the past two decades, particularly for aerospace, underwater and automotive structures. This is largely because many composite materials exhibit high strength-to-weight and stiffness-to-weight ratio, which make them ideally suited for use in weight-sensitive structures. The elastic properties of composite materials maybe significantly different in specimens manufactured under the same general specifications and may be different for the bulk material from those in the laminate. Moreover, the elastic properties of composites may vary as a result of aging, environmental degradation and other effects (e.g., matrix cracking) resulting in overstress and eventual failure of the material. This variability in the properties requires a careful material characterization before composites are used in a structure. Conventional destructive techniques for the determination of the elastic st iffness constants can be cost] y and often inaccurate; this is particularly true for the through-the-thicknes.s properties. Nondestructive determination of the elastic properties allows the performance and reliability of t! structure.

A systematic analytical method proposed by Mal et al [1], employing the leaky Lamb wave (LLW) phenomenon, was found to be an effective method for the characterization of the elastic constants. The model assumes that the composite consist of transversely isotropic layers and the experiment requires the use of water immersion or water injection through squirters for the transmission of the ultrasonic signals, This requirement for water coupling hampers the field applicability of the method and also limits the number of constants that can be measured. Particularly, the constant c_{11} is difficult to determine due to experimental limitations. The application of a contact coupled guided wave method offers the potential for a practical nondestruct ive characterizat ion method.

The theoretical and experimental studies of guided wave propagat ion in composites have grown considerably in recent years [2, 3]. For a homogeneous composite laminate with the symmetric axis parallel to the surfaces (Fig. 1), there are two modes of propagation: symmetric and antisymmetric. The lowest symmetric (Extensional) and ant asymmetric (flexural) modes arc the easiest to measure in an ultrasonic experiment and their velocity value can be used to determine certain material const ants. German et al [4] have developed an ultrasonic technique which is based on



(a) Symmetric Mode (b) Antisymmetric Mode "

Figure 1. Definition of the geometrical variables of the symmetric and antisymmetric guided wave modes.

contact type transducer-pair arrangement that can be used to determine the dispersion curves of the low frequency flexural mode, and the elastic properties. Unfortunately, the flexural wave signals are mixed with reflected signals from the boundary if the lateral dimension of the specimen is small in relation to the wavelength or the structure geometry is complex. In this case, only the Extensional male can be identified clearly.

A systematic parameter study is showing that the stiffness constants c_{11}, c_{22}, c_{23}, and c_{55} have strong influence on the dispersion curves for the lowest symmetric Extensional mode at low frequency range. In Figure 2, the dispersion curve for the symmetric mode and wave propagation along the fiber direction is plotted. In this Figure, the strong effect of varying c_{11} can be easily observed. In this reported study, a detailed analysis of the low frequency symmetric guide waves was conducted and the results were corroborated experiment all y.

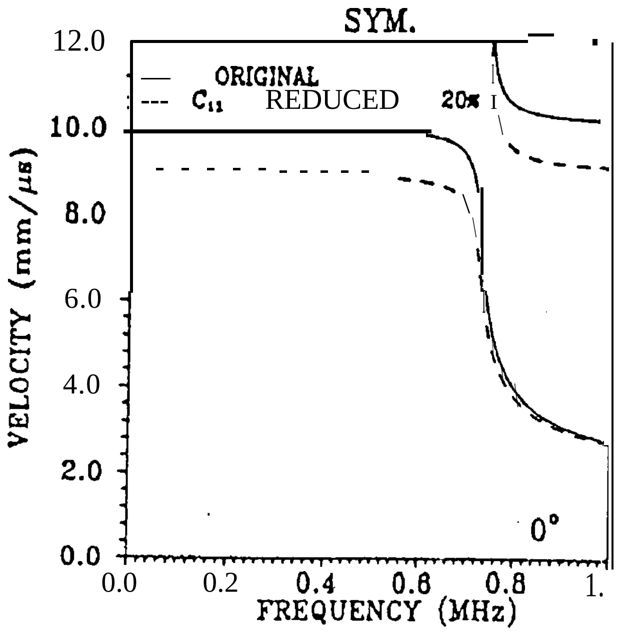


Figure 2: 'he influence of the elastic constant cl 1 on the dispersion curve of the symmetric mode.

