where \( i \) is either \( TE \) or \( TM \) and

\[
Z_{TE} = \frac{j\omega\mu_0}{\gamma_{mn}} \quad Z_{TM} = \frac{\gamma_{mn}}{j\omega\varepsilon} \tag{8}
\]

The tangential fields are sufficient to define the normalized Floquet voltages [9] at a plane \( z \) as they represent power traveling in the \( \pm z \) direction. Note that in defining the \( TE \) and \( TM \) folds, such as shown in (5) and (6), they have been normalized to obtain an asymmetric scattering matrix for reciprocal structures,

\[
\int \int_{\text{unit cell}} \mathbf{E}_i^\pm \times \mathbf{H}_i^\pm \cdot (\pm \hat{z}) \, dS = 1, \quad i = TE \text{ or } TM \tag{9}
\]

Since the normalizing direction, \( \pm \hat{z} \), is dependent on whether the waves are forward or backward traveling. Therefore, the sign of either \( \mathbf{E}_{TE} \) or \( \mathbf{H}_{TE} \) must change dependent on the direction of propagation. As mentioned previously, it is convenient to choose the sign of the electric field components to remain the same, except for the necessary change on \( \gamma_{mn} \),

\[
\mathbf{E}_i^\pm = \overline{\mathbf{E}_i^-} \quad \text{and} \quad \mathbf{H}_i^\pm = -\overline{\mathbf{H}_i^-} \quad \text{for} \quad z = 0, \quad i = TE \text{ or } TM \tag{10}
\]

to guarantee that reflection from a pcc plate will give a reflection coefficient of \(-1\).

The total tangential electric field at a reference plane \( z = z' \) is then weighted by the appropriate coefficients of the normalized Floquet harmonics,

\[
\mathbf{E}_i(m, n) = C_i^+(m, n) \mathbf{E}_i^+(m, n) + C_i^-(m, n) \mathbf{E}_i^-(m, n) \quad i = TE \text{ or } TM \tag{11}
\]

where \( C_i^{+(m, n)} \mathbf{E}_i^+(m, n) \) is analogous to the forward traveling voltage on a transmission line. Likewise, the magnetic field will correspond to the forward traveling current, which is related to the electric field via (7). So that the scattering parameters can be defined at port \( i \) due to an incident Floquet harmonic at port \( j \),

\[
S_{ij} = \frac{C_i^{+(m, n)} \mathbf{E}_i^+(m, n)}{C_j^{+(m, n)} \mathbf{E}_j^+(m, n)} = \frac{C_i^{+(m, n)} e^{j\gamma_{mn} z}}{C_j^{+(m, n)} e^{j\gamma_{mn} z'}} \tag{12}
\]

where the reference plane is located at \( x = y = 0 \) and \( z = z' \).

### C. Power Waves at the Device Under Test (D.U. 7')

It is now necessary to define the scattering parameters at the device ports due to incident Floquet harmonics. For the planar geometry as shown in Fig. 1, the device port locations are not terminated in
transmission line with known characteristic impedance. Therefore it is necessary to choose generalized power waves as described by Kurokawa [12].

\[ a_i = \frac{V_i + Z_i I_i}{2\sqrt{|\text{Re}Z_i|}} \]  

\[ b_i = \frac{V_i - Z_i I_i}{2\sqrt{|\text{Re}Z_i|}} \]  

By convenience, the normalizing impedance \( Z_i \) at the \( i \)th port is chosen to be the input impedance. Thus the scattering parameters as given by (12) are determined with the device ports matched, \( V_i = -Z_i I_i \), so that \( a_i = 0 \).

The scattering parameters between the \( i \)th incident Floquet harmonics and \( k \)th device ports are then computed via

\[ S_{ki} = -I_k \sqrt{|\text{Re}Z_k|} \]  

And by reciprocity, the scattering parameters due to excitation at the device ports are found by using the symmetry of the resulting scattering matrix. The input reflection coefficient at the \( k \)th device location, \( S_{kk} \), is then computed by

\[ S_{kk} = \frac{Z_k - Z_k^*}{Z_k + Z_k^*} \]  

Thus equations (12), (15), and (16) define the generalized scattering matrix for the periodic array. Note that when the supporting superstrate and substrates materials are lossless, the computed generalized scattering matrix satisfies the usual conditions related to power conservation. In the next section waveguide simulation approach and measured results will be presented.

