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Topographic core-mantle coupling and polar motion on decadal time scales

R. Hide[1,2], 11.11. Boggs[1], J. O. Dickey], D. Dong[1], R. S. Gross[1] and A. Jackson[3]

[1] Space Geodetic Science and Applications Group, Jet l'repulsion Laboratory, California Institute of Technology, Pasadena,CA91109-8099 USA

[2]Department of Physics (Atmospheric, Oceanic and Planetary Physics), University of Oxford, Clarendon Laboratory, Parks Road, Oxford OX 1 3PU, England UK

[3]School of Earth Sciences, University of Leeds, Leeds .S2 NJ"]', England, UK

SUMMARY

Associated with non-steady fluid motions in the liquid metallic core of the Earth with typical relative speeds of a fraction of a millimetre per second are fluctuations in dynamic pressure of about $10^3 Nm^{-2}$. Acting on the nonspherical core-mantle boundary (CMB), these pressure fluctuations give rise to a fluctuating net topographic torque $L_i(t)$ (i = 1,2,3)—-where t denotes time---on the overlying solid mantle. Geophysicists now accept the proposal by onc of us (RI 1) that $L_i(t)$ makes a significant and possibly dominant contribution to the total torque l;(t) on the mantle produced directly or indirectly by core motions. Other contributions arc the "gravitational" torque associated with fluctuating density gradients in the core, the "electromagnetic" torque associated with Lorentz forces in the weakly electrical] y-conducting lower mantle, and the "viscous" torque associated with shearing motions in the boundary layer just below the CMB. The axial component $L_3^*(t)$ of l;(t) contributes to the observed fluctuations in the length of the day (LOD, an inverse measure of the angular speed of rotation of the solid Earth (mantle, crust and cryosphere)) and the equatorial components $\left(L_1^*(t)\right.$, I;(I))= $L^*(t)$ to the observed polar motion, as determined from measurements of changes in the Earth's rotation axis relative to its figure axis. In earlier phases of a continuing programme of research based on a method for determining $L_i(t)$ from geophysical data (proposed

independently about ten years ago by Hide and Le Mouel]), it was shown that longitude-dependent irregular CMB topography no higher than about 0.5 km could give rise to values of $L_3(t)$ sufficient to account for the observed magnitude of LOD fluctuations on decadal time scales. Here, we report an investigation of the equatorial components $\left(L_1(t), L_2(t)\right) = L(t)$ of $L_i(t)$ taking into account just onc topographic feature of the CMB—albcit possibly the most pronounced-- namely the axisymmetric equatorial bulge with an equatorial radius exceeding the polar radius by 9.55: 0.1 km (the mean radius of the core being 3485 ± 2 km, 0.547 times that of the whole Earth). A measure of the local horizontal gradient of the fluctuating pressure. field near the CMB can be obtained from the local Eulerian flow velocity in the "free stream" below the CMB by supposing that nearly everywhere in the outer reaches of the core- -the "polosphere" (Hide 1995) -- - geostrophic balance obtains between that pressure gradient and Coriolis forces. The polospheric velocity fields used were those determined by Jackson (1989) from geomagnetic secular variations (GSV) data on the basis of the geostrophic approximation combined with the assumption that on the time scales of the GSV the core behaves like a perfect electrical conductor and the mantle as a perfect insulator. in general agreement with independent calculations by Hulot, Lc 1 Iuy and Lc Mouël (to be published) and others wc found that in magnitude L(t) for epochs from 1840 to 1990 typically exceeds $|L_3(t)|$ by a factor of about ten, roughly equal to the ratio of the height of the equatorial bulge to that strongly implied for irregular topography by determinations of $L_3(t)$ (see Hide et *al.* 1993). But $L(t)$ still apparently falls she] t in magnitude by a factor of up to about five in its ability to account for the amplitude of the observed time series of polar motion on decadal time scales (11PM), and it is poorly correlated with that time series. So wc conclude that unless uncertainties in the determination of the DPM time series from observations-which wc also discuss—-have been seriously underestimated, the action of normal pressure forces associated with core motions on the equatorial bulge of the core-mantle boundary makes a significant but not dominant contribution to the excitation of decadal polar motion. Other geophysical processes such as the movement of groundwater and changes in sea level must also be involved.

Key words: topographic core-mantle coupling, decadal polar motion

INTRODUC11ON

Electric currents generated in the Earth's liquid metallic core arc responsible for the main geomagnetic field and its secular changes (see e.g. Merrill and McElhinny 1983; Jacobs 1987-1991). The currents arc produced by dynamo action involving irregular MHD flow in the core. Concomitant dynamical stresses acting on the overlying mantle have been invoked by geophysicists as the main source of excitation of the so-called "decadal" fluctuations in the speed of rotation of the "solid Earth" (mantle, crust and cryosphere). It is possible that core motions make detectable contribution to polar motion (i. e., the movement of the rotation axis of the solid Earth relative to the figure axis) on these time scales. Studies of rotational manifestations of core motions have important implications, for they bear directly on the improvement of models of the strut.tum, composition and dynamics of the Earth's deep interior, which have to reconcile a wide variety of geophysical observations (for references scc loper and Lay 1995).

