BALANCED ACTUATOR AND SENSOR PLACEMENT FOR FLEXIBLE STRUCTURES

W. Gawronski*
Jet Propulsion Laboratory, California Institute of Technology
Pasadena, CA 9.I1O9

K.B. Lim†
NASA Langley Research Center
Hampton, VA 23665

Abstract

Using properties of multivariable flexible structures, it is shown that the Hankel singular values can be approximately decomposed to the suIn of Hankel sigular values of individual sensor and actuator combination. This decomposition allows one to evaluate each actuator and sensor in terms of the joint controllability and observability. For multimodal systems, placement metrics such as the trace or determinant of Hankel singular value matrix can be formulated and solved. It is shown that for the special case where the trace of the II ankel singular value matrix is used as the placement metric, the actuator placement problem becomes trivial. Several examples arc given to demonstrate the proposed method.

1 Introduction

For the purpose of improving the performance of flexible structure identification and control, it is sometimes useful to investigate various can di elate sensor and/or actuator locations. The freedom to choose their locations is not always given or is limited but if it is allowed, two problems may surface. The first onc is, given its type, determine the sensor and actuator minimal number and placements to meet specified controllability and observability requirements [I]. Second, given a large candidate set of sensor and actuators, a minimal subset is sought which has controllability/observability properties close to the original set. The latter problem is investigated in this paper.

* Technical Staff, Ground Ant ennas & Facilities Engineering Section, MS 144-201, wodek@gdansk.jpl.nasa.gov, Member AIAA

†Research Engineer, Guidance & Control Branch, FDCD, MS 1 61, k.b.lim@larc.nasa.gov

The importance of actuator and sensor placement problems is underscored in many investigations and contributions, only part of which wc have included in our references (see, for example, [2]-[20]). The approach proposed in this paper is an extension and complementary to the earlier results reported in [1]]. The previous approach considered the actuator and sensor placement problems independently via modal grammians in physical (modal) coordinates, while the new approach in this paper allows both independent or simultaneous analysis of actuator and sensor placement via Hankel singular values (l ISV). The Hankel singular values arc then approximately decomposed in terms of individual sensors and actuators for a multivariable flexible structure. Both approaches arc based on the approximate invariance of principal controllability and observability directions for flexible structures.

The approach proposed in this paper is based on an approximation of the grammian matrix and some justification is in order. This approximation for flexible structures allows onc to develop tools that would otherwise be not available. It allows onc to readily and perhaps more importantly, intuitively analyze and design optimal actuator and sensor configurations. It is believed that the reality of our physical world is such that nothing precisely satisfies assumptions associated with mathematical models of a complex dynamical physical process. The approximation for flexible structures used in this study dots not automatic. ally render this tool useless nor unreliable but rather makes it applicable.

2 Flexible Structure

For the purposes of this paper, a flexible structure shall be defined as a time invariant, finite-dimensional, controllable, and observable linear system with small damping and complex conjugate poles. Although this definition narrows the class of linear systems under consideration many interesting properties of structures and their controllers will be derived. A flexible structure is typically represented by the second-order matrix differential equation

$$M\ddot{\xi} + D\dot{\xi} + K\xi = Eu, \; y = F\xi + G\dot{\xi} \tag{1}$$

In this equation ξ is the N_2 x 1 displacement vector, u is the p x 1 input vector, y is the g ×1 output vector, the N_2 x N_2 mass, damping and stiffness matrices are denoted by $M > 0$, $D \geq 0$, and $K \geq 0$ respectively, the input matrix E is N_2 x p, the output displacement matrix F is g x N_2, and the output velocity matrix G is q x N_2.

In finite element models of flexible strucutres, the number of degrees of freedom, N_2, is often unaccept ably high and therefore an order reduction by modal truncation is typically done. The reduced order structure model is given in structural modal coordinates, Ψ, of dimension N_2 x n_2. Define the reduced modal displacement vector, η, of dimension $n_2 \times 1$, where

$$\xi \cong \Psi\eta \tag{2}$$

For flexible structures, typically

$$n_2 \ll N_2 \tag{3}$$

The columns of Ψ denote structural mode shapes obtained from the structural eigenvalue problem. The structural mode shapes have the property that diagonalize the mass and stiffness matrices to produce diagonal modal mass and stiffness matrices, where $M_m = \Psi^T M \Psi = 1$, $K_m = \Psi^T K \Psi = diag(\omega^2)$. If the damping matrix can be diagonalized, i.e., such that $D_m = \Psi^T D \Psi = diag(2\zeta\omega)$, it is called a matrix of proportional damping. The proportionality of damping is commonly assumed for analytical convenience since the nature of damping is not exactly known, and thus its values are typically approximated. The reduced modal equations in second order form can be written as

$$\begin{aligned} M_m\ddot{\eta} + D_m\dot{\eta} + K_m\eta &= \Psi^T Eu \\ y &= F\Psi\eta + G\Psi\dot{\eta} \end{aligned} \tag{4}$$

Let the triple (A, B, C) denote a modal state-space representation of the flexible structure. Following earlier definitions [32, 25, 1 1], define the modal state vector, x, of dimension n x 1, where $n = 2n_2$, such that

$$x = \begin{pmatrix} \dot{\eta}_1 & \omega_1\eta_1 & \ldots & \dot{\eta}_{n_2} & \omega_{n_2}\eta_{n_2} \end{pmatrix}^T \tag{5}$$

