

The Solar Test of the Equivalence Principle

John D. Anderson,¹ Mark Gross,² Kenneth L. Nordtvedt³ and Slava G. Turyshev⁴

Jet Propulsion Laboratory MS 301-230,
California Institute of Technology
4800 Oak Grove Drive - Pasadena, CA 91109 - USA

Abstract

The Earth, Mars, Sun, Jupiter system allows for a sensitive test of the strong equivalence principle (SEP) which is qualitatively different from that provided by Lunar Laser Ranging. Using analytic and numerical methods we demonstrate that Earth-Mars ranging can provide a useful estimate of the SEP parameter η . Two estimates of the predicted accuracy are derived and quoted, one based on conventional covariance analysis, and another (called “modified worst case” analysis) which assumes that systematic errors dominate the experiment. If future Mars missions provide ranging measurements with an accuracy of σ meters, after ten years of ranging the expected accuracy for the SEP parameter η will be of order $(1.2) \times 10^{-4}$. These ranging measurements will also provide the most accurate determination of the mass of Jupiter, independent of the SEP effect test.

Subject headings: celestial mechanics, stellar dynamics gravitation Earth planets and satellites: Mars dark matter

1 Introduction

The question, posed long ago by Newton, of the relation between the gravitational and inertial masses of the same body continues to be the subject of theoretical and experimental investigations. This question arises in most any theory of gravitation. The equality of inertial and passive gravitational masses, often stated as the weak equivalence principle (WEP), implies that different neutral massive test bodies will have the same acceleration of free fall A_0 in an external gravitational field, and therefore in freely falling inertial frames the external gravitational field appears only in the form of a tidal interaction (Singh 1960). Up to these tidal corrections, freely falling bodies behave as if external gravity were absent (Bertotti & Grishchuk 1990). In the construction of the general theory of relativity Einstein went further, postulating that not only mechanical laws of motion, but all non-gravitational laws should behave in freely falling frames as if gravity were absent. If local gravitational physics is also independent of the more extended gravitational environment, we have what is known as the Strong Equivalence Principle (SEP).

Various experiments have been performed to measure the ratios of gravitational to inertial masses of bodies. Experiments on bodies of laboratory dimensions verify the WEP to a fractional precision $\delta A_0/A_0 = \delta(m_g/m) \sim 10^{-11}$ by (Roll, Krotkov & Dicke 1964) and more recently to a precision

¹E-mail: jda@zeus.jpl.nasa.gov

²Department of Physics and Astronomy, California State University, Long Beach CA 90840. E-mail: mgross@csulb.edu

³Northwest Analysis, 118 Sourdough Ridge Road, Bozeman MT 59715

⁴On leave from Bogolyubov Institute for Theoretical Microphysics, Moscow State University, Moscow, 119899 Russia.
E-mail: sgt@zeus.jpl.nasa.gov

$\delta A_0/A_0 \sim 10^{-12}$ by (Braginsky & Panov 1972; Adelberger et al. 1994). The accuracy of these experiments is sufficiently high to confirm equal strong, weak, and electromagnetic interaction contributions to both the passive gravitational and inertial masses of the laboratory bodies. This impressive evidence for laboratory size bodies does not, however, carry over to celestial body scales.

Laboratory size bodies used in the experiments cited above possess a negligible fraction of gravitational self-energy and therefore such experiments indicate nothing about the quality of gravitational self-energy contributions to the inertial and passive gravitational masses of the bodies (Nordtvedt 1968a). Interesting results for celestial bodies are obtained if one includes terms of fractional order (Ω_B/mc^2) , where m is the mass of a body B and Ω_B is its gravitational binding or self-energy:

$$\left(\frac{\Omega}{mc^2}\right)_B = -\frac{G}{2mc^2} \int_{V_B} d^3x d^3y \frac{\rho_B(x)\rho_B(y)}{|x-y|}. \quad (1)$$

This ratio is typically $\sim 10^{-25}$ for bodies of laboratory sizes, so experimental accuracy of a part in 10^2 sheds no light on how gravitational self-energy contributes to the inertial and gravitational masses of bodies.

To test the SEP one must utilize planetary-sized extended bodies in which case the ratio (1) is considerably higher. Numerical evaluation of the integral of expression (1) for the standard solar model (Ulrich 1982) obtains

$$\left(\frac{\Omega}{mc^2}\right)_S \approx -3.52 \cdot 10^{-6} \quad (2a)$$

and the analogous value has been obtained for the Earth (Allen 1985):

$$\left(\frac{\Omega}{mc^2}\right)_E \approx -4.6 \cdot 10^{-10}, \quad (2b)$$

The development of the parameterized post-Newtonian (PPN) formalism (Nordtvedt 1968b; Will 1971; Will & Nordtvedt 1972), allows one to describe within the common framework the motion of celestial bodies in external gravitational fields within a wide class of metric theories of gravity. Within the accuracy of modern experimental techniques, the PPN formalism becomes useful framework for testing the SEP for extended bodies. In that formalism, the ratio of passive gravitational to inertial mass is given by (Nordtvedt 1968a,b)

$$\frac{m_g}{m_i} = 1 + \eta \cdot \frac{\Omega}{mc^2}, \quad (3)$$

in which the SEP violation is quantified by the parameter η . In fully conservative, Lorentz-invariant theories of gravity the SEP parameter is related to the PPN parameters by

$$\eta = 4\beta - \gamma - 3 \quad (4)$$

and is more generally related to the complete set of PPN parameters through the relation

$$\eta = 4\beta - \gamma - 3 - \frac{10}{3} \xi - \alpha_1 + \frac{1}{3} (2\alpha_2 - 2\zeta_1 - \zeta_2). \quad (5)$$