FORMULATION OF SYMMETRIC MODE DISPERSION CURVES

a. Exact Linear Elastic Solution

Generally, the dispersion equations for guided wave propagation in composite materials are very complicate and need to be solve numerically. The exact solution of dispersion curves for Lamb wave propagation in orthotropic composite laminates has been derived by Mal [5], however the derived equations are highly nonlinear and the numerical solution is computational intensive. In the low frequency range approximations can be made to simplify the solution for the lowest Extensional mode. A unidirectional composite laminate is assumed transversely isotropic with symmetric axis along the fiber direction. The symmetry axis is defined as the xl-axis of the coordinate system and the stress-displacement relations is given explicit] y in Ref. [51, wherec11, $c_{12}, c_{22}, c_{23}, c_{55}$ are the five independent real stiffness " constants of the material. We also introduce five constants a_1, a_2, a_3, a_4 and a_5 related to c_{ij} and the density Of the material, p through

$$a_1 = c_{22}/\rho,\ a_2 = c_{11}/\rho,\ a_3 = (c_{12} + c_{55})/\rho$$
$$a_4 = (c_{22} - c_{23})/2\rho,\ a_5 = c_{55}/\rho \qquad (1)$$

The dispersion equation for the symmetric mode can bc expressed [5] as

$$\Delta_1 \cot(\zeta_1 \omega h) + \Delta_2 \cot(\zeta_2 \omega h) + \Delta_3 \cot(\zeta_3 \omega h) = 0 \qquad (2)$$

When frequency times thickness is approaching zero, i.e ωH - 0, then the dispersion equation becomes

$$\frac{\Delta_1}{\zeta_1} + \frac{\Delta_2}{\zeta_2} + \frac{\Delta_3}{\zeta_3} = 0 \qquad (3)$$

This equation can bc factorized as

$$(b_1 - b_2)(\rho V^2 - c_{55} n_1^2)(\rho V^2 - c_{11} n_1^2)\Omega(c_{ij}, n_1, n_2) = 0$$
$$\Omega(c_{ij}, n_1, n_2) = (-c_{12}^2 c_{55} + c_{11} c_{22} c_{55}) n_1^4$$
$$(-2 c_{12}^2 c_{22} + c_{11} c_{22}^2 + 2 c_{12}^2 c_{23} - c_{11} c_{23}^2 - 2 c_{12} c_{22} c_{55} + 2 c_{12} c$$
$$+(c_{22}^2 c_{55} - c_{23}^2 c_{55}) n_2^4 \quad +[(c_{12}^2 - c_{11} c_{22} - 22\ 55^2 + (-c_{22}^2 + c: \qquad (4)$$

Where the equation $\Omega(c_{ij}, n_1, n_2) = 0$ represents the dispersion equation of the limit of the lowest symmetric mode.

a. For propagation along the symmetric axis (0°), the dispersion equation can bc simplified as

$$(\rho V^2 - c_{55})(c_{22} \rho V^2 + c_{12}^2 - c_{11} c_{22}) = 0 \qquad (5)$$

and solved as

$$V_{1,2} = \sqrt{\frac{c_{55}}{\rho}},\ \sqrt{\frac{c_{11} - c_{12}^2/c_{22}}{\rho}} \qquad (6)$$

where V_1 represents the speed of the shear-horizontal SH_0 mode or quasi- transverse mode, and V_2 represent the symmetric mode SO or quasi- longitudinal mode.

b. For propagation along the the direction perpendicular to the symmetric axis (90°), the equation can be simplifies as

$$(\rho V^2 - c_{55})(c_{22}\rho V^2 - c_{22}^2 + c_{23}^2) = o \tag{7}$$

and solved explicitly as

$$V_{1,2} = \sqrt{\frac{c_{55}}{\rho}}, \sqrt{\frac{c_{22} - c_{23}^2/c_{22}}{\rho}} \tag{8}$$

For isotropic material, the solution can be reduced to the well know expression

$$V_{1,2} = \sqrt{\frac{c_{55}}{\rho}}, 2\sqrt{\frac{c_{55}}{\rho}}\sqrt{1 - \frac{c_{55}}{c_{11}}} \tag{9}$$

b. Approximate Plate Theories

For the low frequency range of guided wave propagation in composite laminates, various approximation models were proposed [8]. It is well known that classical plate theories underestimate the deflections as well as the stresses and overestimate the phase velocity of the propagating waves. The error associated with the calculation grows significantly with the increase in plate thickness or in the frequency. Hence, for dynamic analysis of high values of thickness times frequency the classical plate theories are inadequate for the anal ysis. Mindlin and others [7] proposed a improved approximation using the first order shear deformation theory and retaining the transverse shear and rotary inertia of the plate elements. Based on this theoretical approach the dispersion curves of the fist *antisymmetric* mode can be approximated very closely to the exact solutions [8]. According to this theory, the displacement components are assumed to be of the form

