LUNAR MOMENTS, TIDES, ORIENTATION, AND COORDINATE FRAMES

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Abstract. To determine the lunar moments of inertia (A<B<C) it is necessary to determine three quantities. (C–A)/B and (B–A)/C come from Lunar Laser Ranging (LLR) measurements of the lunar orientation, and spacecraft or lunar orbit perturbations provide J_2. Combining four published results gives a normalized polar moment of inertia $C/MR^2 = 0.3930 \pm 0.0008$. Solid-body tides displace the surface about 0.1 m, but can perturb the orbit of a Moon-orbiting spacecraft. The selenocentric coordinates of four lunar retroreflectors are accurately known and can serve as reference points. The orientation and orbit of the Moon are very well known for the time span of the LLR data.

1. Introduction

Analysis of the Lunar Laser Ranging (LLR) data provides information on a variety of phenomena including lunar science (Dickey et al. 1994). The analysis of 3-cm-quality data has given a viewpoint on certain aspects of lunar modeling and science. In this paper we briefly summarize information on the moment of inertia, and we discuss lunar solid-body tides, coordinate systems, orientation, and ephemerides.

2. Lunar Moments

The lunar mass and mean moment of inertia provide two fundamental constraints on any model of the moon's interior density profile. The relative accuracy of the former is quite high ($\sigma(Gm)/Gm < 10^{-6}$), but the latter is much less so. The lunar polar moment is C. When normalized as C/MR^2, where M is the lunar mass and R is its radius, it provides a convenient measure of uniformity. Values less than 0.4 imply an increasing density with depth.

The lunar principal moments of inertia are A<B<C. The Moon's three-dimensional rotation (physical libations), and hence the LLR data, are sensitive to the differences of the moments through the parameters $\beta = (C-A)/B$ and $\gamma = (B-A)/C$. A lunar-orbiting satellite is sensitive to the second-degree gravitational harmonics $J_2 = (2C-A-B)/2MR^2$ and $C_{22} = (B-A)/4MR^2$ which also involve moment differences. Of these seven parameters only three are independent. The relative accuracy of β and γ is an order of magnitude better than for the second-degree gravity harmonics. J_2, β, and γ make a good set of parameters for deriving the moments, in particular the normalized polar moment C/MR^2.

The entries in the following table are selected from published values for the normalized polar moment and the J_2 values (unnormalized) upon which they are based. In the table, most of the variation in the normalized moment comes from J_2, but some comes from older values of β and γ and different treatments of tides.

Table 1. Lunar Polar Moment and J_2

Study	C/MR^2	σ	J_2 10^{-6}	σ 10^{-6}
Blackshear and Gapcynski 1977	0.391	0.002	202.19	0.91
Ferrari et al. 1980	0.3905	0.0023	202.15	1.2
Konopliv et al. 1993	0.3939	0.0011	203.80	0.57
Dickey et al. 1994	0.3940	0.0019	204.0	1.0

Weighted means give $C/MR^2 = 0.3930\pm0.0008$ and $J_2 = (203.32\sim0.41) \times 10^{-6}$. A more extensive listing of earlier results is given in Ferrari et al. (1980). Using the mean moment $I = (A+B+C)/3$, then I/MR^2 is smaller than C/MR^2 by 0.00014.

The J_2 values for the first three entries in the table depend on the analysis of tracking data from lunar-orbiting spacecraft. Blackshear and Gapcynski (1977) used Explorers 35 and 49 to estimate zonal harmonics. Ferrari et al. (1980) combined selected undisturbed long arcs of high altitude Lunar Orbiter V Doppler data with LLR data to get a low-degree lunar gravity field (5x5 plus 6, 6 and 7, 7), but J_2 came only from the Doppler data. Konopliv et al. (1993) used extensive Doppler tracking of Lunar Orbiters I-V and Apollo 15 and 16 subsatellites to derive a 60x60 field. We have converted their forma] errors for J_2 to more realistic errors by multipl ying by 10 and the error for C/MR^2 has been calculated from that. Because the Moon keeps one face toward the Earth the harmonics C_{21}, S_{21}, and S_{22} arc near zero (Ferrari et al. 1980) and were used to determine this factor. Konopliv et al. also give $C/MR^2 = 0.3926\pm0.0020$ (our scaled error) based on C_{22} and a joint 0.3933itl.001 (their error). Dickey et al. (1994) is a pure LLR solution with β and y determined from the physical librations and J_2 from perturbations of the lunar orbit. From this solution comes $\beta = (631.72\pm0.15) \times 10^{-6}$ and $y = (227.88\pm0.02) \times 10^{-6}$. Part of the β and γ errors come from the third- and fourth-degree gravity field. Improvements can be anticipated.