A difference between gravitational and inertial masses produces observable perturbations in the motion of celestial bodies in the Solar System. By analyzing the effect of a non-zero η on the dynamics of the Earth-Moon system moving in the gravitational field of the Sun, Nordtvedt (1968c) found a polarization of the Moon's orbit in the direction of the Sun with amplitude $\delta r \sim \eta C_0$, where C_0 is a

constant of order 13nt. We call this effect, generalized to all similar three body situations, the ‘‘SEP polarization effect’’. The most accurate test of this effect is presently provided by Lunar Laser Ranging (LLR) (Williams 1976; Shapiro et al. 1976; Dickey et al. 1989), and in the most recent results (Dickey et al. 1994; Williams et al. 1995) the parameter η was determined to be

$$\eta = -0.0005 \pm 0.0011 . \quad (6)$$

Other tests of SEP violation have been discussed. An experiment employing misting binary pulsar data has been proposed by Damour and Schäfer (1991). A search for the SEP polarization effect in the motion of the Trojan asteroids was suggested in (Nordtvedt 1968a) and carried out by (Orelana & Vucetich 1992). Also results are available from numerical experiments with combined processing of LLR, spacecraft tracking, planetary radar and Very Long Baseline Interferometer (VLBI) data (Chandler et al. 1994).

It has been observed previously that a measurement of the Sun’s gravitational to inertial mass ratio can be performed using the Sun-Jupiter-Mars or Sun-Jupiter-Earth system (Nordtvedt 1970; Shapiro et al. 1976). This is the first paper from a planned series addressing the above problem. The question we would like to answer first is how accurately can we do this ranging experiment? We emphasize that the Sun-Mars-Earth-Jupiter system, though governed basically by the same equations of motion as Sun-Earth-Moon system, is significantly different physically. For a given value of SEP parameter η the polarization effects on the Earth and Mars orbits are almost two orders of magnitude larger than on the lunar orbit. In this work we examine the SEP effect on the Earth-Mars range, which has been measured as part of the Mariner 9 and Viking missions. Moreover, future Mars missions, now being planned as joint U. S.-Russian endeavours, should yield additional ranging data.

The dynamics of the four-body Sun-Mars-Earth-Jupiter system in the Solar system barycentric inertial frame were considered. The quasi-Newtonian acceleration of the Earth (E) with respect to the Sun (S) is straightforwardly calculated to be:

$$\begin{aligned} \vec{A}_E - \vec{A}_S = & -\mu_{SE}^* \cdot \frac{\vec{R}_{SE}}{R_{SE}^3} + \left(\frac{m_g}{m_i}\right)_E \sum_{k=M,J} \mu_k \left[\frac{\vec{R}_{kS}}{R_{kS}^3} - \frac{\vec{R}_{kE}}{R_{kE}^3} \right] + \\ & + \eta \left[\left(\frac{\Omega}{mc^2}\right)_S - \left(\frac{\Omega}{mc^2}\right)_E \right] \sum_{k=M,J} \mu_k \frac{\vec{R}_{kS}}{R_{kS}^3} = \vec{A}_N + \vec{A}_{tid} + \vec{A}_\eta \end{aligned} \quad (7)$$

where $\mu_{SE}^* \equiv \mu_S + \mu_E + \eta \left[\mu_S \left(\frac{\Omega}{mc^2}\right)_E + \mu_E \left(\frac{\Omega}{mc^2}\right)_S \right]$ and $\mu_k \equiv Gm_k$. The subscripts (M) and (J) indicate Mars and Jupiter, respectively. Also $\vec{R}_{BC} \equiv \vec{R}_C - \vec{R}_B$ is the vector from body B to body C and \vec{A}_N is the Newtonian acceleration term. A_η is the SEP acceleration term, which is of order $1/c^2$. While it is not the only term of that order, the other post-Newtonian $1/c^2$ terms (suppressed in eq. (7)) do not affect the determination of η until the second post-Newtonian order ($\sim 1/c^4$). Finally A_{tid} is the Newtonian tidal acceleration term. Note, that $A_\eta/A_N \sim 71.10^{-10}$ and $A_{tid}/A_N \sim 7.10^{-6}$. Given that level of accuracy, we ignore the mutual attraction of the two planets, Earth and Mars. The SEP acceleration is treated as a perturbation on the restricted three-body problem, and the SEP effect is evaluated as an alteration of the planetary Keplerian orbit.