The triple for the corresponding modal state equations take the form $A = diag(A_1, \ldots, A_{n_2})$, $B = (B_{1*}^T, \cdots, B_{n_2*}^T)^T$ and $C = (C_{*1}, \ldots, C_{*n_2})$, with

$$A_i = \begin{bmatrix} -2\zeta_i\omega_i & -\omega_i \\ \omega_i & 0 \end{bmatrix}, \; B_{i*} = \begin{bmatrix} b_i \\ 0 \end{bmatrix}, \; C_{*i} = \begin{bmatrix} c_{ri} & \frac{1}{\omega_i}c_{di} \end{bmatrix} \tag{6}$$

where $i = 1, \ldots, n_2$, $b_i = \psi_i^T E$, $c_{di} = F\psi_i$ and $c_{ri} = G\psi_i$. Notice that for small damping ratio the above choice of the state vector gives approximately normal state matrix and hence approximately orthogonal eigenvectors. For flexible structures with distinct natural frequencies, the steady-state controllability and observability grammians asymptotically (as $\zeta \to 0$) approach 2-by- 2 block diagonal matrices and are given in [32, 25]

$$W_{c_{ii}} \cong \frac{\beta_i^2}{4\zeta_i\omega_i} I_2 \tag{7}$$

$$W_{o_{ii}} \cong \frac{\theta_i^2}{4\zeta_i\omega_i^3} I_2 \tag{8}$$

where $i = 1, \ldots, n_2$, denotes the range of indices of all structural modes to be controlled whose set is denoted by

$$S \equiv \{1, \cdots, n_2\} \tag{9}$$

In the above equation, $\theta_{di}^2 = c_{di}^T c_{di}$, $\theta_{ri}^2 = c_{ri}^T c_{ri}$, $\theta_i^2 = \theta_{di}^2 + \omega_i^2\theta_{ri}^2$, $\beta_i^2 = b_i b_i^T$, and I_2 is a 2 x 2 identity matrix. Note from Eq. (5) that x consists of modal velocities and frequency weighted modal displacements. Accordingly, this modal state is considered to be a physical coordinate because of its direct physical link to tile structural mode shape.

The above approximate grammian relationships indicate that high frequency modes with larger damping are among the least controllable and observable. The terms associated with each mode, β_i, θ_{ri}, and θ_{di} are called modal grammian coefficients (MGC) [1 1] due to their physical significance, i.e., they link principal directions directly to structural modes. It should be noted that in the context of actuator placement problem, relative actuator contributions (in terms of singular values of grammians) to a particular mode depends only on the MGC since the frequency and damping effect on the singular values are the same for all actuators for this mode. Therefore, for the class of problems where a set of modes to be controlled are given a priori based on physical grounds, it is only necessary to analyze M G C. For other classes of problems where only the most control lable and observable modes are to be controlled (for example, when actuator and sensor limitations are severe), the frequency and damping weights in the grammians given by Eqs.(7,8) must be considered, as in the balanced approach proposed in this study.

3 Balanced Flexible Structure

The controllability (W'.) and observability (W_o) grammians arc convenient forms of characterzing a system's controllability and observability. They arc obtained from solutions of the following Lyapunov equations

$$AW_c + W_c A^T + BB^T = 0, \; A^T W_o + W_o A + C^T C = 0 \quad (10)$$

For stable A and minimal (A, B, C), the solutions are positive definite and the geometric interpretations are well understood (scc [11]). The system triple is balanced if its controllability and observability grammians are equal and diagonal [30]

$$W_c = W_o = \Gamma^2 \tag{11}$$

where $\Gamma^2 > 0$ is the diagonal matrix of Hankel singular values of the system.

The controllability and observability properties of flexible structures arc analyzed in [22, 24, 25, 26, 27, 19, 32, 33]. These properties for flexible structures in the context of actuator and sensor placement are investigated in [1 1]. Two important properties of a flexible structure arc extensively used in this paper. First, states of a flexible structure in modal coordinates are almost orthogonal, and second, the state matrix A in the balanced coordinates is diagonally dominant. The terms, "almost orthogonal" and "diagonally dominant," arc alternative expressions for an approximate equality in the sense of small approximation error as measured by a spectral norm.

Assuming small damping, such that $\zeta \ll 1 (\zeta = \max(\zeta_i), i = 1, \ldots n_2)$, the balanced and modal representations of flexible structures of the forms given in Eqs.(5,6), arc closely related, as it is expressed in the following properties:

Property 1: In modal coordinates controllability arid observability grammians arc diagonally dominant,

$$W_c \cong diag(w_{c1} I_2, \ldots, w_{cn_2} I_2) \quad (12)$$
$$W_o \cong diag(w_{o1} I_2, \ldots, w_{on_2} I_2) \quad (13)$$

Sine.c the eigenvalues of the product arc the Hankel singular values

$$\Gamma^4 = diag(\lambda_1(W_c W_o), \ldots, \lambda_n(W_c W_o)) \cong W_c W_o \tag{14}$$

Property 2: By resealing the modal representation (A, B, C) one obtains almost balanced representation. Let (A_b, B_b, C_b) be the balanced representation, then

$$(A_b, B_b, C_b) \cong (A, R^{-1} B, CR) \tag{15}$$

where

$$R = diag(\frac{w_{c1}}{w_{o1}} I_2, \ldots \frac{w_{cn_2}}{w_{on_2}} I_2)^{1/4} \tag{16}$$

Note that the diagonal transformation leaves A unchanged and it approximates the exactly balanced A_b. The rows of B and columns of C which are associated with each mode arc individually scaled by the transformation.