Consider a set of body-fixed axes x_i (r' = 1, 2, 3) aligned with the principal axes of the solid Earth and rotating about the centre of mass of the whole Earth with angular velocity

$$\hat{\omega}_i = \hat{\omega}_i(t) = (\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3) = \Omega\,(\hat{m}_1, \hat{m}_2, 1 + \hat{m}_3), \qquad \textbf{(1)}$$

where t denotes time and Ω the mean speed of rotation of the solid Earth in recent times (0.7292 115 x 10^{-4} radians per second) (see e.g. Munk and MacDonald 1960; Lambeck 1980; Rochester 1984; Moritz and Mueller 1987; Wahr 1988). Over time scales that are short compared with those characteristic of geological processes, the rotation of the Earth departs only slightly from steady rotation about the polar axis of figure, so that $\hat{m}_1, \hat{m}_2, \hat{m}_3$ are all very much less than unity and $|\dot{\hat{m}}_1| << \Omega$ where $\dot{\hat{m}}_1 \equiv d\hat{m}_1 / dt$. Periodic variations in ω_i on time scales less than a few years are forced by periodic lunar and solar tidal torques and related changes in the moment of inertia of the solid Earth. Irregular sub-decadal variations are evidently produced largely by atmospheric and oceanic torques due to tangential stresses in boundary layers and normal (pressure) stresses acting on surface topography. These fluctuating stresses are associated with seasonal, intraseasonal and interannual fluctuations in the total angular momentum of the atmosphere (for references see Hide 1986; Hide and Dickey 1991; Eubanks 1993; Rosen 1993). When these rapid variations have been removed from the observational data, the smoothed time series

$$\omega_i(t) = \Omega\,(m_1(t), m_2(t), 1 + m_3(t)) \tag{2}$$

that remains (see Figures 1 and 2) represents the contributions of decadal variations to $\hat{\omega}_i(t)$. Errors and uncertainties in these determinations of $m_1(t)$ and $m_2(t)$ are discussed in Appendix A.

Figure 1 and 2 near here

It is convenient at this stage to introduce the vector $L_i^{+}(t)$, defined as the hypothetical torque acting on the solid Earth that would account for observed Earth-rotation fluctuations on decadal time scales in the absence of other processes, such as changes in the inertia tensor of the solid Earth associated with the stresses responsible for

$L_i^+(t)$ and also with the movement of groundwater, melting of ice, etc. Using standard methods (see e.g. Munk and Macdonald 1960; Lambeck 1980) it is readily shown that to sufficient accuracy here (cf. Hide 1989):

$$L^+(t) \equiv \left[L_1^+(t), L_2^+(t)\right] = \Omega^2\left(C^{(m)} - A^{(m)}\right)\left[m_2(t), -m_1(t)\right] \tag{3}$$

where $C^{(m)} - A^{(m)} = 2.63 \times 10^{35}\ kg\, m^2$, $C^{(m)}$ and $A^{(m)}$ being the principal moments of inertia of the solid Earth (about polar and equatorial axes through the Earth's centre of mass respectively). $C^{(m)}$ is 0.895 times the polar moment of inertia of the whole Earth, including the metallic core. Time series of $L_1^+(t)$ and $L_2^+(t)$ are given in Figure 2 below.

In this paper we investigate, as part of a continuing programme of research on Earth rotation fluctuations, the extent to which the observed DPM can be accounted for by the fluctuating topographic torque $L(t) \equiv [L_1(t), L_2(t)]$ associated with normal pressure forecasting on the equatorial bulge of the CMB. Our findings (see below) are in general agreement with those of an independent and parallel study by Hulot, Le Huy and Le Mouël (1995, to be published), extending earlier work by Hinderer *et al.* (1987), which was kindly brought to our attention by its authors at a late stage of our investigation. According to Dr. G. Hulot (private communication), they are also in agreement with the findings of another independent study, by Lefftz and Legros, reported in Hulot *et al.* (1995), in which effects of irregular latitude-dependent topography as well as electromagnetic coupling and induced changes in the inertia tensor of the imperfectly - rigid mantle are also investigated.

THE TOPOGRAPHIC TORQUE EXERTED BY THE CORE ON THE MANTLE

The fluctuating torque $L_i^*(t)$ exerted by the core on the overlying mantle is produced by: (a) tangential stresses at the CMB associated with shearing motions in the thin (probably less than 1 m) viscous boundary layer just below the CMB, (b) normal dynamic pressure forces acting on the equatorial bulge and other (possibly) smaller and more irregular departures from sphericity of the shape of the CMB, (c) Lorentz forces due to the flow of electric currents in the weakly-conducting lower mantle generated by the electromotive forces associated with the geodynamo processes in the core, and (d) gravitational forces associated with horizontal density variations in the core and mantle and especially with CMB topography (Jault and Le Mouël 1993; Eubanks 1993). Dynamical arguments show that the pressure coupling associated with quite modest CMB topography could predominate over other effects (Hide 1969). While the contribution made by viscous stresses is negligible if, as is likely, the effective coefficient of kinematic viscosity of the core is significantly less than about $10^4 m^2 s^{-1}$, electromagnetic coupling might suffice if the (unknown) electrical conductivity of the lower mantle were sufficiently high. Gravitational coupling may turn out to be significant as well (see Jault and Le Mouël 1993; Hulot *et al.* 1995).

The axial component $L_3(t)$ of the topographic torque and its manifestation in decadal LOD variations have been considered in previous studies (e.g. Jault and Le Mouël 1990; Hide *et al.* 1993). Here we discuss the. equatorial components $L_1(t)$ and $L_2(t)$ and their contribution to the decadal polar motion $m_1(t)$ and $m_2(t)$, a problem which, as noted above, has also been studied independently by Hinderer *et* al. (1987) and Hulot *et al.* (1995), with findings in general agreement with those of the present study. If p_s is the dynamic pressure associated with core motions $\boldsymbol{u} = \boldsymbol{u}_s = (u_s, v_s, w_s)$ in the free stream just below the viscous boundary layer at the CMB ($\boldsymbol{u}$ being the Eulerian flow velocity relative to a reference frame fixed to the solid Earth) and the CMB is the locus of points where the distance from the Earth's centre of mass is $\boldsymbol{r}$ = c + h (θ, ϕ) (c being the mean radius of the CMB and (θ, ϕ) the co-latitude and longitude of a general point P), then