Using expression (3) and noticing that $\mu_M/R_{SM} \ll \mu_J/R_{SJ}$, we obtain from eq.(7),

$$\begin{aligned} \vec{A}_E - \vec{A}_S \approx & \\ \approx & -\mu_{SE}^* \cdot \frac{\vec{R}_{SE}}{R_{SE}^3} + \mu_J \left[\frac{\vec{R}_{JS}}{R_{JS}^3} - \frac{\vec{R}_{JE}}{R_{JE}^3} \right] + \eta \left(\frac{\Omega}{mc^2}\right)_S \mu_J \frac{\vec{R}_{JS}}{R_{JS}^3} \end{aligned} \quad (8)$$

Corresponding equations for Mars are obtained by replacing subscript E by M in eqs. (7) and (8). To good approximation the SEP acceleration \vec{A}_η has constant magnitude and points in the direction from Jupiter to the Sun, and since it depends only on the mass distribution in the Sun, the Earth and Mars experience the same perturbing acceleration. The responses of the trajectories of each of these planets due to the term \vec{A}_η determines the perturbation in the Earth-Mars range and allows a detection of the SEP parameter η through a ranging experiment.

The presence of the acceleration term \vec{A}_η in the equations of motion results in a polarization of the orbits of Earth and Mars, exemplifying the planetary SEP effect. We investigate here the accuracy with which the parameter η can be determined through Earth-Mars ranging and approach the problem with a series of successive approximations. In Sections II and III the "tidal term" \vec{A}_{tid} in equation (8) is neglected. In Section II the perturbation theory about circular coplanar reference orbits for Earth and Mars is performed. A covariance analysis is carried out to estimate the accuracy to which the SEP parameter η can be determined from a large number of Mars ranging measurements, each of accuracy σ meters. In Section III the calculations of Section II are improved by employing numerical integration rather than perturbation theory. The agreement between the two approaches is good, and the eccentricity corrections are found to improve the accuracy of the analytic approximation significantly. In Section IV the tidal acceleration term A_{tid} is restored, requiring the addition of the mass of Jupiter M_J to our set of covariance parameters. This mass can be determined more accurately from a few years of Earth-Mars ranging than from the Pioneer 10,11 and Voyager 1, 2 flybys combined, independent of the η measurement. In Section V we summarize and suggest further avenues for testing SEP violation.

2 Perturbation About a Circular Reference Orbit

Here and in the next section the problem is simplified by ignoring the tidal term \vec{A}_{tid} in equation (8) and the corresponding equation for Mars. We examine the effect of the SEP acceleration term \vec{A}_η in (8) on the orbits of Earth and Mars by carrying out first-order perturbation theory about the zeroth order orbits of Earth and Mars, taken to be circular. Jupiter's orbit is also taken as circular and coplanar with Earth and Mars. With these approximations a nine parameter covariance analysis is carried out to estimate how precisely Mars ranging can determine the SEP parameter η . These approximations give a standard deviation for η which closely agrees with the later numerical integration result of Section III.

A heliocentric reference frame rotating with Jupiter at constant angular frequency $\omega = \sqrt{\mu_S/R_{SJ}^3}$ is assumed. The SEP perturbation is represented by a constant acceleration g_η directed from Jupiter to the Sun. Locating Jupiter on the x axis, the following Hamiltonian for both Earth and Mars in polar coordinates results:

$$H = \frac{1}{2} \left(p_r^2 + \frac{p_\theta^2}{r^2} \right) - \omega p_\theta - \frac{\mu_S}{r} + g_\eta r \cos\theta, \quad (9)$$

where

$$g_\eta \equiv \eta \frac{\Omega}{mc^2} \frac{\mu_J}{R_{JS}^2}, \quad p_r = \dot{r}, \quad p_\theta = r^2(\dot{\theta} + \omega). \quad (10)$$

We take the reference orbits for Earth and Mars to be circular Keplerian with $r = a$ and $g_\eta = 0$. Then $p_\theta \equiv \sqrt{\mu_S a}$ and $n \equiv \sqrt{\mu_S/a^3}$ is the orbital angular frequency. A covariance analysis can be performed to estimate the accuracy to which one can determine the SEP parameter through Earth-Mars ranging. Earth-Mars range ρ_{EM} is defined as follows:

$$\rho_{EM}^2 = r_E^2 + r_M^2 - 2r_E r_M \cos(\theta_M - \theta_E). \quad (11)$$

The variation of $\delta\rho_{EM}$ is then:

$$\delta\rho_{EM}(t) = \sum_{k=1}^9 \delta q_k \frac{\partial \rho_{EM}}{\partial q_k}(t), \quad (12)$$

where $\delta\rho_{EM}$ does not depend on $\delta\omega$ and \mathbf{q} is the following vector:

$$\mathbf{q} \equiv (r_{E0}, r_{M0}, p_{E\tau_0}, p_{M\tau_0}, p_{E\theta_0}, p_{M\theta_0}, \theta_{E0} - \theta_{M0}, \mu_S, g_\eta). \quad (13)$$

The partial derivatives are easily computed and for example,

$$\frac{\partial \rho_{EM}}{\partial r_{E0}} = \frac{1}{\rho_{EM}} \left[a_E \cos(n_E t) - \frac{3a_M}{2} \cos(\theta_E - \theta_M - n_E t) + \frac{a_M}{2} \cos(\theta_E - \theta_M + n_E t) \right] \quad (14)$$

By definition, the covariance matrix α_{jk} has elements:

$$\alpha_{jk} = \sum_{i=1}^N \frac{1}{\sigma_i^2} \frac{\partial \rho_{EM}}{\partial q_j}(t_i) \frac{\partial \rho_{EM}}{\partial q_k}(t_i), \quad (15)$$

where N ranging measurements have been made at times $t_i, i = 1, \dots, N$, and have uncertainties σ_i . The uncertainty in the estimations of the SEP acceleration term g_η and parameter η are then:

$$\sigma_{g_\eta} = \sigma_\eta \left(\frac{\Omega}{mc^2} \right)_S \frac{\mu_J}{R_{JS}^2} = \sqrt{(\alpha^{-1})_{g_\eta g_\eta}}, \quad (16)$$

with the fractional binding energy $(\Omega/mc^2)_S$ given by expression (2a).