Property 3: Denote $\alpha_i = -4\gamma_i \zeta_i \omega_i$ where the approximate ith Hankel singular value for flexible structure is given by

$$\gamma_i^2 = \frac{\theta_i \beta_i}{4\zeta_i \omega_i^2} \quad i \in S \tag{17}$$

For balanced representation one obtains

$$B_b B_b^T \cong C_b^T C_b \cong -\Gamma(A + A^T)$$
$$\cong diag(\alpha_1, 0, \ldots, \alpha_{n_2}, 0) \tag{18}$$

which, for the ith block, it translates to

$$B_{bi} B_{bi}^T \cong C_{bi}^T C_{bi} \cong -\gamma_i (A_i + A_i^T) = diag(\alpha_i, 0) \tag{19}$$

where B_{bi}, is the two-row block of B_b, and C_{bi} is the two-column block of C_b.

The first property follows from the definition of Hankel singular values and the well known diagonal dominance of grammians in modal coordinates. It gives an approximate but explicit formula for HSV of flexible structures. The second property follows by noting that the grammians in modal coordinates are already diagonally dominant so that only a scaling via a diagonal state transformation is required to attain grammians that arc equal and diagonally dominant. The third property follows directly from the diagonal dominance of the matrix A in the balanced coordinates. All the above properties arc corollaries of earlier results reported in [25, 32, 26].

Diagonal dominance of the grammians in modal coordinates imply that the principal directions for controllability, observability, and balanced principal direct ions arc approximately the same as eigenvectors, hence "modal grammians" as discussed in [1 I]. This dominance leads to particularly useful simplifications for flexible strn ctures, and in the context of actuator and sensor placement, the following points arc noted:

- individual structural modes can be associated with principal directions of controllability and observability and balanced principal vectors.

- Different sets of actuators and sensors give approximately the same principal directions; only principal values are significantly affected by the choice of actuators and sensors.

Although the balanced and modal coordinates almost coincide for very lightly damped flexible structure, the important difference between them lies in their scaling. It is well known that the modal coordinates are not unique, since they depend on the scaling of the natural modes, which is arbitrary. The scaling of the balanced coordinates is unique, it is such that the condition in Eq.18 holds. Physically, this means that each modal state is scaled such that its controllability and observability becomes equal while maintaining the eigenvector shape and the principal direction.

4 Actuator/Sensor Placement

The actuator and sensor placement methodology proposed in this paper is based on the balanced representation of flexible structures. The Hankel singular values are used to construct various forms of metric that quantifies the degree of controllability and observability for a given set of sensor and actuator configuration in balanced coordinates. Although the use of HSV to analyze the degree of controllability and observability of a linear system is well established, especially in model reduction applications [30, 26], the approximate decomposition of the squares of the HSV with multiple sensors and actuators in terms of the sum of squares of HSV of all combinations of sensor and actuator pair, is new. This result significantly simplify the design problem of selecting the most effective set of sensors and actuators for flexible structures.

4.1 Decomposition of HSV

Hankel singular values quantify the joint controllability and observability properties of a system, thus they can serve as a metric for sensor and actuator locations. Although it is known that adding sensors and actuators will in general increase control lability and observability in all principal directions [11] and hence all HSV, an explicit relationship between the sensor and actuator locations and its HSV has never been derived due to its complexity. The earlier results, as summarized in Properties 1 to 3, give explicit approximation of HSV in terms of modes and the contribution from individual actuator and sensor have not been explored. In this section, an approximate but explicit relationship in terms of HSV of individual actuator and sensor is given for flexible structures.

Consider the placement of p actuators and q sensors. In this case the input B and the output C matrices consist of p columns and q rows, respectively

$$B = [B_1, \ldots, B_p], \quad C^T = [C_1^T, \ldots, C_q^T] \qquad (20)$$

Notice that previously in Eqs. (6) and (18), the input and output matrices, B and C, are decomposed in terms of modes, i.e., $B_{1*}, \ldots, B_{n_2*}$, and $C_{*1}, \ldots, C_{*n_2}$. However, in the following derivation, the decomposition is in terms of actuators and sensors (see Eq.(20)). These matrices can be decomposed as

$$BB^T = \sum_{i=1}^{p} B_i B_i^T, \quad C^T C = \sum_{j=1}^{q} C_j^T C_j \qquad (21)$$

so that the controllability and observability grammians are a sum of grammians for each individual sensor and actuator

$$W_c = \sum_{i=1}^{p} W_{ci}, \quad W_o = \sum_{j=1}^{q} W_{oj} \qquad (22)$$

where W_{ci} and W_{oj} denotes the $n \times n$ controllability and observability grammians for the ith input and jth output, respectively.