$$L_i(t) = -c^2 \int_0^{2\pi}\int_0^{\pi} (\boldsymbol{r} \times p_s \nabla_s h)_i \sin\theta \, d\theta \, d\phi \qquad (4)$$

if $|h| \ll c$ and $|\nabla_s h| \ll 1$. Here $\nabla_s \equiv c^{-1}(\hat{\theta}\partial/\partial\theta, \hat{\phi}\operatorname{cosec}\theta\,\partial/\partial\phi)$, where $\hat{\theta}$ and $\hat{\phi}$ are unit vectors in the direction of increasing θ and ϕ respectively, and $\mathbf{r}$ is the vector distance from the Earth's centre of mass. Because $\mathbf{r} \times \nabla_s(hp_s)$ is a toroidal vector, with no radial components, its integral over the whole CMB is equal to zero. This leads to a more useful expression for $L_i(t)$ in the present context (see below), namely

$$L_i(t) = c^2 \int_0^{2\pi}\int_0^{\pi} (\mathbf{r} \times h\nabla_s p_s)_i \sin\theta \, d\theta d\phi\,, \tag{5a}$$

with components

$$L_1 = -c^2 \int_0^{2\pi}\int_0^{\pi} h\left[\sin\theta\sin\phi\frac{\partial p_s}{\partial\theta} + \cos\theta\cos\phi\frac{\partial p_s}{\partial\phi}\right] d\theta d\phi, \tag{5b}$$

$$L_2 = c^2 \int_0^{2\pi}\int_0^{\pi} h\left[\sin\theta\cos\phi\frac{\partial p_s}{\partial\theta} - \cos\theta\sin\phi\frac{\partial p_s}{\partial\phi}\right] d\theta d\phi\,, \tag{5c}$$

$$L_3 = c^2 \int_0^{2\pi} h\left[\sin\theta\frac{\partial p_s}{\partial\phi}\right] d\theta d\phi\,. \tag{5d}$$

Given $h(\theta,\phi)$ and determinations of $p_s(\theta,\phi,t)$ on the CMB, $L_i(t)$ could be calculated directly using either Eq. (4) or Eq. (5). However, as in the case of the Earth's core and in other situations where p_s is *not* known from direct measurements but other information is available, such as $\mathbf{u}_s(\theta,\phi,t)$, it is still possible to estimate $L_i(t)$ by using the equations of fluid dynamics to relate the horizontal pressure gradient to "observable" quantities, Owing to the Earth's rotation, Coriolis forces exert a strong influence on core flow (Elsasser 1939; Frenkel 1945; Inglis 1955). They should be in close "geostrophic" balance with horizontal pressure gradients nearly everywhere within those regions of the core where Lorentz forces, which produce the strongest "ageostrophic" effects (Hide

1956), arc comparatively weak, and in "magnetostrophic" balance in any regions where Lorentz forces arc comparable in magnitude with Coriolis forces. Now the magnetic field in the core can be decomposed into "toroidal" and "polo oidal" parts (Elsasser 1947; cf. Eq. (12) below), where the former has no radial component and may exceed the latter in magnitude by as much as a factor of about ten and give rise to magnetospheric flow throughout most of the liquid core—the "torosphere" (see Hide 1995)-–but not in the outer reaches of the core--- -the "polosphere"-- -where by definition the toroidal magnetic field is no stronger than the poloidal field (see Le Mouël 1984; Hide 1995). With an error of no more than about 10-"2, polospheric flow can be assumed geostrophic nearly everywhere, satisfying the equation

$$2\bar{\rho}_s \Omega \cos\theta\,[-w_s, \mathrm{v},] = -- c^{-1}[\partial p_s/\partial\theta, \operatorname{cosec}\theta\, \partial p_s/\partial\phi]. \qquad (6)$$

Here (v_s, w_s), the (0, ϕ) components of $\boldsymbol{u}_s$, are typically much greater in magnitude than u_s, the r-component of $\boldsymbol{u}_s$. It follows from Eqs. (5) and (6) that on the time scales of interest here, over which $\boldsymbol{u}$ may change significantly but h dots not,

$$[L_1(t), L_2(t)] = 2\bar{\rho}_s \Omega\, c^3 \int_0^{2\pi}\int_0^{\pi} h(\theta, \phi)[f_1(\theta,\phi,t), f_2(\theta,\phi,t)] \sin\theta \cos\theta \, d\theta \, d\phi \qquad (7a)$$

where

$$[f_1, f_2] \equiv [v_s \cos\theta \cos\phi - w_s \sin\phi,\; v_s \cos\theta \sin\phi + w_s \cos\phi] \qquad (7b)$$

(Hide 1989, 1995; Jault and Le Mouël 1989; Voorhies 1991).

The basic theoretical relationships needed here are give.n by Eqs. (3–7). The integral on the right-hand side of Eq. (7a) involves the CMB topography $h(\theta, \phi)$. The ϕ-independent mean equatorial bulge $h_0(\theta)$ (say) of the CMB corresponds to a 9.5± 0.1 km difference between the equatorial and polar radii of the CMB (see Denis and Ibrahim 1981; Gwinn *et al.* 1986; Herring *et al.* 1991; cf. Moritz and Mueller 1987). If this is substantially larger than the typical vertical amplitude of irregular topographic features of the CMB (and there is evidence that this might be so from the study of decadal variations in the length of day, involving the evaluation of $L_3(t)$, see Jault and Le Mouël (1990); Hide *et al.* (1993)), then to a good first approximation we can replace $h(\theta, \phi)$ in Eq. (7) by $h_0(\theta)$ (see Eqs. (17) and (18) below). The other quantity needed in the evaluation of $L_i(t)$ is *either* the field of dynamical pressure $p_s(\theta, \phi, t)$ (see Eqs. (4) and (5) and Appendix B below) *or* the field of horizontal flow $(v_s(\theta, \phi, t), w_s(\theta, \phi, t))$, which is related to p_s through Eq. (6). Geomagnetic secular variation data have been used by various workers to infer (v_s, w_s) by a method that invokes geostrophic balance in combination with the equations of electrodynamics appropriate to the case when the mantle can be treated as a perfect electrical insulator of uniform magnetic permeability and the core as a perfect conductor, as wc shall now discuss.