The results obtained are presented in Figure 1, with the thin dashed curve showing the result of evaluating equations (15) and (16) for σ_η assuming N daily range measurements have been taken during the mission, each with the same uncertainty σ , measured in meters. The initial angles between Earth and Jupiter and Mars and Jupiter were taken from the DE242 ephemeris at time 2441272.75, the beginning of the Mariner 9 ranging measurements. The quantities n_E and n_M are taken to be the mean motions of Earth and Mars, a_E and a_M their mean distances from the Sun, and the frequency ω is taken to be the mean motion of Jupiter. The uncertainty in η first drops very rapidly with time and then after a few years approaches the asymptotic behavior $\sim N^{-1/2}$. This result gives a lower bound on the uncertainty as predicted by conventional covariance analysis. For a mission duration of order ten years, the uncertainty behaves as

$$\sigma_\eta \sim .0028 \sigma / \sqrt{N} \quad (17)$$

This result, eq.(17), assumes Gaussian random ranging errors with a white spectral frequency distribution. But past ranging measurements using the Viking Lander have been dominated by systematic error (Chandler et al. 1994). One approach to accounting for systematic error is to multiply the formal errors from the covariance matrix by \sqrt{N} (Nordvedt 1978). With this approach, the expected error decreases rapidly near the beginning of the data interval, but for large N approaches an asymptotic value as demonstrated by Fig.1. However, we believe this is overly conservative. A more optimistic error estimate would include a realistic description of the time history of the systematic error. For example if we knew that the systematic error was a sinusoid of known angular frequency, we could add its amplitude and phase as additional variables in the covariance analysis. By contrast

the worst-case error analysis treats each variable of concern equally by multiplying its computed error by \sqrt{N} .

A realistic systematic error budget for ranging data to Mars, or for Mercury as considered by a group at University of Colorado (Ashby et al. 1995), is not presently available. But it is unlikely that we will be so unfortunate that the frequency spectrum of the signal will correspond to the spectrum of the systematic error. However, we reduce the upper error bound determined by the \sqrt{N} multiplier ($\sigma_\eta = 0.00280$) by a numerical factor. The ranging experiment proposed by Ashby et al. (1995) for Mercury is quite similar to our proposed experiment using Mars. We therefore follow the Colorado group and reduce the worst-case error estimate by a factor of three and call the result the modified worst-case analysis. This yields an asymptotic value for the error of $\sigma_\eta = 0.00090$, in our opinion a realistic estimate of the upper bound on the error.

In the case of the existing Mars ranging derived from the Mariner 9, Viking, and Phobos missions, the rms ranging residual referenced to the best-fit Martian orbit was 7.9 m. We computed the covariance matrix with assured daily range measurements for Mariner 9 (actual data interval JD 2441272.750 to JD 2441602.504) and Viking (actual data interval JD 2442980.833 to JD 2445286.574). Additionally, one ranging measurement from Phobos (actual time JD 2447605.500) was included, although it had negligible effect on the result. With $\sigma = 7.9111$, a formal error $\sigma_\eta = 0.0005$ is obtained from the covariance matrix. If 7.2 years of Mars ranging is assumed, though not continuous we reach the asymptotic limit of the modified worst-case analysis (as shown by Fig. 1), a realistic error $\sigma_\eta = 0.009$ which is about 17 times the formal error. This is a factor of eight larger than the realistic error set by Chandler et al. (1994) from an analysis of the actual combined LLR and Mars ranging data. We conclude that the best determination of η is provided by the 1,1,1 data, but the existing Mars ranging can provide an independent solar test with a realistic accuracy interval of

$$\sigma_\eta \approx 0.0005 - 0.009 \quad (\text{Mariner 9, Viking, Phobos}). \quad (18)$$

Expression (35) establishes the interval for expected accuracy σ_η with the lower and upper bounds estimated for existed data by conventional covariance analysis and “modified worst case” analysis respectively. This interval will be narrowed by ongoing upgrades to GPS instrumentation and better modelling of the antenna and spacecraft ranging systems (Anderson et al. 1985). Future Mars Orbiter and Lander missions are expected to achieve an rms systematic ranging error between 0.5 and 1.0 m. Then after a few years of ranging, the realistic error on η should fall to around

$$\sigma_\eta \approx (0.00006 - 0.0011) \sigma \quad (\text{Future! Mars missions}). \quad (19)$$

3 Numerical Integration Without the Tidal Term \vec{A}_{tid} .

To obtain a more accurate estimation of σ_η and to check the results of Section 11, a numerical integration of the Sun-Earth-Mars-Jupiter system in heliocentric coordinates is performed. Equation (8) without the \vec{A}_{tid} term was used for Earth and Mars (with the interchanging of the subscripts $(E) \rightarrow (M)$). The equation of motion for Jupiter did not include a SEP term or Newtonian perturbation from other planets. The DE242 ephemeris was again used for the initial conditions, and the same 9 parameters of equation (30) were used in the covariance analysis.