111. Waveguide Simulation

Fig. 3 shows the one-port dipole unit cell to be simulated in a waveguide simulator. The substrate is supported by Rogers RT/duroid 5880, \( \varepsilon_r = 2.2 \), that is 0.32 cm thick. To support two orthogonal modes for two-port device measurements, the unit cell was placed in square waveguide. The dimensions of one edge of the square unit cell is 4.74 cm, and the dipole has a length of 1.0 cm and width of 0.44 cm. The dipole is loaded with a chip resistor of length 0.1 cm, width 0.5 cm, and height of 0.04 cm.
Fig. 4 shows a simplified diagram of the one port device in the waveguide. In this simplified diagram, the generalized scattering matrix computed by the procedure outlined above results in a three port scattering matrix relating the field quantities at ports 1 and 3, to the power wave representation at the device port. Upon calibration at the waveguide ports, the computed generalized scattering matrix then forms the embedding network to which the chip resistor is attached. Since the scattering matrix for this embedding network is known, the scattering parameters of the chip resistor load can be determined from measurements at waveguide ports 1 and 2.

In the more general case which uses a square unit cell the measurement of the two orthogonal TE modes, $TE_{01}$ and $TE_{10}$, in the square waveguide are required. For measurement of these two orthogonal modes, orthomode junctions become part of the embedding network as shown in Fig. 5. Therefore, the generalized scattering matrix for the unit cell, $S^{num}$, is represented by a 5 port generalized scattering matrix for a one port device, and a 6 port generalized scattering matrix for a two port device. The composite scattering matrix resulting from the combination of $S^{num}$ with the measured scattering parameters of the orthomodes forms a 4 port embedding network. Measurement of the resulting 4 port scattering matrix then allowed for extraction of $S^{dat}$.

A. Test Setup Characterization

Addition of the orthomode junctions into the measurement setup requires measurement of the $S$ parameters of these 4 port devices. Determination of the $S$ parameters of the orthomodes is complicated by the need to place standards at the square waveguide port that match one of the TE modes, while reflecting the orthogonal mode with known magnitude and phase. This was accomplished by using a stepped transition in one dimension from 4.74 cm to 3.27 cm, and then from 3.27 cm to rectangular waveguide of 2.21 cm by 4.74 cm which only supports one dominant TE mode. A mode-matching solution was used to calculate the magnitude and phase of the stepped transition so that it could be used as a standard in determination of the $S$ parameters of the orthomode junctions.

In order for the relatively narrow band stepped transitions to exhibit greater than 20 dB return loss, and to avoid introduction of the higher order $TE_{11}$ and $TM_{11}$ modes, it was necessary to limit the measurement range from 4.0 GHz to 4.4 GHz. Note that the addition of the orthomodes creates a source of uncalibrated
error in the measurement system, which is calibrated in rectangular waveguide. Assessment of errors due to the presence of the orthomodes was done by connecting two "identical" orthomodes so that their square waveguide ports are together. The external ports presented then allow for a four port measurement in rectangular waveguide. 'J'bus the perfect "thru" junction between the two orthomodes can be de-embedded. This measurement indicated a maximum of 8% error in magnitude of one propagating mode, and 3% error in the magnitude of the orthogonal square waveguide mode with phase error less than 2.2 degrees for both modes.

The use of square waveguide for simulation of a periodic array requires the solution of the periodic array problem for the two Floquet modes composing the TE01 mode. These plane waves are incident at the angle $\theta$ which varies from 46° to 52° over the frequency range of measurement. This large angle of incidence results in additional reflection of the $m = -1$ Floquet harmonic, which must be properly combined with the two original exciting modes when forming $S^{num}$. In order to satisfy the boundary conditions of the square waveguide, a rot at ion of $\phi = 90$ degrees then computes the parameters for the $TE_{10}$ mode. Thus a 10 port representation of $S^{num}$ using Floquet modes is reduced to a 6 port scattering matrix for waveguide simulation.

### B. Results for a One Port D. U. T.

Fig. 6 shows the results of de-embedding the S parameters of the unit cell shown in Fig. 3 when the load is replaced by a conducting patch. The de-embedded S parameter shown is normalized to the antenna input impedance. The superstrate layer for this unit cell consists of an air layer of thickness 1 cm. The inclusion of the air superstrate layer allows the generalized scattering matrix for the array to be properly referenced to the interface of the support fixture for the unit cell, and the face of the orthomode junction. The support fixture used consisted of a square waveguide section of length 1.32 cm in which the unit cell was press fitted. This allowed for removal and replacement of the device under test.

Fig. 7 shows the de-embedded S parameter for the unit cell shown in Fig. 3 when it is loaded with a 51 $\Omega$ chip resistor. The theoretical curve is computed by using the computed input impedance $Z_{in}$, which is the normalizing impedance. The frequency dependence of these curves is due to the slight variation in the
input impedance which varies from \((164.8 - j \times 8.2) \Omega\) to \((178.8 + j \times 6.1) \Omega\) from 4.0 to 4.4 GHz. The maximum error in the measured magnitude of the device S parameter is 8.6%.