VELOCITY AND PRESSURE FIELDS IN THE CORE

Denote by $\boldsymbol{B}(r, \theta, \phi, t)$ the value of the main geomagnetic field at a general point (r, θ, ϕ) at time t, and by $\dot{\boldsymbol{B}} \equiv \partial \boldsymbol{B}/\partial t$ the geomagnetic secular variation (GSV). Determinations of $\boldsymbol{B}$ made at and near the Earth's surface at various epochs can be used to infer $\boldsymbol{u}_s$, the Eulerian flow velocity just below the CMB (for references see Bloxham and Jackson 199 1; Hulot *et al.* 1992). In turn, estimates of the associated horizontal pressure gradient $c^{-1}(\partial p_s/\partial\theta, \operatorname{cosec}\theta\, \partial p_s/\partial\phi)$ there can be deduced by using Eq. (6). The first of the three reasonable key assumptions that underlie the method used is that the electrical conductivity of the mantle and magnetic permeability gradients there arc negligibly small, so that $\boldsymbol{B}$ can be written as the gradient of a potential V satisfying Laplace's equation $\nabla^2 V = 0$. This facilitates the downward extrapolation of the observed field at and near the Earth's surface in order to obtain $\boldsymbol{B}$ and $\dot{\boldsymbol{B}}$ at the CMB (see e.g. Jacobs 1987–1991). The second assumption concerns the time scales of the GSV and the electrical conductivity of the core. Dynamo theory requires high but not perfect electrical conductivity, for it is impossible to change the magnetic flux linkage of a perfect conductor (Bondi and Gold 1950), in accordance with Alfvén's "frozen flux" theorem. But when dealing with fluctuations in $\boldsymbol{B}$ on time scales very much less than that of the ohmic decay of magnetic fields in the core (which is several thousand years for global-scale features), Alfvén's "frozen flux" theorem, in our notation

$$\partial \boldsymbol{B}/\partial t = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}), \tag{8}$$

should provide a good leading approximation, implying that the lines of magnetic force emerging from the core are advected by the horizontal flow (v_s, w_s) just below the CMB (Roberts and Scott 1965; Backus 1968). Accordingly if $\boldsymbol{B} = (B_r, B_\theta, B_\phi)$ then B_r at the CMB satisfies

$$\frac{\partial B_r}{\partial t} + \frac{v_s}{c}\frac{\partial B_r}{\partial \theta} + \frac{w_s}{c\sin\theta}\frac{\partial B_r}{\partial \phi} = B_r\left[\frac{\partial u}{\partial r}\right]_{r=c}. \tag{9}$$

The last equation alone does not permit the unique determination of $\boldsymbol{u}_s$ from knowledge of $\boldsymbol{B}$ and $\dot{\boldsymbol{B}}$ at the CMB, so a third physically plausible assumption is needed. Effective uniqueness can be secured by making use of the assumption expressed by Eq. (6) above, namely that to first approximation, polospheric flow is in geostrophic balance (hills 1979; Le Mouël 1984; Le Mouël *et al.* 1985; Backus and Le Mouël 1986; Gire and

Le Mouël 1990; see also Bloxham and Jackson 1991; Hulot *et al.* 992). This gives the additional equation

$$\cos\theta\left[\frac{\partial u}{\partial r}\right]_{r=c}+\frac{\sin\theta}{c}v_s=0, \tag{10}$$

which is obtained by eliminating p_s from Eq. (6) and using the mass continuity equation

$$\nabla\bullet\boldsymbol{u}=0 \tag{11}$$

for flow in an effectively incompressible fluid.

Various groups of geomagnetic workers have lu-educed maps of (v_s, w_s) based on this (and other) assumptions and investigated the errors and uncertainties encountered in practice (Bloxham and Jackson 1991; Hulot *et* al. 1992). Fields of B_r and $\dot{B}_r$ at $r=c$ deduced by Jackson (1989) for epochs going back to the year 1840 AD provided the basic $\boldsymbol{B}$ and $\dot{\boldsymbol{B}}$ input data for the present study. These were used to produce hypothetical "geostrophic" flow fields $\boldsymbol{u}_s$ constructed using spherical harmonic expansions (see Eqs. (1 6--20) below) up to degree and order 14, which account for more than 90% of the observed GSV (Jackson 1989).

TORQUE ON THE EQUATORIAL BULGE

For an effectively incompressible fluid the Eulerian flow field $\boldsymbol{u}$ is solenoidal, in virtue of Eq. (11), and can therefore be decomposed into a toroidal part (with no radial component) and a poloidal part as follows:

$$\boldsymbol{u}=\boldsymbol{u_T}+\boldsymbol{u_P}=\nabla\times(T';)+\nabla\times\nabla\times(P^*\hat{\boldsymbol{r}}) \tag{12}$$

where $\hat{\boldsymbol{r}}$ is a unit vector in the direction of increasing r (cf. Eq. (4)). If (u, v, w) are the (r, θ, ϕ) components of u, then

$$u = \frac{-1}{r^2}\left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial P^*}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 P^*}{\partial\phi^2}\right\}, \tag{13a}$$