Assuming frequent Earth-Mars range measurements as in Section 11, the results are shown in Figure 1 (thick dashed curve). The results agree fairly well with the analytical ones of Section 11 (thin dashed curve). For ten years of observations we obtain

$$\sigma_\eta \sim .0027\sigma/\sqrt{N}, \quad (20)$$

with range measurement σ in meters. Note that this result compares favorably with the cruder analytic result presented by expression (17).

An estimate of how well η could be estimated from existing ranging data, as discussed at the end of Section II, yields:

$$\sigma_\eta \approx (0.00055 \text{ -- } 0.009) \quad (\text{Mariner 9, Viking, Phobos}), \quad (21)$$

which is slightly smaller than the analytic result given in eq.(18).

While numerical integration is expected to be more accurate than analytic approximations, one might wish to gain more understanding of the planetary SEP effect by doing a realistic analytic calculation which improves on the "circular orbit" approximation of Section 11. It is natural to eliminate the largest sources of error of that approximation. Mars has an orbital eccentricity of .093, and there is no fundamental barrier to using elliptical reference orbits. The analytic calculation of Section 11 was redone with elliptical reference orbits for Earth and Mars, working to first order in the eccentricity. WC also used a more sensible way to include the eccentricity corrections, the method of variation of parameters (Robertson & Noonan 1968), and were able to solve the variation of parameters equations for the perturbed orbits of Earth and Mars to fourth order in the eccentricity. Figure 2 shows plots of the partial derivative of the Earth-Mars range with respect to the SRI acceleration. Comparison of the three curves shows that the eccentricity correction plays a more fairly significant role, than one might expect. One reason for this is that the eccentricity corrections turn out to include more "scalar" matrix elements which are proportional to the time t . Such elements dominate at large times, and the eccentricity corrections thereby qualitatively change the nature of the solution in the linear approximation.

4 Numerical Integration With the Tidal Term \vec{A}_{tid} .

Jupiter's mass μ_J needs treatment as an adjustable parameter to be fit with the ranging data. This is because the octopolar tide of Jupiter acting on the orbits of Earth and Mars produces polarizations similar to those produced by the SEP effect, but fortunately having a different Earth-Mars ratio and therefore separable from the desired SEP effect. If Jupiter's mass were uncertain by 4 parts in 10^8 , its tidal polarization of Mars' orbit would be uncertain by that would be produced by an $\eta \sim .001$, for example. But Jupiter's mass is only known to a part in a million, so we must include μ_J as a free parameter with its own partial.

In this Section we outline the most accurate calculation. The full equations (8) (and analogue for Mars) were numerically integrated including the tidal term \vec{A}_{tid} , previously was neglected in Sections II and III. To the parameters $r_{P_0}, r_{M_0}, p_{Er_0}, p_{Mr_0}, p_{E\theta_0}, p_{M\theta_0}, \theta_{P_0}, \theta_{M_0}, \mu_S$ and g_η , then, we add μ_J in the covariance analysis, otherwise, the analysis is identical to that of Section 11.

The results are shown in Figure 1, with the solid curve giving the result for σ_η from N ranging measurements, each with error σ meters. For a mission time of the order ten years we find

$$\sigma_\eta \sim .0039\sigma/\sqrt{N}. \quad (22)$$

The estimation of how well η can be determined from existing ranging data (as discussed in Section 11) is:

$$\sigma_\eta \approx (0.0012 \text{ -- } 0.02) \quad (\text{Mariner 9, Viking, Phobos}), \quad (23)$$

about double the result from Section 11.

The covariance analysis gives the expected formal error in μ_J as well, analogous to eq.(16), with the result shown in Fig. 3. For a mission time of order ten years we find $\sigma_\mu \sim 5.7\sigma/\sqrt{N}$ in $km^3 s^{-2}$ where

N , as before, is the number of daily ranging measurements taken during the mission. For $\sigma = 7.9m$, σ_{μ_J} falls below the present accuracy determined from Pioneer 10,11 and Voyager 1,2, namely, $\mu_J = 100 \text{ km}^3/\text{s}^2$ (Campbell & Symmott 1985) - within two years. This surprising result suggests that Earth-Mars ranging can provide an extremely accurate value for the mass of Jupiter, independent of the any determination of a SEP effect.

Following Section 11, in order to obtain the realistic estimates for σ_η and σ_{μ_J} one multiplies these results by $\sqrt{N}/3$, giving values of the upper bound of realistic errors for η and μ_J as $\sigma_\eta \sim .0012\sigma$ and $\sigma_{\mu_J} \sim 1.9\sigma \text{ km}^3/\text{s}^2$ correspondingly. With mission duration of order ten years this result gives the interval for the uncertainties σ_η and σ_{μ_J} during the future Mars missions:

$$\begin{aligned}\sigma_\eta &\approx (0.000) - - 0.0012) \sigma, \\ \sigma_{\mu_J} &\approx (0.09 - 1.9) \sigma \text{ km}^3/\text{s}^2.\end{aligned}\tag{24}$$

5 Discussion

The planet Mars has become an object of intensive investigation by many scientists around the world. The next flight opportunity during 1996-97 will mark the initiation of a number of new space missions to that planet from which we expect to obtain a rich set of data, including spacecraft tracking and planetary radar measurements, and allowing precise relativistic celestial mechanics experiments.