The load impedance magnitude and phase for the one port device can be calculated sinew then normalizing impedance, the input impedance at the device port \(Z_{in}\), is known.

\[
Z_{in} = \frac{S_{dat} Z_{in} - Z_{in}}{1 - S_{dat}}
\] (17)

The nonlinear nature of this conversion may increase or decrease the error in the measured S parameter \(S_{dat}\), but puts the results in a more readable form. Fig. 8 shows the measured load impedance magnitude and phase as computed from (17). The measured load magnitude is in error by approximately 8.4%. As expected the relative error when represented by (17) was minimal when the loading chip resistor was more closely matched to the input impedance presented at the device. Although it was not done here, incorporation of error correction terms into the measurement system would alleviate these problems.

Before moving to results for two port devices it is noted that measured transmitted and reflected parameters of the loaded unit cell, such as those depicted by \(S_{21}\) and \(S_{12}\) in Fig. 4, where also measured and compared to theoretical results. These 4 port parameters were, in general, of similar accuracy to the load de-embedding results presented. This provided another method of verifying a composite \(S_{num}\) formed from the combination of the Generalized scattering parameters for the periodic array and the S parameters of the device.

C. Results for a Two Port D. U. T.

Fig. 9 shows the two port unit cell used for waveguide simulation. The substrate and individual dipole dimensions are the same as the single dipole shown in Fig. 3. By separating orthogonal chip resistors by a thin layer of polyimide, each individual dipole was loaded independently with no connection to the other device. In this configuration equation (17) can be used to de-embed the load impedance values from the measured data. Fig. 10 shows the measured results when two 120 \(\Omega\) chip resistors are used. The average error on the vertical loaded port, port 5, is 4.4% while the average error on the horizontal port is 15.8%. This is more error than expected due to a worst case linear summation of the two polarizations of the orthomodes, 11%. The maximum phase angle measured was 24 degrees.
D. Results for a Differential Pair HEMT TwoPort D. U. T.

Since it is difficult to measure the differential pairs often used in grid amplifiers [3-5], waveguide simulation of a single unit cell provides a means of measuring these parameters. The unit cell configuration shown in Fig. 9 was used to measure a differential pair HEMT wire bonded in the coil configuration shown in Fig. 1. Upon measurement of the 4 port scattering matrix at the external rectangular waveguide ports of the two orthomode junctions, the 2 port S parameters of the differential HEMT pair were extracted for different bias levels.

The differential pair is in common source configuration with DC ground applied to the sources through a 50Ω resistor. The gates are self-biasing. Fig. 11 shows the de-embedded two port S parameters of the differential pair HEMT. The devices are biased with a drain bias of 5 V and 8 mA. To put the generalized scattering parameters in a more standard form, Fig. 11 shows scattering parameters normalized to 50 ohms. Note the large mis-match of the devices to both input and output ports, as represented by S11 and S22, for this normalizing impedance. Even with the mis-matches on input and output the necessary conditions for stability

$$|S_{11}| < 1, \ |S_{22}| < 1$$

are satisfied. Fig. 12 shows the scattering parameters normalized to 50 ohms when the drain bias becomes 0 V.

Fig. 13 shows the magnitude of the generalized scattering parameters as computed, where the normalizing impedances are the input impedances at the ports. This corresponds to the actual device loading presented by the grid during the waveguide measurement. The result is a more desirable match to the device at higher frequencies as shown. And as the input match improves, the transducer power gain, S21, increases until a maximum of 0.8 dB is reached at 4.4 GHz.

IV. CONCLUSIONS

A generalized scattering matrix for modeling of grid amplifiers and oscillator arrays has been developed and verified experimentally. The method incorporates a generalized scattering matrix representation for the periodic array, where Floquet harmonics represent ports of the array. These additional ports are combined
with power wave representations of at the device ports which are in the plane of the array. The advantages of this approach are that magnetic and electric wall assumptions are not needed. Also the Floquet mode representation allows for the combination of polarizers and other periodic structures commonly used in quasi-optical setups by cascading of scattering matrices [9], while allowing for non-normal incidence and inclusion of coupling effects. A single unit cell was measured in a waveguide simulator for verification of the approach. Results for loaded dipoles with one port and two port devices were presented. Also measured scattering parameters of a single unit cell of a grid amplifier using a differential pair 11 EMT were presented.

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References


14


Figure Titles

Fig. 1. Overview showing distinction between the generalized scattering matrix of the grid, $S^{num}$, and the differential pair device under test. Note that input and output are separated by orthogonal polarizations at normal incidence.

Fig. 2. General form of generalized scattering matrix containing $n$ Floquet modes in region 1 and region 2.

Fig. 3. Unit cell of one port device for waveguide simulation. The substrate is RT/duroid 5880 with $\epsilon_r = 2.2$ and a thickness of 0.32 cm (0.125"). The port is terminated with a chip resistor of length 0.1 cm, width 0.05 cm and height of 0.04 cm.