$$v = -\frac{1}{r\sin\theta}\frac{\partial T^*}{\partial\phi}+\frac{1}{r}\frac{\partial S^*}{\partial\theta}, \tag{13b}$$

$$w = -\frac{1}{r}\frac{\partial T^*}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial S^*}{\partial\phi} \tag{13c}$$

where $S^* \equiv \partial P^*/\partial r$. If $\boldsymbol{u} = \boldsymbol{u}_s = (u_s, v_s, w_s)$ in the free stream just below the CMB where, for the purpose of this part of the calculation (but see Hide 1995, Eq. (4.5)), u_s can be set equal to zero and r equal to c, the mean radius of the CMB (cf. Eq. (4)), and T* = T and S* = S on $r = c$, then we have

$$v_s = -\frac{1}{c\sin\theta}\frac{\partial T}{\partial\phi}+\frac{1}{c}\frac{\partial S}{\partial\theta}, \tag{14a}$$

$$w_s = -\frac{1}{c}\frac{\partial T}{\partial\theta}+\frac{1}{c\sin\theta}\frac{\partial S}{\partial\phi}, \tag{14b}$$

By the geostrophic relationship Eq. (6)

$$\frac{\partial p_s}{\partial\theta} = (2\bar{\rho}\Omega c\cos\theta)\, w_s = (2\bar{\rho}\Omega)\left[-\cos\theta\frac{\partial T}{\partial\theta}+\frac{\cos\theta}{\sin\theta}\frac{\partial S}{\partial\phi}\right], \tag{15a}$$

$$\frac{\partial p_s}{\partial\phi} = -(2\bar{\rho}\Omega c\cos\theta\sin\theta)\, v_s = (2\bar{\rho}\Omega)\left[-\cos\theta\frac{\partial T}{\partial\phi} - \cos\theta\sin\theta\frac{\partial S}{\partial\theta}\right], \tag{15b}$$

from which it follows that T and S satisfy:

$$\sin\theta \frac{\partial T}{\partial\phi} = (\cos^2\theta - \sin^2\theta)\frac{\partial S}{\partial\theta} + \cos\theta\sin\theta \frac{\partial^2 S}{\partial\theta^2} + \frac{\cos\theta}{\sin\theta}\frac{\partial^2 S}{\partial\phi^2} \tag{15c}$$

The scalar quantities $T(\theta,\phi,t)$ and $S(\theta,\phi,t)$ and the topography $h(\theta,\phi)$ can be expressed as spherical harmonic expansions:

$$q = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(q_{n,m}^{c}\cos m\phi + q_{n,m}^{s}\sin m\phi\right)P_{n,m}(\cos\theta)\,, \tag{16}$$

where

$$\left(q_{n,m}^{c}\,,q_{n,m}^{s}\right) = \left(\tau_{n,m}^{c}\,,\tau_{n,m}^{s}\right) \text{ when } q = T\,, \tag{17}$$

$$\left(q_{n,m}^{c}\,,q_{n,m}^{s}\right) = \left(s_{n,m}^{c}\,,s_{n,m}^{s}\right) \text{ when } q = S\,, \tag{18}$$

$$\left(q_{n,m}^{c}\,,q_{n,m}^{s}\right) = \left(h_{n,m}^{c}\,,h_{n,m}^{s}\right) \text{ when } q = h\,. \tag{19}$$

Here $P_{n,m}(\cos\theta)$ are the associated Legendre polynomials of degree n and order m with the Schmidt semi-normalization

$$\int_0^{\pi} P_{n,m}(\cos\theta)P_{n',m'}(\cos\theta)\sin\theta\, d\theta = 2(2-\delta_{m,o})\delta_{n,n'}/(2n+1) \tag{20}$$

where $\delta_{n,n'} = 1$ when $n = n'$ and $\delta_{n,n'} = 0$ when $n \neq n'$. (See Appendix B for a discussion of the spherical harmonic expansion of the pressure field p_s in terms of the coefficients of the expansions of T and S.)

In the case when the equatorial bulge is the sole topographic feature of the CMB to be considered, we have $h(\theta,\phi) = h_0(\theta)$ where

$$h_0(\theta) = h^c_{2,0} P_{2,0}(\cos\theta) = h^c_{2,0}\left[\frac{1}{2}\left(3\cos^2 - 1\right)\right]. \tag{21}$$

This gives $-3h^c_{2,0}/2$ for the difference between the equatorial and polar radii (9.5 ± 0.1 km, scc above), so that

$$h^c_{2,0} = -(6.3 \pm 0.1) \times 10^3 m. \tag{22}$$

The general expressions for (L_1, L_2) obtained when Eq. (7) are combined with Eqs. (12–16) arc quite complicated and will not be written down here (but see Appendix B). When $h(\theta,\phi) = h_0(\theta)$ given by Eq. (21), the expressions simplify to the following:

$$(L_1, L_2) = -2\pi\bar{\rho}\Omega c^2 h^c_{2,0}\left\{\frac{72\sqrt{3}}{35}\left(s^c_{2,1}, s^s_{2,1}\right) + \frac{32\sqrt{10}}{21}\left(s^c_{4,1}, s^s_{4,1}\right)\right\}. \tag{23}$$

The polar motion due the action of geostrophic pressure forces acting on the equatorial bulge in the CMB in the case when the mantle is perfectly rigid could be derived by combining this equation with the equation obtained by setting $(L_1, L_2) = (L_1^+, L_2^+)$ in Eq. (3). Alternatively, wc can compare the time series of $(L_1(t), L_2(t))$ as given by Eq. *(23)* with those of $(L_1^+(t), L_2^+(t))$ implied by the observed polar motion through Eq. (3).