Anticipating these events, we have analyzed the ability for testing SEP violation with Earth-Mars ranging. The expected accuracy of the future ranging experiments would put significant constraints on theoretical models, including a possible inequality of the Sun's inertial and gravitational masses. Using analytic and numerical methods we have shown from covariance analysis that Earth-Mars ranging can provide a quality estimate of η . Indeed, for N ranging measurements with an accuracy of σ meters, the SEP parameter η according to covariance analysis can be determined within the accuracy $\sigma_\eta \approx 3.9 \times 10^{-3} \sigma / \sqrt{N}$. The "realistic" estimate for σ_η based on a "modified worse case" analysis sets a conservative limit on the accuracy and indicates that even in unfavorable cases the Sun-Earth-Mars-Jupiter system allows for a sensitive test of the Strong Equivalence Principle, qualitatively different from that provided by LLR. The mass of Jupiter, μ_J , can be determined more accurately from a few years of Earth-Mars ranging than from Pioneer 10,11 and Voyager 1,2 combined. This analysis shows a rich opportunity for obtaining new scientific results from the program of ranging measurements to Mars.

Efforts are under way at JPL to determine η from the Mariner 9, Viking and Phobos ranging data. This research will modify the theoretical model to include effects due to Saturn. We will perform the numerical experiments with combined data collected from the planetary missions, LLR and VLBI. And, as the data do not presently include any direct ranging to the Sun, it will be interesting to include ranging results to the spacecrafts of the joint US-Russian Solar probe missions scheduled for launch in the year 2001, or shortly thereafter. A preliminary determination from combined solar-system data, including Mars ranging and Mar-laser ranging, has been reported at a Division of Dynamical Astronomy meeting by Chandler et al. (1994). However, we have found that the inclusion of the \vec{A}_η acceleration of CC[.7) in the JPL planetary ephemerides system as postulated (Standish et al. 1993), has not been straightforward. The total SEP range signal in Earth-Mars ranging is so complex and unique, one should be very conservative about physical interpretation of the results obtained. The full scale research of this important experiment is currently underway at JPL. The results reported here provide insights into what is being measured, and hence they minimize the possibility of error in implementing the planetary SEP effect in the complicate software system. And by concentrating on covariance analysis, information needed for the planning of gravitational experiments on future Mars

missions is obtained. We intend that this paper serve as one of a collection of guides for scientific goals and priorities on Mars missions over the next decades.

Finally we mention that the analysis of solar ranging data might provide the opportunity for another fundamental test, namely a Solar system search for dark matter (Nordtvedt 1994; Braginsky 1994; Nordtvedt 1995). Suppose that dark matter weakly interacts with ordinary matter in a manner depending on the specific properties of the matter composing the bodies. The Sun, having an internal structure and matter composition which is considerably different from the rest of the inner bodies in the Solar system, might then have a different coupling to dark matter, and a corresponding anomalous cosmic force $\vec{F}_c = m_S A_c$ acting on the Sun, would produce extra terms in the heliocentric equations of motion for the planets - like A_η , but fixed in direction, in equation (8). Interesting limits on the size of any SEP violation for extended bodies in the Solar system falling toward dark matter may be obtainable. This research will be the subject of a subsequent publication.

We are indebted to our colleagues John Armstrong, Eunice Lau and Skip Newhall for many useful and stimulating conversations. MG acknowledges the partial support by an AWU-JPL sabbatical fellowship. KLN was supported in part by National Aeronautic and Space Administration through-out Contract NASW-4840. SGT was supported by National Research Council, Resident Research Associateship award. This work was carried out in part at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with National Aeronautic and Space Administration.

6 References

- Adelberger E. G. et al. 1994, Phys. Rev., D50, 3614
- Allen, C. W. 1985, Astrophys. Quantities (London: Athlone press)
- Anderson, J. D., Levy, G. S. & Renzetti, N. A. 1986, in: Relativity in Celestial Mechanics and Astrometry (IAU Symposium 114), eds. J. Kovalevsky & V. A. Brumberg (Reidel: Dordrecht), 329
- Ashby, N., Bender, P. L. & Wahr, J. M. 1995, private communication
- Braginsky, V. B. & Panov, V. I. 1972, SOv. Phys. JETP, 34, 463
- Braginsky, V. B. 1994, Class. Quantum Grav., 11, A1
- Bertotti, B. & Grishchuk L. P. 1990, Class. Quantum Grav., 7, 1733
- Campbell, J. K. & Symnott, S. P. 1985, AJ, 90, 364
- Chandler, J. P., Reasenberg, R. D. & Shapiro I. I. 1994, BAAS, Vol. 26, No.2, 1019
- Damour, P. & Schäfer G. 1991, Phys. Rev. Lett., 66, 2549
- Dickey, J. O., Newhall, X X & Williams, J. G. 1989, Advance in Space Research, 9, 75
- Dickey, J. O. et al. 1994, Science, 265, 482
- Orellana, R. B & Vucetich H. 1993, A&A, 273, 313.
- Nordtvedt, K., Jr. 1968a, Phys. Rev., 169, 1014
- _____. 1968b, Phys. Rev., 169, 1017
- _____. 1968c, Phys. Rev., 170, 1186
- _____. 1970, Icarus, 12, 91
- _____. 1973, Phys. Rev., D 7, 2347
- _____. 1978, in "A Close-Up of the Sun" (JPL Pub. 78 70), 58
- _____. 1994, ApJ, 437, 529
- _____. 1995, Icarus, 114, 51
- Robertson, H. P. & Noonan, T. W. 1968, Relativity and Cosmology, (Philadelphia: Saunders)