Fig. 4. Waveguide method of one port S parameter cie-embedding using a single unit cell. Note that $S_{31}$, $S_{21}$, ... are computed by numerical analysis.

Fig. 5. Waveguide method of two port S parameter de-embedding using a single unit cell and two orthomode junctions to verify $S^{num}$ and measure $S^{det}$.

Fig. 6. De-embedded $S^{det}$ parameters using the unit cell shown in Fig. 3 when the load is a conducting patch, (a) measured $S_{11}$ magnitude and (b) measured $S_{11}$ phase.

Fig. 7. De-embedded $S^{det}$ parameters using the unit cell shown in Fig. 3 when the load is a 51 ohm chip resistor, (a) measured $S_{11}$ magnitude and (b) measured $S_{11}$ phase.

Fig. 8. De-embedded load impedance of the unit cell shown in Fig. 3 when the load is a 120 ohm chip resistor, (a) measured load magnitude and (b) measured load phase.
Fig. 9. Unit cell of two-port device for waveguide simulation. The substrate is RT/duroid5880 with \( \varepsilon_r = 2.2 \) and a thickness of 0.32 cm (0.125"). The port is terminated with a differential pair HEMT designed and fabricated at JPL.

Fig. 10. De-embedded two port load impedances of the unit cell shown in Fig. 9 when the loads are two 120 ohm chip resistors, (a) measured load magnitude and (b) measured load phase.

Fig. 11. Magnitude of de-embedded device S parameters, normalized to 50 ohms, when the unit cell shown in Fig. 9 is loaded with a differential pair HEMT biased at 5 V, 8.0 mA.

Fig. 12. Magnitude of de-embedded device S parameters, normalized to 50 ohms, when the unit cell shown in Fig. 9 is loaded with a differential pair HEMT biased at 0 V.

Fig. 13. De-embedded device S parameters, normalized to input impedances, when the unit cell shown in Fig. 9 is loaded with a differential pair HEMT biased at 5 V, 8.0 mA.
Fig. 1. Overview showing distinction between the generalized scattering matrix.

TWO PORT DEVICE: $\Sigma$
Fig. 2. General form of generalized scattering matrix containing n+m Floquet modes in region 1 and region 2.

Nominal polarization vertical for normal incidence

Nominal polarization horizontal for normal incidence

Grid Array
Fig. 3. Unit cell of one port device for waveguide simulation. The substrate is RT/duroid 5880 with ε' = 2.2 and a thickness of 0.32 cm (0.125"), and a chip resistor of 0.04 cm. The width of the device is 0.1 cm, and the height is 0.05 cm.

Around the terminations: Nickel barrier/chrome
- Height of 0.016"
- Width of 0.020"
- Length of 0.039"

International Manufacturing Services, Inc. (IMS) chip resistors:
0.125" thick.
3.2 Ω EDC copper
1 oz. RT/duroid 880
Substrate:
Photo of one-port dipole JPL-22651-ac
unit cell. Note that $S_{31}, S_{21}, \ldots$ are computed by numerical analysis.

FIG. 4. Waveguide method of one port $S$ parameter de-embedding using a single

One port at D.U.T.
Fig. 6. Waveguide method of two port S parameter de-embedding using a single unit cell and two orthomode junctions to verify Snum and measure Sout.
(b) Measured S11 phase

The load is a conducting patch, (a) measured S11 magnitude and (b) phase. Figure 3 shows the unit cell shown in Fig. 3 when...
Fig. 7. De-embedded S parameters using the unit cell shown in Fig. 3 when the load is a 51 ohm chip resistor. (a) Measured $S_{11}$ magnitude and (b) measured $S_{11}$ phase.

Frequency (GHz)

Theory

Measured

Phase of $S_{11}$, degrees

Magnitude of $S_{11}$
Fig. 8. Detoembedded load impedance of the unit cell shown in Fig. 3 when the measured load phase load is a 120 ohm chip resistor; (a) measured load magnitude and (b)
Fig. 9. Unit cell of two port device for waveguide simulation. The substrate is RT/duroid® 5880 with \( \varepsilon_r = 2.2 \) and a thickness of 0.32 cm (0.125"). The port is terminated with a differential pair HEMT designed and fabricated at JPL.
Fig. 9 When the loads are two 120 ohm chip resistors, (a) measured load magnitude and (b) measured load phase.

Fig. 10 De-embedded two port S-parameters load impedances of the unit cell shown in (a).
Fig. 11. Magnitude of de-embedded device S parameters, normalized to 50 ohms, when the unit cell shown in Fig. 9 is loaded with a differential pair HEMT biased at 5 V, 0.0 mA.
pair HEMT biased at 0 V, when the unit cell shown in Fig. 9 is loaded with a differential.

**Fig. 12.** Magnitude of de-embedded device S parameters, normalized to 50

**Frequency (GHz)**

Frequency (GHz) versus magnitude.