Accurate tracking of future lunar-orbiting spacecraft would provide the opportunist y to improve the accuracy of the polar moment by making an accurate determination of the second-degree harmonics.

3. Tides on the Moon

The attraction of the Earth raises solid-body tides on the Moon. The Moon keeps one face toward the Earth, but there are still time-varying tides due to 0.1 radian variations in the apparent direction of the Earth (both latitude and longitude) and variation in the distance. The time-varying radial tidal displacement is about 0.1 m. Each of the three causes has a 0.1 m maximum effect at a different location on the Moon. The two dominant periods are 27,55 d (period of mean anomaly) and 27.21 d (period of node crossings). This is large enough to affect the LLR data and is accounted for in the analysis, but it is less of a concern for altimetry.

The tides also affect the gravity field. The following calculations use the lunar Love number $k_2 = 0.0302+0.0012$ from Dickey et al. (1994). At the surface of the Moon the radial acceleration varies about ±0.02 regal due to tides. The pattern is the same as for the radial displacements: two dominant periods and different terms maximizing at different locations. The tidal variation in J_2 is dominated by changes in distance with an anomalistic period of 27.55 d and has an amplitude of 0.02x 10--6 (the model static contribution is 0.1 1x10^{-6}). The 0.03x10^{-6} amplitude of variation in C_{21} has a 27.21 d period. C_{22} varies ±0.01x10^{-6} due to distance changes (model static contribution 0.055 x10-6). S_{22} varies by ±0.01x10^{-6} with a 27.55 d period. The model static contributions use the above Love number for convenience, but real static tides, which are unobservable, depend on a much larger Love number. How the model static contribution is defined would affect more accurate values of J_2 and C/MR^2. Scc Ferrari et al. (1980) for onc treatment.

An orbiter determination of the Love number k_2 would be welcome since the LLR-determined value may be influenced by core flattening (Dickey et al. 1994).

4. Reflector Coordinates

LLR has determined accurate selenocentric locations for four retroreflectors. Differential VLB1 on the ALSEP radio signals has given baselines between the ALSEP locations as well (King et al. 1976). Known differences between the LLR retroreflectors and ALSEPs allows the baselines to be put on a selenocentric coordinate system. Several of these sites have been located on photographs and used as reference points for a lunar control network (Davies, Colvin, and Meyer 1987).

Torques from harmonics above second degree. do not average to zero because of the Moon's synchronous rotation. Consequently, the average direction of the principal axis associated with A is somewhat displaced from the mean direction to the Earth (x axis), and the principal axis associated with C is displaced from the mean rotation axis z. As the harmonics change with each new solution, coordinates with respect to the principal axes change much more than those with respect to the mean Earth/rotation axes. The numerically integrated physical libations usc principal axes and the constant rotations to mean axes arc not explicit. Analytical solutions (Eckhardt 1981) give the rotations and can bc corrected for changes in the third-degree harmonics. Fourth-degree corrections arc a concern. Here wc adopt consistency with the frame of a previously published set of coordinates by fitting the rotations to the mean Earth/rotation axes coordinates of Williams, Newhall, and Dickey (1987). A shift in the x-axis values has been allowed for, since the X coordinates trade off with the mean distance as GM(Earth+Moon) is refined. The tabulated selenocentric coordinates in the mean Earth/rotation axes frame are from the solution which poduced the lunar science results in Dickey et al. (1994). The three rotations (in arcseconds) from principal (P) to mean (M) axes are $M = R_1(-0.15'') R_2(-79.12'') R_3(-66.48'') P$.