The results arc shown in Figures 2 and 3, where in evaluating $m_1(t)$ and $m_2(t)$ we have taken:

Figure 3 near here

$$\bar{\rho} = 0.99 \times 10^4 \text{ kg m}^{-3}, \Omega = 7.29 \times 10^{-5} \text{ rad s}^{-1},$$

$$c = 3.48 \text{ X } 106111, C^{(m)} - A^{(m)} = 2.63 \text{ X } 10^{35} \text{ kg m}^2$$

(see e.g. Stacey 1992; Eubanks 1993), so that

$$2\pi\bar{\rho}\,\Omega c^2 h^c_{2,0} = -3.48 \times 10^{17}\ \mathrm{kg\,s^{-1}} \quad \text{and} \quad \left[\Omega^2\left(C^{(m)} - A^{(m)}\right)\right]^{-1} = 8.12 \times 10^{-28}\ \mathrm{kg^{-1}m^{-2}s^2}$$

are the numerical values of these factors in Eqs. (23) and (3) respectively.

DISCUSSION AND CONCLUDING REMARKS

The axial component $L_3(t)$ of the net torque $L_i(t)$ on the mantle due to the action on topographic features $h(\theta, \phi)$ of the CMB of normal (pressure) stresses associated with core motions could, as shown in previous work, make a significant and possibly dominant contribution to observed decadal LOD fluctuations, even with longitudinal variations in h that are no bigger than 1 km in vertical amplitude and possibly even slightly less (Hide 1969; Jault and Le Mouel 1990; Hide *et al. 1993).* The equatorial bulge of about 10 km is likely to be the main topographic feature involved in producing the equatorial component $L(t)$ of $L_i(t)$, and it is not surprising therefore, as the calculations presented in this paper show (see Figure 2 and Hide *et* al., 1993), that $|L(t)|$ typically exceeds $|L_3(t)|$ by a large factor. However, it is clear from Figure 2 that $L(t)$ is about 5 times smaller in magnitude than the equatorial torque $L^+(t)$ inferred from the observed polar motion, Further analysis reveals that the series are uncorrelated as well.

In our study, the equatorial bulge is taken (for simplicity) to be the sole topographic CMB feature, and the resulting expression for the equatorial torque involves only the second and fourth degree spherical harmonic coefficients of the velocity field (see Eq. 23). The second degree harmonic clearly dominates (Figure 3) with little cancellation between these terms, so it is unlikely that uncertainties in the velocity fields used could account for the discrepancy. Our general findings concerning the inadequacy of topographic coupling in the excitation of decadal polar motions agree qualitatively with those of the abovementioned work by Hulot et *al.* (1995), where effects due to irregular latitude-dependent topography, gravitational torques associated with the nonspherical shape of the CMB, and changes in the inertia tensor of the solid Earth produced by fluctuating horizontal pressure variations in the upper reaches of the core are also taken into account. The fluctuating electromagnetic torque on the mantle associated with core motions (see above) could, of course, be stronger than the topographic torque, but only under extreme assumptions concerning the (unknown) distribution and magnitude of the electrical conductivity of the lower mantle and of the toroidal part of the geomagnetic field just below the core mantle boundary.

It is an old suggestion that the movement of air and water at and near the Earth's surface on relevant time scales are involved in the excitation of DPM (see Wilson 1993), but quantitative studies are hard to make. The spectral characteristics of water movement are similar to that of DPM, red, and increasing sharply at decadal periods, whereas the air mass excitation spectrum is flat or white (Kuehne and Wilson 1991). DPM observations imply an excitation showing linear polarization along the direction that would result from a uniform rise or fall of sea level, implying forcing is due, at least in part, to the re-distribution of water mass (Wilson 1993). It will be necessary in future work, as better data become available, to re-investigate all possible excitation mechanisms, for it seems that the decadal polar motion is most likely caused by a variety of geophysical processes with the topographic torque induced by the equatorial bulge producing a significant but not a dominant contribution.

APPENDIX A: OBSERVATIONS OF POLAR MOTION

Studies of decadal variations in polar motion (see Figure 1) rely heavily upon daily observations of the latitude of each of the. five stations of the International Latitude Service (ILS) obtained by the technique of optical astrometry (see e.g. Yumi and Yokoyama 1980). Determinations of polar motion $(m(t), m_2(t))$ (see Eq. (2)) from monthly averages of these observations span the 80 years period 1899– 1979. Analyses of this 11.S polar motion time series show clear evidence of variability on decadal time scales (e.g. Wilson and Vicente 1980; Dickman 1981). But the reality of this variability has been called into question out of concern about contaminating effects of possible systematic errors occurring at individual stations (see e.g. Eubanks 1993). In particular, Zhao and Dong (1988) noticed that when observations from the Ukiah 11.S station arc not used, the recovered DPM variations, although still present with about the same amplitude, arc not as regular as when the polar motion is determined from all available observations. From this, they concluded that the apparent regularity of the decadal polar motion variations (i.e., the so-called 30-year Markowitz wobble) is an artifact of systematic errors in the Ukiah observations. We emphasize here that the study of Zhao and Dong (1988), although widely cited as concluding that the DPM variations are not real, in fact only questions the *regularity* of the observed variations, not their presence.

Gross (1982, 1990), recognizing the corrupting influence of systematic errors, developed a technique in which polar motion is recovered from the ILS variation of latitude observations by simultaneously solving for station-dependent systematic errors, leaving the polar mot ion parameters free of such effects. The decadal variations evident in the polar motion series he obtained by this technique arc nearly identical to that exhibited in the 11.S series, leading him to conclude that these variations arc real and not an artifact of systematic errors in the observations,

$pacc-geodetic determinations of polar motion began with the launch of the Lageos I satellite in May, 1976, and now span nearly 20 years. The POLE93 polar motion series anal yzed here is a Kalman filter-based combination of the ILS polar motion series (spanning 1899-1 979), the BIH (Bureau International dc 1'1 leure) optical astrometric series (spanning 1962–1 982), and space-geodetic polar motion measurements made by the techniques of SLR (satellite laser ranging) (spanning 1976–1 993), VLBI (very long baseline interferometry) (spanning 1979--1993), and the GPS (global positioning system) (spanning 1991-1 993). Since 1982, the polar motion values in PO] .E93 are therefore based solely upon modern space-geodetic. measurements. As seen in Figures 1 and 2, there is clear evidence of decadal-scale variability in the 1'01 .E93 polar motion series since 1982. Furthermore, the post-1 983 variability is consistent with similar-scale variability evident in this series during earlier epochs, thereby giving credence to this earlier variability.