For a multivariable linear system, the HSV are defined by the eigenvalues of the product of the grammians and by using Eq.(22)

$$\begin{aligned} \gamma_k^4 &= \lambda_k(W_c W_o) \\ &= \lambda_k\left(\sum_{i=1}^{p}\sum_{j=1}^{q} W_{ci} W_{oj}\right) \end{aligned} \qquad (23)$$

where $k = 1, \ldots, n$ and $\lambda_k(\cdot)$ denotes the kth eigenvalue. Except for the special class of low order systems of four or less, this equation cannot be explicitly solved in general.

Fortunately, for lightly-damped multivariable flexible structures, its balanced representation is almost independent of its actuator and sensor locations and the grammians are diagonally dominant (see Properties 1 to 3) so that

$$W_{ci} W_{oj} \cong \tilde{\Gamma}^4(i,j) \cong \Gamma^4(i,j) \qquad (24)$$

where

$$\tilde{\Gamma}(i,j) = diag(\tilde{\gamma}_1(i,j) I_2, \ldots, \tilde{\gamma}_{n_2}(i,j) I_2) \qquad (25)$$

$$\Gamma(i,j) = diag(\gamma_1(i,j), \ldots, \gamma_n(i,j)) \qquad (26)$$

denotes a matrix of approximate and exact HSV for the ith actuator and jth sensor pair respectively. Henceforth, the tilde symbol will denote the approximate value of the variable. Substituting the diagonal dominance of Eq. 24 in 23, we arrive at

$$\gamma_k^4 \cong \sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{\gamma}_k^4(i,j), \quad k \in S \tag{27}$$

or

$$\Gamma^4 \cong \sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{\Gamma}^4(i,j) \equiv \tilde{\Gamma}^4 \tag{28}$$

where

$$\tilde{\Gamma} = (5'112, \ldots, \tilde{\gamma}_{n_2} I_2) \tag{29}$$

Note that the n approximate HSV in Eq. 29 occur in pairs (see properties 1 to 3) so that there will only be n_2 distinct approximate HSV in general. The following important distinction is also emphasized: Γ and $\tilde{\Gamma}$ refers to the HSV for a set of actuators and sensors while $\tilde{\Gamma}(i,j)$ and $\Gamma(i,j)$ refers to the HSV for the ith actuator and jth output.

The importance of Eq.(27) lies in the decomposition of the HSV of the multivariable flexible structure in terms of the sum of approximate HSV of each actuator and sensor pair. observe from Eqs. 23 and 24 that the HSV for a multivariable flexible structure cannot be decomposed in terms of the exact HSV, $\Gamma(i,j)$, of its individual actuators and sensors. Nevertheless, in the context of actuator and sensor placement problem, if HSV are used to construct a placement metric, the contribution of each sensor and actuator pair appears in a very convenient form (cf. Eq.23 and Eq.27). In additon, the kth balanced mode for the ith actuator and jth sensor pair can be independently evaluated and is denoted by $\tilde{\gamma}_k(i,j)$.

4.2 Placement Indices

Three separate problems can be distinguished: actuator placement only, sensor placement only, and joint actuator and sensor placement. The decomposition of HSV applies to all three classes of problems as long as flexible structures are considered. Hence, the problem is formulated only for actuator placement where a set of sensors are assumed fixed. For simplicity, the dependence of HSV on the fixed set of sensors will not be stated explicitly for the following actuator placement formulation.

Let N denote a candidate set of actuator locations. In this case, Eq. (27) simplifies for the kth balanced mode to

$$\gamma_k^4(N) \cong \sum_{i \in N} \tilde{\gamma}_k^4(i), \quad k \in S \tag{30}$$

or

$$\Gamma^4(N) \cong \sum_{i \in N} \tilde{\Gamma}^4(i) \tag{31}$$

where

$$\tilde{\Gamma}(i) = diag(\tilde{\gamma}_1(i) I_2, \ldots, \tilde{\gamma}_{n_2}(i) I_2) \tag{32}$$

is the matrix of approximate HSV for the ith actuator location and

$$\Gamma(N) = diag(\gamma_1(N), \ldots, \gamma_n(N)) \tag{33}$$

is the matrix of HSV for the set of actuators, N

Following [11], it is assumed that typically a designer does not know exactly how many actuators to use and where to locate them. However, a larger but redundant set of candidate locations, denoted here by N, is usually known. The scalar $\tilde{\gamma}^4(N)$ denotes the maximum achievable joint controllability and observability corresponding to all actuator candidate locations, and it is a sum of $\tilde{\gamma}_k^4(i)$ over all candidate actuators and all modes

$$\tilde{\gamma}^4(N) = \sum_{k \in S}\sum_{i \in N} \tilde{\gamma}_k^4(i) \tag{34}$$

In order to derive the placement strategy, a normalized index $\hat{\gamma}_k^4(i)$ of the ith actuator for the kth mode is defined. It is a ratio of the fourth power of HSV of the ith actuator for the kth mode, $\tilde{\gamma}_k^4$ (i), over the fourth power of HSV for the whole set

$$\hat{\gamma}_k^4(i) = \frac{\tilde{\gamma}_k^4(i)}{\tilde{\gamma}^4(N)} \leq 1 \tag{35}$$

Definition 1: Define a subset of actuators, N_a, so that $N_a \subseteq$ N. The joint controllability and observability of the kth mode for the actuator set, N_a, are characterized by the kth modal index, $\rho_m(k)$.