Table 2. Reflector Coordinates Using Mean Earth/Rotation Axes

Reflector	X	Y m	Z m	R m	E Long	Lat
Apollo 11	1591? 49.20	691218.22	20395.7'/	1735472.31	23.472930	0.673372
Apollo 14	1652816.84	-520458.83	-110362.66	1736335.45	-17.478799	-3.644215
Apollo 15	1554938.27	98601.50	764410.65	1735476.57	3.628373	26.133332
Lunakhod 2	1339390.87	802307.78	755846.92	1734638.36	30.922007	25.832229

The internal accuracy is high, but the rotations to the mean axes may bc in error by several meters. The uncertainty of the lunar center of mass in the x direction is 0.8 meter duc to GM(Earth+Moon).

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5. Lunar Ephemeris and Physical Librations

The lunar ephemeris and physical libations are simultaneously integrated, and the starting conditions for the numerical integration comes from a fit to the LLR data. The planetary data arc also fit and integrated at the same time. A lunar ephemeris and its associated physical libations arc internally consistent in orientation. The physical libations (three Euler angles) are computed for the principal axis frame.

If there arc range data to lunar-orbiting spacecraft, then a high-quality geocentric lunar ephemeris is necessary. The lunar orbit is highly perturbed (thousands of kilometers), but distance variations are known well due to the 3-cm-quality LLR data. The major range uncertainty is a scale factor, duc to uncertainty in GM(Earth+Moon), which corresponds to a mean distance uncertainty of 0.8 m. GM(Earth+Moon) and the reflector X coordinates arc difficult to separate.

High-quality lunar ephemerides and physical libations can be generated for the time span of observations. The errors of extrapolated ephemerides and librations will increase nonlinearly outside of the observation span clue to tidal friction and 18.6 yr (node precession period) effects for the orbit and multi-decade 'period effects for the libations.

6. Conclusions

Accurate tracking of lunar-orbiting spacecraft would provide an opportunity to improve the accuracy of the lunar moments of inertia and make an independent determination of the Love number k_2. The four LLR retroreflectors can serve as reference points for a selenocentric coordinate frame. The accuracy of the lunar orientation and orbit is adequate for future missions so long as LLR data is acquired and analyzed.

Acknowledgments

Wc acknowledge and thank the staffs of the CERGA, Haleakala, and University of Texas McDonald observatories, and the LLR associates. The research described in this paper was carried out at the Jet Propulsion Laboratory of the California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

References

Blackshear, W. T., and J. P. Gapcynski, An improved value of the lunar moment of inertia, J. Geophys. Res. 82, 1699-1701, 1977.

Davies, M. E., T. R. Colvin, and D. L. Meyer, A unified lunar control network: the near side, J. Geophys. Res. 92, 14177-14184, 1987,

Dickey, J. O. et al., Lunar laser ranging: a continuing legacy of the Apollo program, Science 265,482-490, 1994.

Eckhardt, D. H., Theory of the libration of the Moon, The Moon and Planets 25, 3-49, 1981.

Ferrari, A. J., W. S. Sinclair, W. L. Sjogren, J. G. Williams, C. F. Yoder, Geophysical parameters of the Ear(h-Moon system, J. Geophys. Res. 85, 3939-3951, 1980.

King, R. W., C. C. Counselman 111, and 1. I. Shapiro, Lunar dynamics and selenodesy: results from analysis of VLBI and laser data, J. Geophys. Res. 81, 6251-6256, 1976.

Konopliv, A. S., W. L. Sjogren, R. N. Wimberly, R. A., Cook, A. Vijayaraghavan, in Proceedings of AAS/AIAA Astrodynamics Specialist Conference, published for AAS by Univelt, 1275-1294, 1993.

Williams, J. G., X X Newhall, and J. O. Dickey, Lunar gravitational and reflector coordinates, in Figure and Dynamics of the Earth, Moon, and Planets, Prague, pp. 643-648, 1987.