The POLE93 series is used here since it is the most complete polar motion series currently available. The above discussion and previously cited studies indicate that the influence of systematic errors on the decadal variability evident in this series may affect the shape of the variations, but not the amplitude.

APPENDIX B: SPHERICAL HARMONIC EXPANSION OF PRESSURE FIELD

In virtue of the equivalence of Eq. (4) and Eq. (5), which lead to Eq. (7) for $L_i(t)$ in terms of $h(\theta,\phi)$ and u_s (θ, ϕ, t), it is possible to determine L,(t) without going through the stage of evaluating $p_s(\theta,\phi,t)$ directly (see Hide 1989). But for the sake of completeness and other reasons it is useful to relate, through Eq. (6), the coefficients in the spherical harmonic expansions for $u_s(\theta$, $,?) (see Eq. (14), (17) and (18)) to those of the following expression for $p_s(\theta,\phi,t)$:

$$p_s(\theta,\varphi,t)=\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(p_{n,m}^{c}(t)\cos m\varphi+p_{n,m}^{s}(t)\sin m\varphi\right)P_{n,m}(\cos\theta), \tag{B1}$$

where the $P_{n,m}$(cos O) are defined by Schmidt normalization given by Eq. (20). The coefficients $p_{n,m}^{s}$ and $p_{n,m}^{c}$ can be expressed in terms of the spherical harmonic coefficients of $u_s(\theta,\phi,t)$ as follows (see Eq. (17) and (18), cf. Girt and Le Mouël, 1990):

$$p_{n,m}^{c}=-2\bar{\rho}\,\Omega\left[C_{n,m}^{(-)}\;s_{n-2,m}^{s}+C_{n,m}^{(o)}\;s_{n,m}^{s}+C_{n,m}^{(+)}\;s_{n+2,m}^{s}\right] \tag{B2}$$

$$p^s_{n,m} = 2\rho\,\Omega\left[C^{(-)}_{n,m}\, s^s_{n-2,m} + C^{(0)}_{n,m}\, s^s_{n,m} + C^{(+)}_{n,m}\, s^c_{n+2,m}\right] \tag{B3}$$

when $m>0$, and

$$p^c_{n,0} = -2\rho\,\Omega\left[\frac{n-1}{2n-1}\tau^c_{n-1,0} + \frac{n+2}{2n+3}\tau^c_{n+1,0}\right], \tag{B4}$$

the coefficients $s^c_{n,m}$, $s^s_{n,m}$, $t^c_{n,m}$, $t^s_{n,m}$ being equal to zero when $n<m$ and $m>0$, in equations (B2) and (B3)

$$C^{(-)}_{n,m} = \frac{(n-1)(n-2)[(n-m)(n+m)(n-1-m)(n-1+m)]^{1/2}}{m(2n-1)(2n-3)}, \tag{B5}$$

$$C^{(0)}_{n,m} = \frac{n(n+1)}{m(2n+1)}\left[\frac{(n+1-m)(n+1+m)}{(2n+3)} + \frac{(n-m)(n+m)}{(2n-1)}\right], \tag{B6}$$

and

$$C^{(+)}_{n,m} = \frac{(n+2)(n+3)[(n+1-m)(n+1+m)(n+2-m)(n+2+m)]^{1/2}}{m(2n+3)(2n+5)} \tag{B7}$$

in terms of the coefficients of pressure field, the torque acting on the main bulge at the CMB can be expressed as follows:

$$(I_1, I_2) = \frac{4\sqrt{3}\pi}{5} r^2 h_{2,0}(-p^s_{2,1}, p^c_{2,1}) \tag{B8}$$

(see Hinderer *et al.* 1990). It is readily demonstrated using Eq. (B2) and (B3) that Eq. (B8) is consistent with the expression for the torque in terms of the velocity field given by Eq. (23). Taking $h_{2,0}$ as - $2c\alpha_c/3$, where α_c denotes the ellipticity at CMB, our Eq. (B8) is identical to the formula (22b) of Hulot et *al.* (1995).