$$\rho_m(k) = \sum_{i \in N_a} \hat{\gamma}_k^4(i) \leq 1, \quad k \in S \tag{36}$$

Definition 2: The joint controllability and observability of the ith actuator are characterized by the ith actuator index, $\rho_a(i)$.

$$\rho_a(i) = \sum_{k \in S} \hat{\gamma}_k^4(i) \leq 1, \quad i \in N_a \tag{37}$$

The actuator index, ρ_a (i), is a non-negative contribution of the ith actuator summed over all modes. The modal index, $\rho_m(k)$, is a non-negative contribution of the kth mode summed over all actuators. This summation property is an important feature of the indices, since the total contribution to the system are decomposed into a sum of non-negative contributions of each individual actuator and mode. In fact, $\hat{\gamma}_k^4(i)$ can be viewed as a matrix of non-negative numbers whose column or row sum corresponds to $\rho_a(i)$ or $\rho_m(k)$. This table of numbers is very informative and can be used to define a suitable metric.

Given a set of actuators, N_a, and a fixed set of sensors, two metrics which are based on the sum and products of principal values, ρ_m (k), are

$$J_1(N_a, N) = \sum_{k\in S} \rho_m(k) = \sum_{k\in S}\sum_{i\in N_a} \hat{\gamma}_k^4(i) = \sum_{i\in N_a} \rho_a(i) \tag{38}$$

$$J_2(N_a, N) = \prod_{k\in S} \rho_m(k) = \prod_{k\in S}\sum_{i\in N_a} \hat{\gamma}_k^4(i) \tag{39}$$

The metric $J_1(N_a, N)$ is a sum of all principal values, i. e., the *trace* of the HSV matrix. The physical implication of the first index is that the modes are weighted in their order of degree of controllability and observability. hence when used as a placement metric, the least controllable and observable modes arc ignored in the actuator and sensor placement.

The second metric, $J_2(N_a$, N), is a product of all principal values over all modes. Alternately, this product is also the *determinant* of the HSV matrix. Physicall y, this means that the least controll able an observable mode (or principal direction) is as important as the most controllable and observable mode. Geometrically, this product is directly proportional to the volume of an n_2 dimensional hyperellipsoid whose principal axes are given by the principal directions and values. in all cases, the physics of the particular application would dictate the selection of the most physically appropriate metric.

4.3 Placement Strategy

Auy placement strategy obviously depends on the metric chosen, which in turn must be based on the needs of the physical problem. For example, if the physical problem of interest suggests that the index $\rho_a(i)$ which characterizes the importance of the ith actuator over all modes is suitable and in addition the sum of this index represents the combined actuator metric, then, $J_1(N_a, N)$ can be used. The advantage of this metric. is that the contribution of each actuator appears linearly and independently and the optimization problem then reduces to one that can be solved by inspection. Hence, actuators with small values of pa(i) can be removed as the least significant ones.

The balanced modal index $\rho_m(k)$ can be useful when modes at the required controllability and observability level arc required. Indeed, it characterizes the significance of the kth balanced mode for the given locations of sensors and actuators. The controllability and observability of the least significant modes (those with the small index $\rho_m(k)$) can either be enhanced by adding and/or reconfiguring actuators and/or sensors. Based on the products of $\rho_m(k)$ over all modes, a second metric, $J_2(N_a$, N), can be defined. Notice that the placement metric $J_2(N_a, N)$ is not as convenient for optimization as the former metric.

The index $\rho_a(N_a)$ achieves its maximum for N_a = N, giving $\rho(N_a) = 1$. Another extreme situation appears when a single actuator controls a single structural mode [7]. In this case 11 ,= 1, N_a = 1, N > 1, and $\rho_a(i) = \hat{\gamma}_1^4(i) = \hat{\gamma}_1^4(i)/\hat{\gamma}^4$ (N), so that the location with the largest amplitude gives the largest index $\rho_a(i)$, for i = 1, ... N. Thus for this simplest case, selection of actuator location based on the peak amplitude location of tile mode shape [7] is equivalent to largest modal grammian coefficient [11] and largest Hankel singular value.

5 Examples

in tile following examples, the trace of the HSV matrix is used as the actuator placement metric. All the outputs used throughout the examples to generate balanced coordinates or modes arc assumed fixed.

5.1 Truss structure with internal actuator forces

The truss from Fig. is considered. Its outputs are

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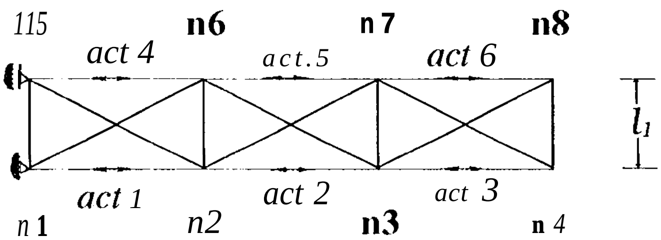

Figure 1: Actuator Configuration for Example 1.

rates measured at vertical direction at nodes $n4$ and $n8$. The following actuators are considered: (1) force in the bar connecting node $n2$ and the support node 711, (2) force in the bar connecting node n3 and n2, (3) fore.c in the bar connecting node $n4$ and n3, (4) force in the bar connecting node n6 and the base node $n5$, (5) force in the bar connecting node n7 and $n6$, (6) force in the bar connecting node n8 and rr7. The task is to find the two inputs within the given six candidates with the best controllability and observability properties.