$$\begin{aligned} u_1 &= u_1^0(x_1, x_2, f) + x_3\psi_1(x_1, x_2, t) \\ u_2 &= u_2^0(x_1, x_2, t) + x_3\psi_2(x_1, x_2, t) \\ u_3 &= u_3^0(x_1, X_2, t) \end{aligned} \tag{lo}$$

where u^0_1, u^0_2 and u^0_3 are the displacement components of a point in the mid-plane, and ψ_1 and ψ_2 are the rotations of a line element, originally perpendicular to the longitudinal plane about the x_2 and xl axes, respectively. However, based on this assumption, the lowest symmetric modes are nondispersive and are the same as the results from the classical plate. This is the result of ignoring that u_1 and U_2 are even functions of x_3, and u_3 is an odd function of x_3 for the symmetric mode (Fig. 1 a). In order to obtain a high order approximate symmetric mode dispersion curve, a term $x_3\psi_3$ is included in the out of plane displacement u_3.

$$\begin{aligned} u_1 &= u_1^0(x_1, x_2, t) \\ u_2 &= u_2^0(x_1, x_2, t) \\ u_3 &= x_3\psi_3(x_1, x_2, t) \end{aligned} \tag{11}$$

Hence the governing equation for the symmetric mode can be written as

$$\begin{vmatrix} A_{11}\frac{\partial^2}{\partial x_1^2}+A_{55}\frac{\partial^2}{\partial x_2^2} & (A_{12}+A_{55})\frac{\partial^2}{\partial x_1\partial x_2} & A_{12}\frac{\partial}{\partial x_1} \\ (A_{12}+A_{55})\frac{\partial^2}{\partial x_1\partial x_2} & A_{55}\frac{\partial^2}{\partial x_1^2}+A_{22}\frac{\partial^2}{\partial x_2^2} & A_{23}\partial x_2 \\ A_{12}\frac{\partial}{\partial x_1} & A_{23}\frac{\partial}{\partial x_2} & D_{55}\frac{\partial^2}{\partial x_1^2}+D_{44}\frac{\partial^2}{\partial x_2^2}+ \end{vmatrix} \qquad (12)$$

Assume plane wave solutions as follows

$$\begin{aligned} u_1^0 &= U_1^0 e^{i(k_1x_1 + k_2x_2 - \omega t)} \\ u_2^0 &= U_2^0 e^{i(k_1x_1 + k_2x_2 - \omega t)} \\ \psi_3 &= \Psi_3 e^{i(k_1x_1 + k_2x_2 - \omega t)} \end{aligned} \qquad (13)$$

where k_1, k_2 and k_3 represent the wavenumbers along the x_1, x_2 and x_3 directions, respectively, and ω is the circular frequency. Hence, the dispersion equation can be derived from the following eigenvalue solution:

$$\begin{vmatrix} -(A_{11}k_1^2+A_{55}k_2^2)+I_1\omega^2 & -(A_{12}+A_{55})k_1k_2 & iA \\ -(A_{12}+A_{55})k_1k_2 & -(A_{55}k_1^2+A_{22}k_2^2)+I_1\omega^2 & iA \\ iA_{12}k_1 & iA_{23}k_2 & D_{55}k_1^2+D_{44} \end{vmatrix} \qquad (14)$$

where $n_1 = \cos\phi$ and $n_2 = \sin\phi$; ϕ *is the* wave propagating angle, and $k_1 = \omega/V\, n_1, k_2 = \omega/V\, n_2$, V *is the* phase velocity. A_{ij} B_{ij} and D_{ij} are commonly used the generalized elastic parameters for composite laminate. If the laminate is a transversely isotropic material, then from

$$\begin{aligned} & I_1 = \rho H,\ I_3 = \rho H^3/12 \\ & A_{11} = c_{11}H,\ A_{12} = c_{12}H,\ A_{22} = c_{22}H,\ A_{55} = c_{55}H, \\ & D_{55} = c_{55}I_3/\rho,\ D_{44} = c_{44}I_3/\rho,\ c_{44} = (c_{22}-c_{23})/2 \end{aligned} \qquad (15)$$