- Roll, P. G., Krotkov, R. & Dicke, R. H. 1964, *Ann. Phys. (N.Y.)*, 26, 442
- Singe, J. L. 1960, *Relativity: the General Theory* (Amsterdam: North-Holland)
- Shapiro, I. A. et al. 1976, *Phys. Rev. Lett.*, 36, 555
- Standish, E. M. et al. 1992, in: *Exp. Suppl. to the Astron. Almanac*, (Mill Valley: Univ. Sci. Books), 279
- Ulrich, R. K. 1982, *ApJ*, 258, 404
- Will, C.M. & Nordtvedt, K. 1972, *ApJ*, 177, 757
- Will, C.M. 1971, *ApJ*, 169, 125
- _____ 1993, *Theory and Experiment in Gravitational Physics* (Cambridge: Cambridge Univ. Press)
- Williams, J. G. et al. 1976, *Phys. Rev. Lett.*, 36, 551
- Williams, J. G., Newhall X X & Dickey, J. O. 1995, Submitted to *Phys. Rev., D*

7 Figure Legends

Fig. 1. Variation of the uncertainty in the SEJ' parameter η with N , assuming N Earth-Mars range measurements, each with uncertainty σ meters. The thin dashed curve is the result of the analytic approximations of Section 11. The thick dashed curve is the numerical integration result of Section 111. The solid curve comes from the numerical integration described in Section IV where the term A_{tid} was restored and μ_J was included in the covariance analysis.

Fig. 2. Investigation of first order eccentricity corrections (thick dashed curve) to the linear approximation of Section 11 (thin dashed curve). Both of these curves should be compared to the more accurate numerical results of Section 111 (solid curve). All three of these calculations used ml.(8) without the \vec{A}_{tid} term. The initial conditions taken were different from those in figures 1 and 3.

Fig. 3. Plot of the uncertainty in the mass of Jupiter versus N from the numerical integration of equation (8) plus covariance analysis. See Section IV.