The controllability and observability properties of each actuator are characterized by the indices pa(i), $i = 1,...,6$. These indices are obtained from the HSV of each individual actuator, which contributes to the total HSV. First the accuracy of the HSV of the system with all six inputs is checked by computing the exact HSV using Eq. (23) and then the same HSV through Eq.(31). The results shown in figure 2 confirm that

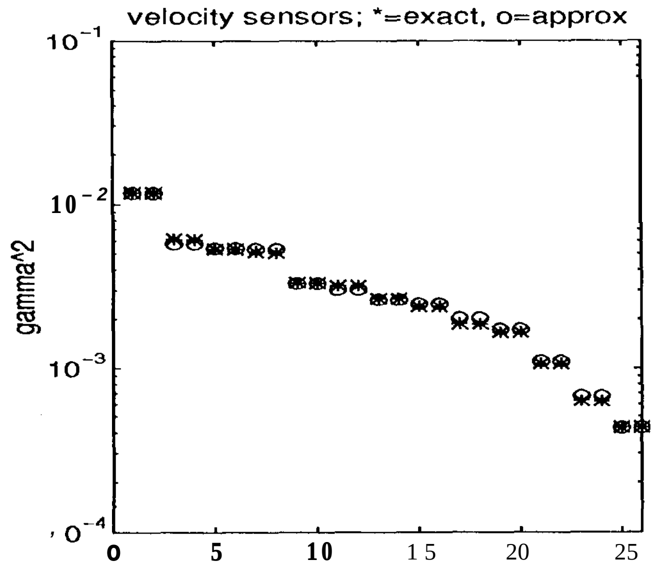

Figure 2: Exact (*) and approximate (0) Hankel singular values of truss.

Eq.(31) holds with satisfactory accuracy with only small discrepancies. These errors appear acceptable for the actuator placement purposes.

The indices $\rho_m(k), k = 1,...,13$ and $\rho_a(i), i = 1,...,6$ are given in figure 3, which indicate that actuator 1 (force in the bar connecting node $n2$ to the base) and actuator 4 (force in the bar connecting node $n6$ and the basis) are the most appropriate for the actuator locations. These choice of locations are intuitively correct since internal strains and displacements at the root of a beam are largest for beam tip vertical motion.

5.2 Truss structure with external actuator forces

The same truss with the same outputs are considered. However, a different set of eight candidate external actuator locations are considered: (1) horizontal force at node $n3$, (2) vertical force at node $n3$, (3) horizontal force at node $n4$, (4) vertical force at node $n4$, (5) horizontal force at node $n7$, (6) vertical force at node $n7$, (7) horizontal force at node $n8$, (8) vertical force at node $n8$, as shown in figure 4. The task is to find the best two inputs within the candidate locations.

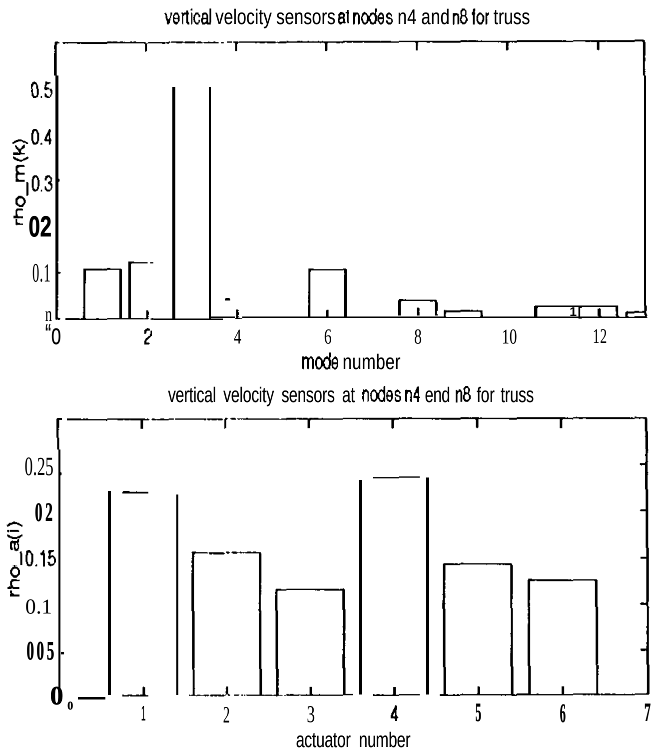

Figure 3: Modal index, $\rho_m(k)$, aud actuator index, $\rho_a(i)$ for truss structure example 1

The accuracy of the HSV of the system with all eight inputs are checked by computing the exact HSV using Eq.(23) and then the same HSV through Eq. (31). The results shown in figure 5 again confirm that Eq.(31) holds with satisfactory accuracy with only small discrepancies.

The modal and actuator indices, $\rho_m(k)$, $k = 1,...,13$ and $\rho_a(i), i = 1,...,8$ are shown in figure 6. The results indicate that the locations 4 (vertical force at node $n4$), and 8 (vertical force at node $n8$) are the best choices. This is not surprising since the given outputs are in the same location and direction.