the approximated dispersion equation can be expressed as

$$\begin{aligned} & [(-c_{12}^2c_{55} + c_{11}c_{22}c_{55})n_1^4\ (-2c_{12}^2c_{22} + c_{11}c_{22}^2 + 2c_{12}^2c_{23} - \\ & 2c_{12}c_{23}c_{55})n_1^2\ ^2 r+(c_{22}^2c_{55} - c_{23}^2c_{55})n_2^4 \\ & +[(c_{12}^2 - c_{11}c_{22} - c_{22}c_{55})n_1^2+(-c_{22}^2 + c_{23}^2 - c_{22}c_{55})n_2^2]\rho V^2 + C_2 \end{aligned} \qquad (16)$$

When at the limit $\omega H \to 0$ this equation is the same as equations (4). Note that high order approximations Such as

$$\begin{aligned} & u_1 = u_1(x_1, x_2, t) + x_3\eta_1(x_1, x_2, t) \\ & u_2 = u_2^0(x_1, x_2, t) + x_3^2\eta_2(x_1, x_2, t),\ u_3 = x_3\psi_3(x_1, x_2 ; \end{aligned} \qquad (17)$$

can lead to more accurate results but they will increase the complexity of the dispersion equations.

The measured wave velocity can be either the phase or the group velocity depending on the experimental set and the value of the two maybe very different, particularly in composite materials. The group velocity V_e can be calculated from the characteristic equation of the phase velocity by

$$V_e = -\frac{\partial\Omega/\partial n}{\partial\Omega/\partial V}$$

EXPERIMENTAL

The experiment? consists of a contact pitch catch arrangement, where the pulse source is induced by breaking a pencil lead on the surface of the test composite. Three identical receiving transducers are placed in contact with the composite laminate along one line that defines the angle of propagation and spaced at a distance of 25 mm apart. The transducers are broadband type with 5 MHZ center frequency (Digital Waves, Model B 1000). For data acquisition, Fracture Wave Detector (Digital Waves, F4000) with four signal conditioning modules were used. Each of the transducers was connected to a wideband preamplifier through a signal conditioning module and the signals were digitize and recorder at a rate of 3.125 MHZ to 25 MHZ. A schematic view of the experimental setup is shown in Fig. 3.

The use of the pencil lead breaking method as the source. of signals was chosen since it forms signals with a low frequency broadband spectra at the range of 50 to 100 kHz. The data for each signal was transferred to a personal computer for analysis and measurement were made along different directions

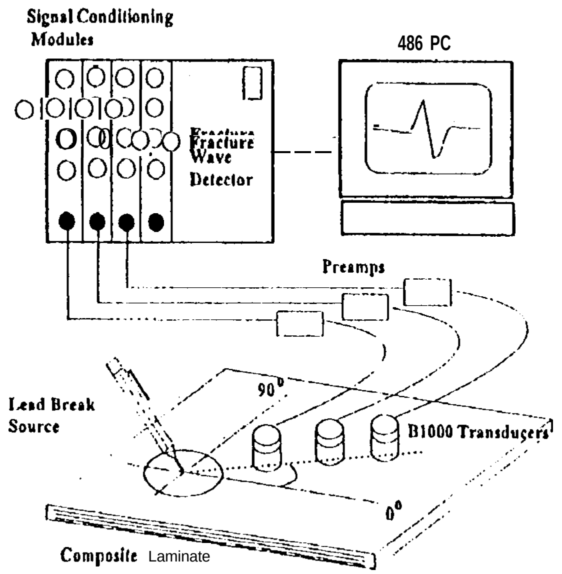


Figure 3, A schematic description of the experimental setup.

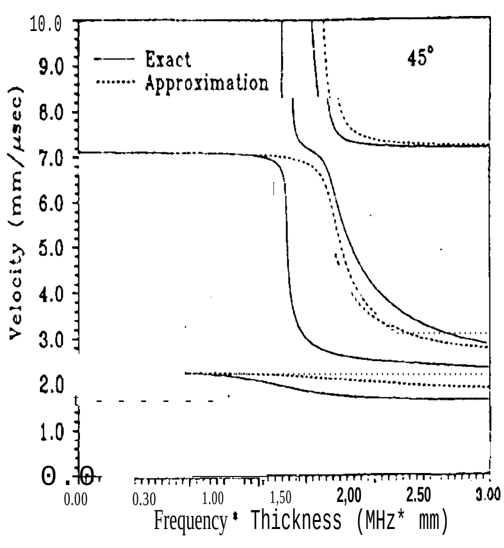


Figure 4. Comparison of dispersion curves for a 1 mm thick graphite/epoxy plate between exact theory and shear deformation theory for wave propagation at 45° to fibers.

with the fibers in 15° intervals from 0" to *90°*. The transducers were placed along the propagation direction with distance 25 mm 50 mm, and 75 mm from the source. At the various directions of the transducers placement, the group velocity was determined from the time-of-flight measurement using the time arrival of the first received signal.