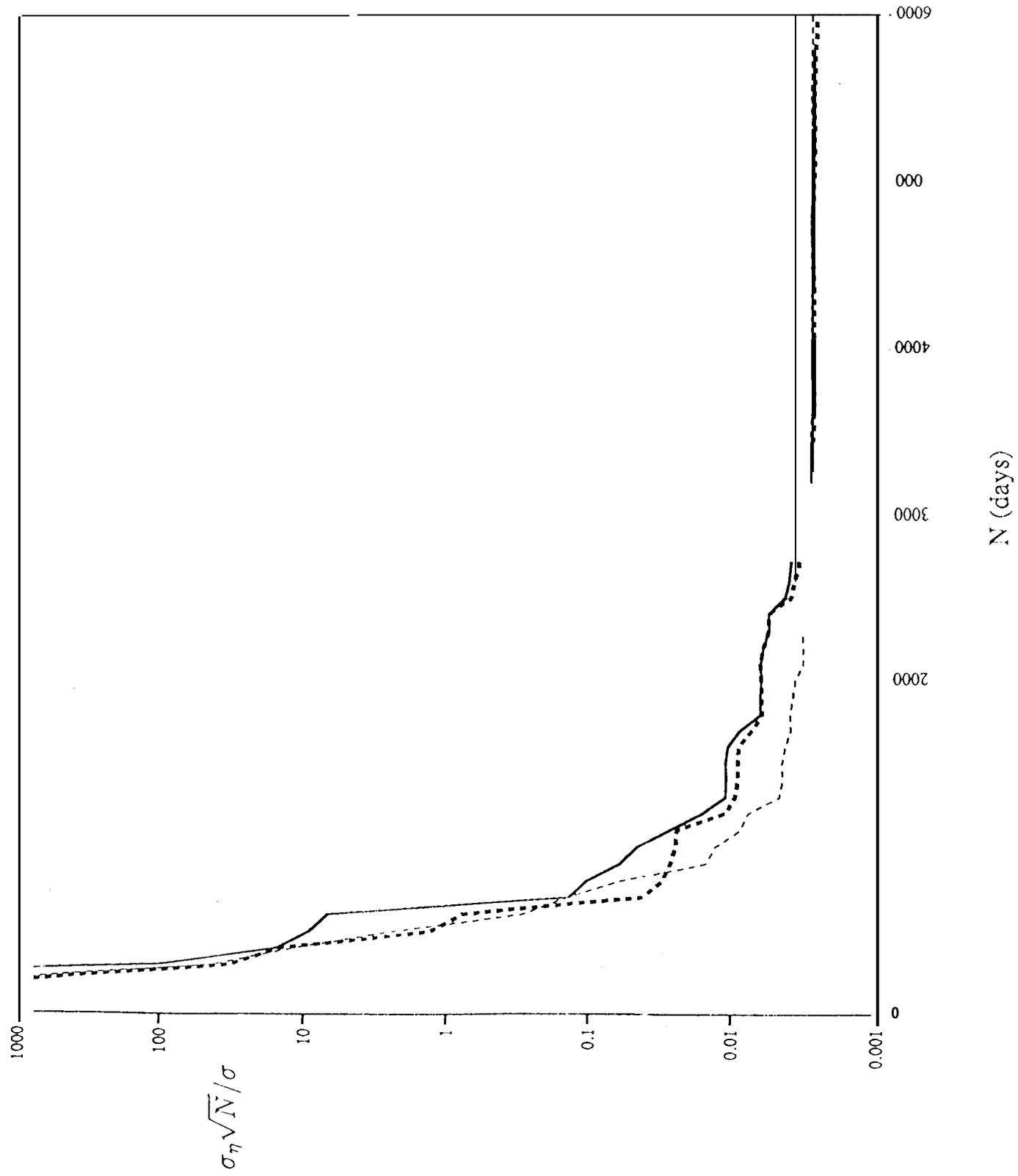


Figure 1

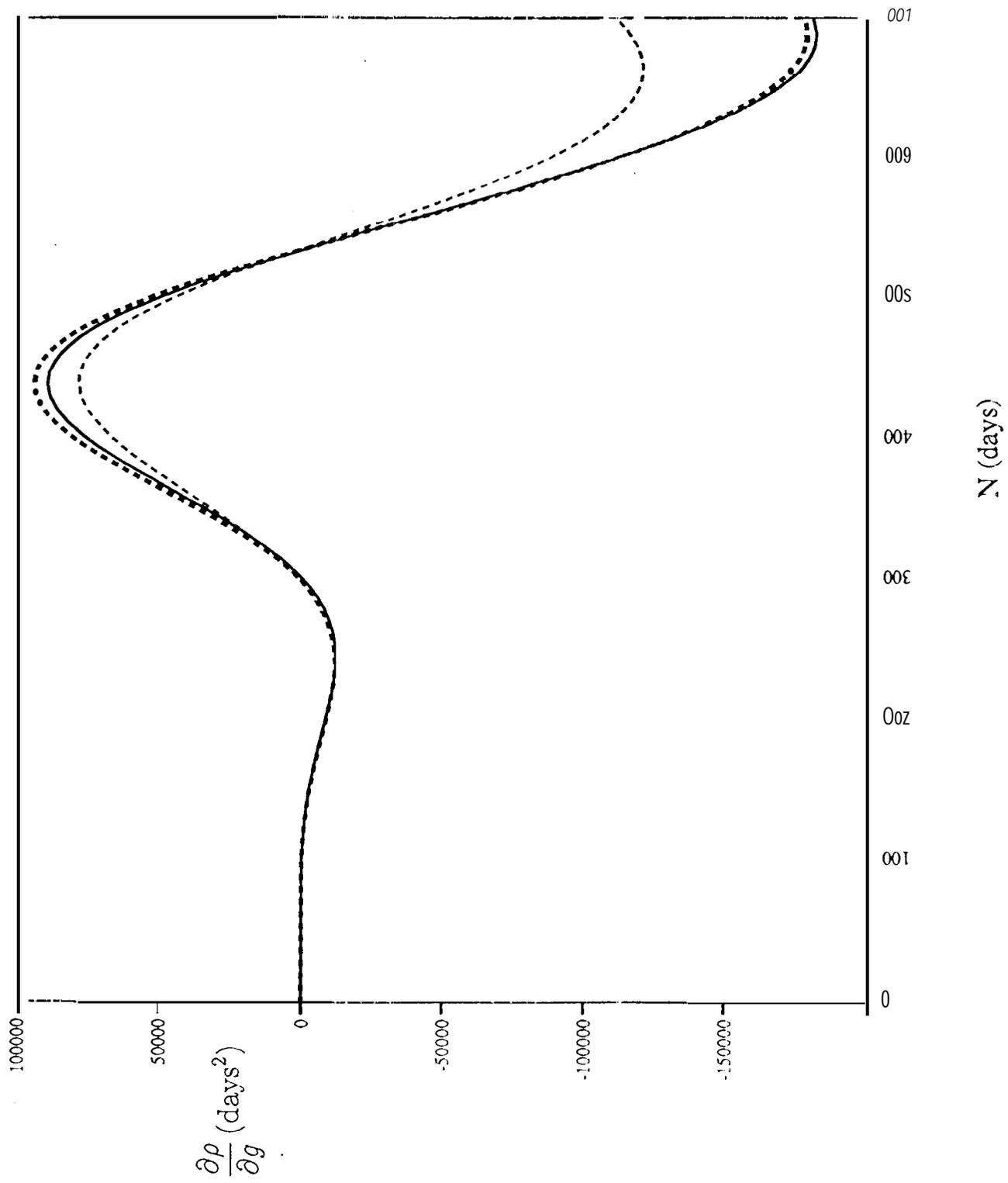


Figure 2

Figure 3