5.3 Actuator placement for CEM

The actuator placement procedure is applied to the experimental structure called the Control-Structures Interaction Evolutionary Model (CEM), shown in Fig.7. A total of $N = 50$ candidate locations for the air thrusters is selected and shown. The structural model consists of $n_2 = 12$ modes whose first six modes are suspension modes. The frequencies are closely spaced and lightly damped, which is a typical phenomenon for

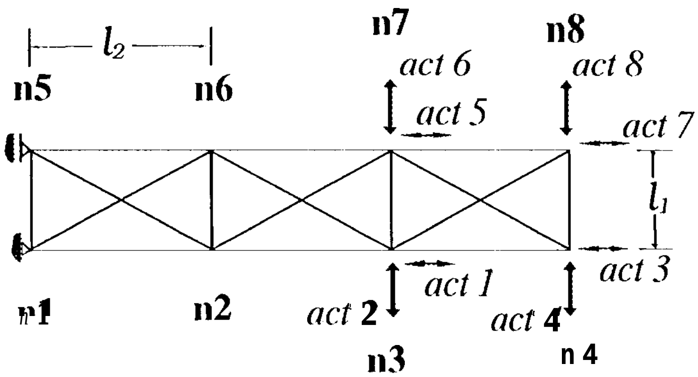

Figure 4: Actuator Configuration for Example 2.

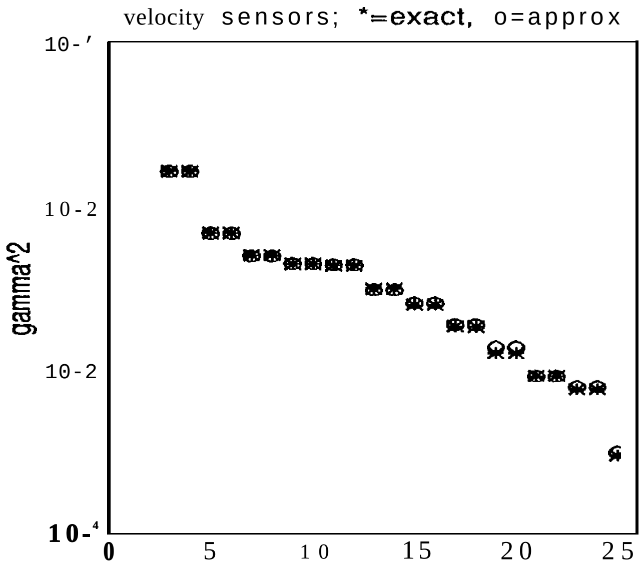

Figure 5: Exact (*) and approximate (o) Hankel singular values of truss.

this kind of structure. The main purpose of the CEM actuators is to suppress the suspension induced and structural vibrations due to external disturbances by using feedback control. The report in [21] gives a more detailed description of the structural model. Based on the set of actuators 1 to 8, the results of several control designs are reported in [23, 11, 12, 13].

The placement of actuators depends on the sensor location. We consider three velocity sensor locations. In the first case, sensors No. 9,37, and 46 were used, all of them sensing the CEM dynamics in y-direction. The accuracy of the approximate decomposition of the HSV is checked by computing the true HSV of a system with all 50 actuators, and the approximate Hankel singular values obtained from Eq. (31) as the sum of the HSV of systems with single actuator. The plot in figure 8 shows that the exact and the approximate HSV are

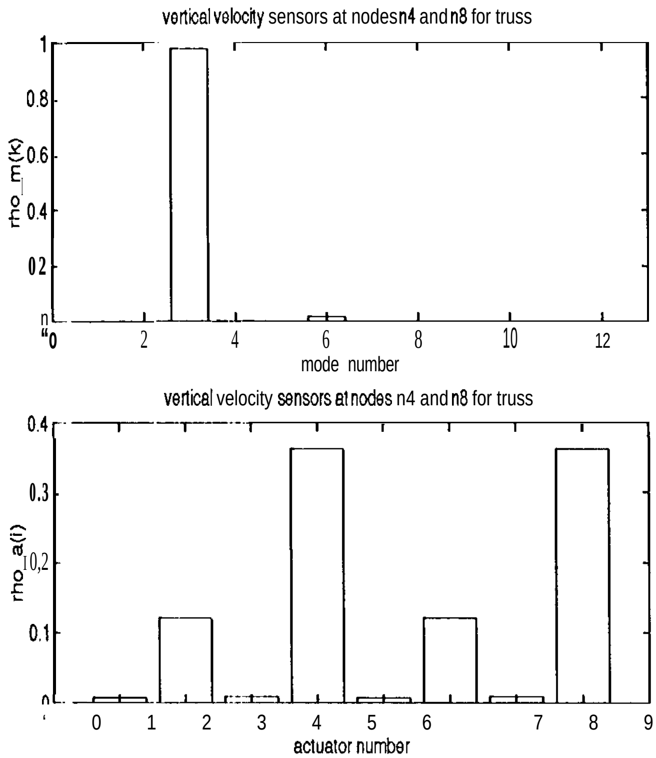

Figure 6: Modal index, $\rho_m(k)$, and actuator index, $\rho_a(i)$ for truss structure example 2

close enough to proceed with the placement procedure.