A $[0]_{16}$ 12 x 12 cm^2 unidirectional AS4/3502 (Hercules) graphite/epoxy laminate was used in the experiment. The laminate was produced using standard hot-press curing technique leading to a laminate thickness of 3.175 mm. For the inversion of c_{11} and c_{12}, the material density of p =1.56 g/cm^3 was used and the matrix dominated material constants c_{22}, c_{23}, and c_{55} were predetermined using the inversion technique that is described in Ref. [1] as

$$c_{22} = 15.6,\ c_{23} = 7.89,\ c_{55} = 5.00\ \text{(GPa)}$$

RESULTS AND CONCLUDING REMARKS

Dispersion curves for the exact and approximate solution of the symmetric mode are shown in Figure 4. This Figure'is showing the phase velocity of wave propagation in a unidirectional graphite/epoxy along 45° with the fibers. It can be seen that the shear deformation approximate solution agrees with the exact solutions for the frequency times the thickness is below 0.7 MHZ-mm. Further, this approximation allows the calculation of modes that can not be obtained using the classical plate theory.

The measured and calculated group velocity for wave propagation along the 0° to 90° with the fibers are present cd in Fig. 5. The elastic constants c_{11}, c_{12} were det ermined by inversion of the measured group velocity and they are:

$$c_{11} = 155.01,\ c_{12} = 6.44,\ \text{(GPa)}$$

It can bc seen that the calculated curves fit the experimental data quite well. However, it is known that the group velocity of the Extensional mode in this frequency range may not be sensitive to some of the elastic constants. In order to characterize the material constants from the measured group velocity, a parametric study was carried out and are presented in Fig. 6. From this Figure, onc can easil y sec that c_{11} has the strongest effect on the group velocity curve near the 0" with X-axis and decreasing toward zero at about 45°.

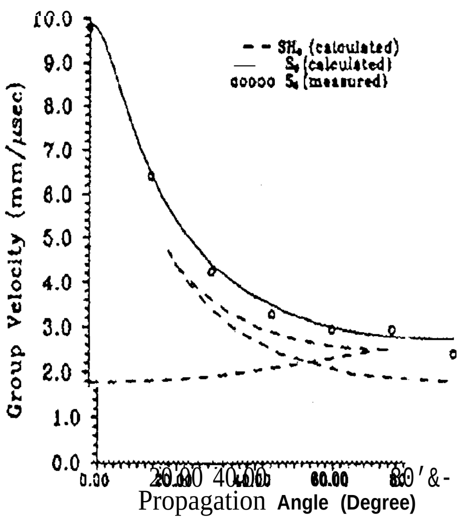


Figure 5. Measured and calculated group velocity for the Extensional mode waves propagating in a unidirectional graphite/epoxy plate of 3.175 mm thickness from O" to 90° to the fibers.

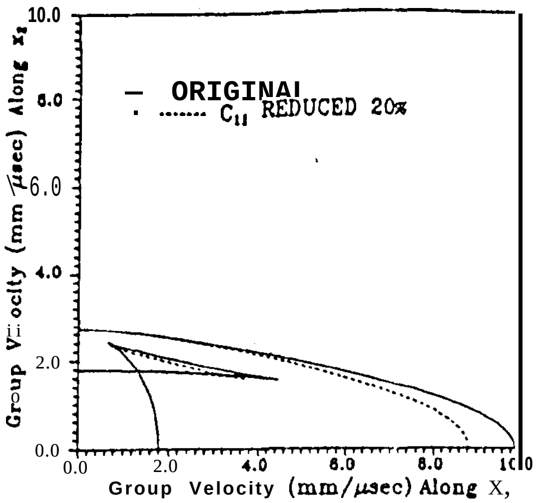


Figure 6. Influence of the stiffness constants c_{ij} on group velocity for the lowest symmetric mode of a unidirectional graphite/epoxy laminate.

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