The modal indices $\rho_m(k), k = 1,\ldots,12$, and the actuator placement indices $\rho_a(i), i = 1, \ldots, 50$ were determined and plotted in figure 9 respectively. The modal indices show that the second mode primarily participates in the output. The actuator indices show that actuators placed at locations 1, 3, 6, 8, 9, 11, 17, 19, 21, 23, 25, 27, 30, 33, 35, 37, 43, and 46 most influence the output. All of them act in y-direction, i.e., they are oriented in the same direction as the outputs.

6 Conclusions

The approach in this paper extends and complements the approximate decomposition of the singular values of the controllability and observability grammian matrices of a multivariable flexible structure to Hankel singular values. The main result of this study is that for lightly-damped flexible structures, the Hankel singular values of individual pairs of actuators and sensors can be summed to approximate the HSV of the multivariable flexible structure. The Hankel singular values can be used to construct various forms of metrics that quantifies the degree of controllability and observability

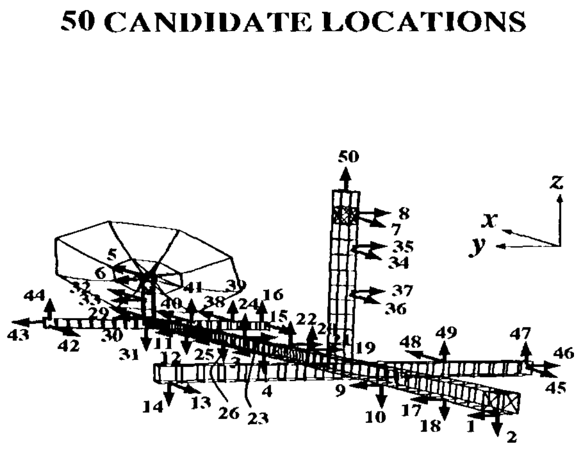

Figure 7: Actuator configuration of CEM.

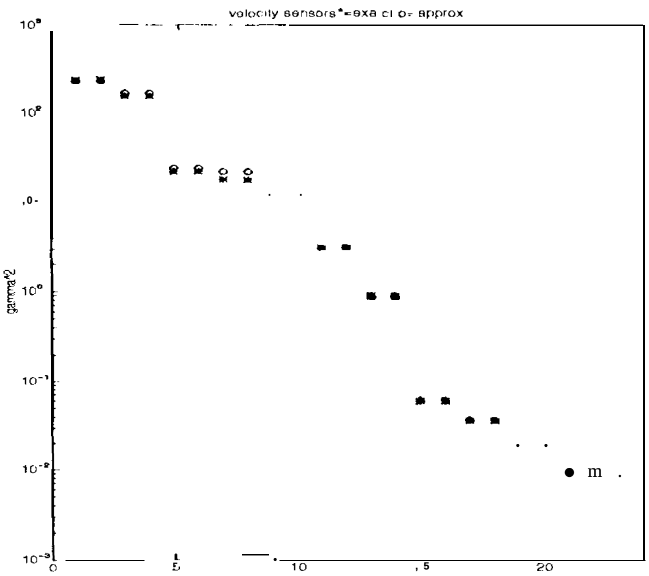

Figure 8: Exact(∗) and approximate (o) Hankel singular values of CEM.

in balanced coordinates. Based on the metric chosen, corresponding placement methodology can be derived. It is shown that for the special case where the trace of the Hankel singular value matrix is used as the metric, the optimal actuator placement problem becomes very simple.

FURTHER D ESIGN AND SIMULATION 1 S NECESSARY...

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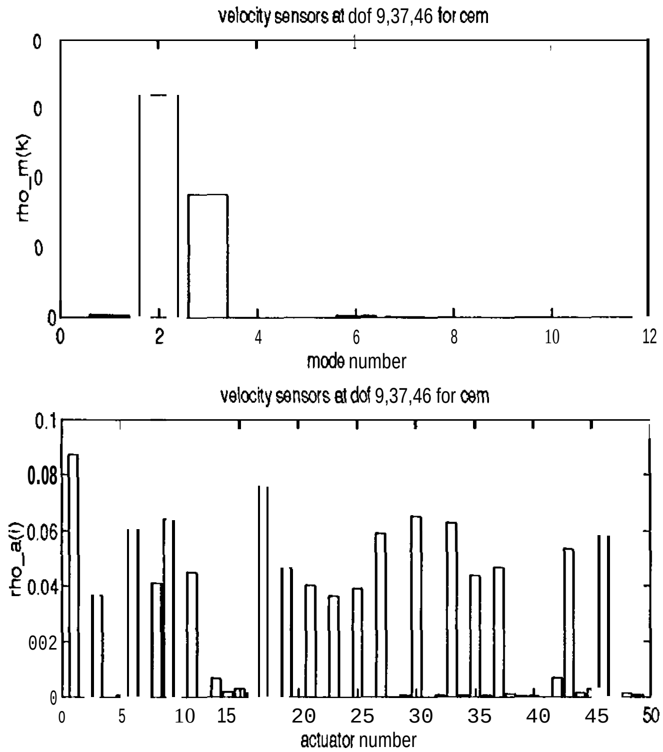

Figure 9: Modal index, ρ_m (k), and actuator index, $\rho_a(i)$ for CEM will, velocity outputs at 9, 37, 46

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