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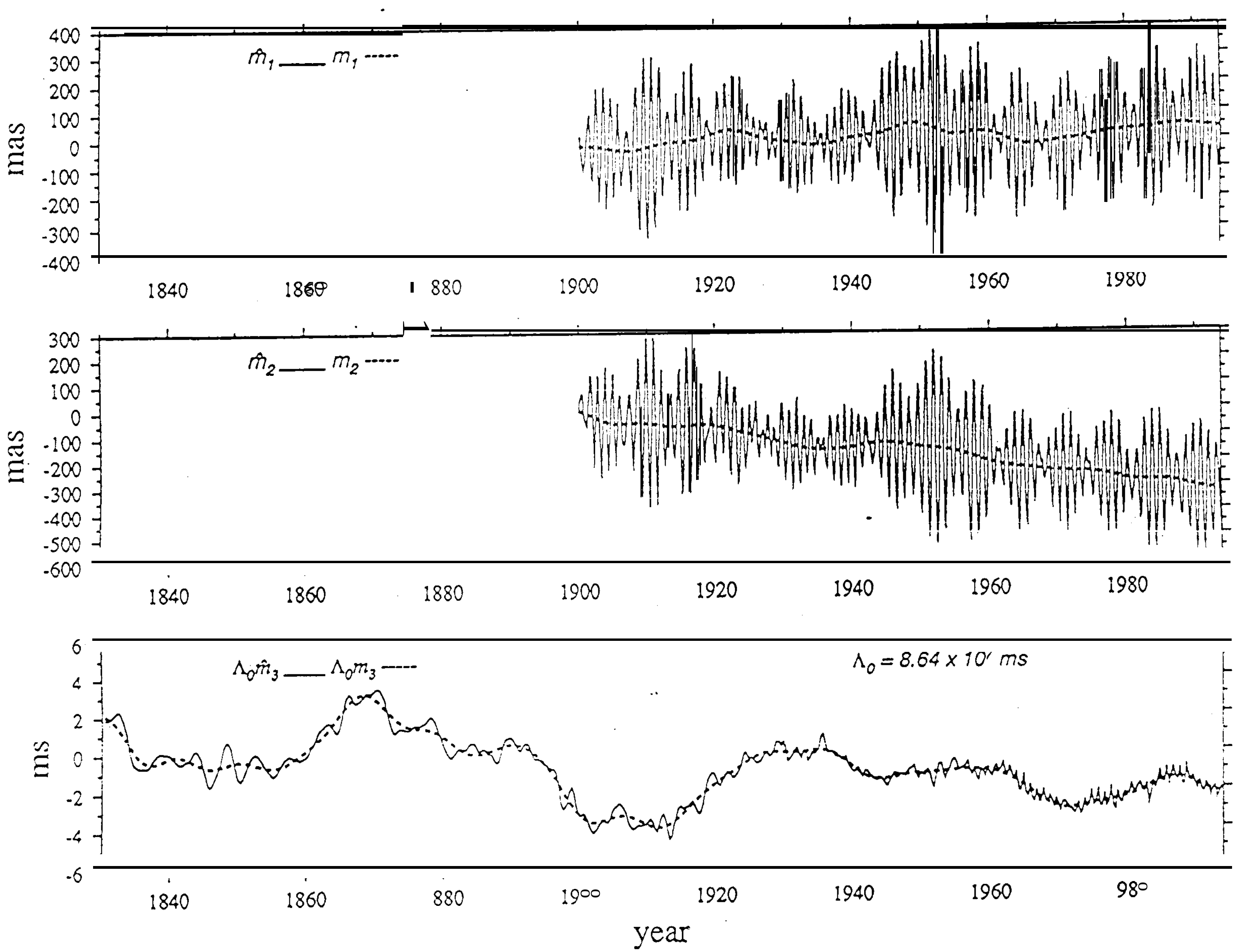
LEGENDS FOR DIAGRAMS

Figure 1. Timeseries (solid lines) of the observed fluctuations in the Earth's rotation vector $\hat{\omega} \equiv \Omega[\hat{m}_1(t), \hat{m}_2(t), 1 + \hat{m}_3(t)]$ *(see* Eq. (1) and Appendix A). The top panel shows the x-component of polar motion, Al(t), defined to be positive toward the Greenwich meridian, the middle panel shows the y-component of polar motion, $\hat{m}_2(t)$, defined to be positive toward 90° E longitude, and the bottom pane] shows changes in the length-of-day $\hat{\lambda}(t)$ which is related to $\hat{m}_3(t)$ by $\hat{\lambda}(t) = -\Lambda_0 \hat{m}_3(t)$ where Λ_0 *is* the nominal length-of-day of 86400s. The dashed lines show the decadal variations obtained by applying a low pass filter (cutoff period = 10 years) to the observed series (see also Figure 2).

Figure 2. Timeseries of the components $\left(L_1^+(t), L_2^+(t), L_3^+(t)\right)$ of the equivalent torques implied by decadal components of $(\hat{m}_1(t), \hat{m}_2(t), 1 + \hat{m}_3(t))$ of $(\hat{m}_1(t), \hat{m}_2(t), 1 + \hat{m}_3(t))$ for a perfectly-rigid mantle (see Eqs. (2) and (3) and Hide (1989)). Also given (clashed lines) arc the estimated time series of the contribution made. to $(\hat{m}_1(t), \hat{m}_2(t))$ by the equatorial torque $(L_1(t), L_2(t))$ (see Eq. (23) due to the action on the equatorial bulge of the CMB of normal pressure forces associated with core motions in the case of a perfectly-rigid mangle (see Figure 3). A low pass filter with a cutoff period of 10 years has been applied to all time series.

Figure 3. Timeseries of the components $L_1(t)$ and $L_2(t)$ of the topographic torque produced by the action on the equatorial bulge of the CMB of normal pressure forces associated with core motions in the case of a perfectly rigid mantle (see Eq. (23)). Of the terms in the full spherical harmonic expansion of the pressure field (see Appendix B), only those corresponding to terms of degree $n = 2$ and $n = 4$ in the poloidal part of the associated velocity field make non-zero contributions to $(L_1(t), L_2(t))$, namely $(L_1(t; n = 2), L_2(t; n = 2))$ and $(L_1(t; n = 4), L_2(t; n = 4))$, indicated in the. diagram by clashed and dotted lines respectively. A low pass filter with a cutoff period of 10 years has been applied to all time series.

(End of the main text plus legends, diagrams i.e., Figures 1, 2 and 3, given on next three pages.)



$\hat{m}_1$ —— m_1 -----
mas
$\hat{m}_2$ —— m_2 -----
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$\Lambda_0\hat{m}_3$ —— $\Lambda_0 m_3$ -----
$\Lambda_0 = 8.64 \times 10^7$ ms
ms
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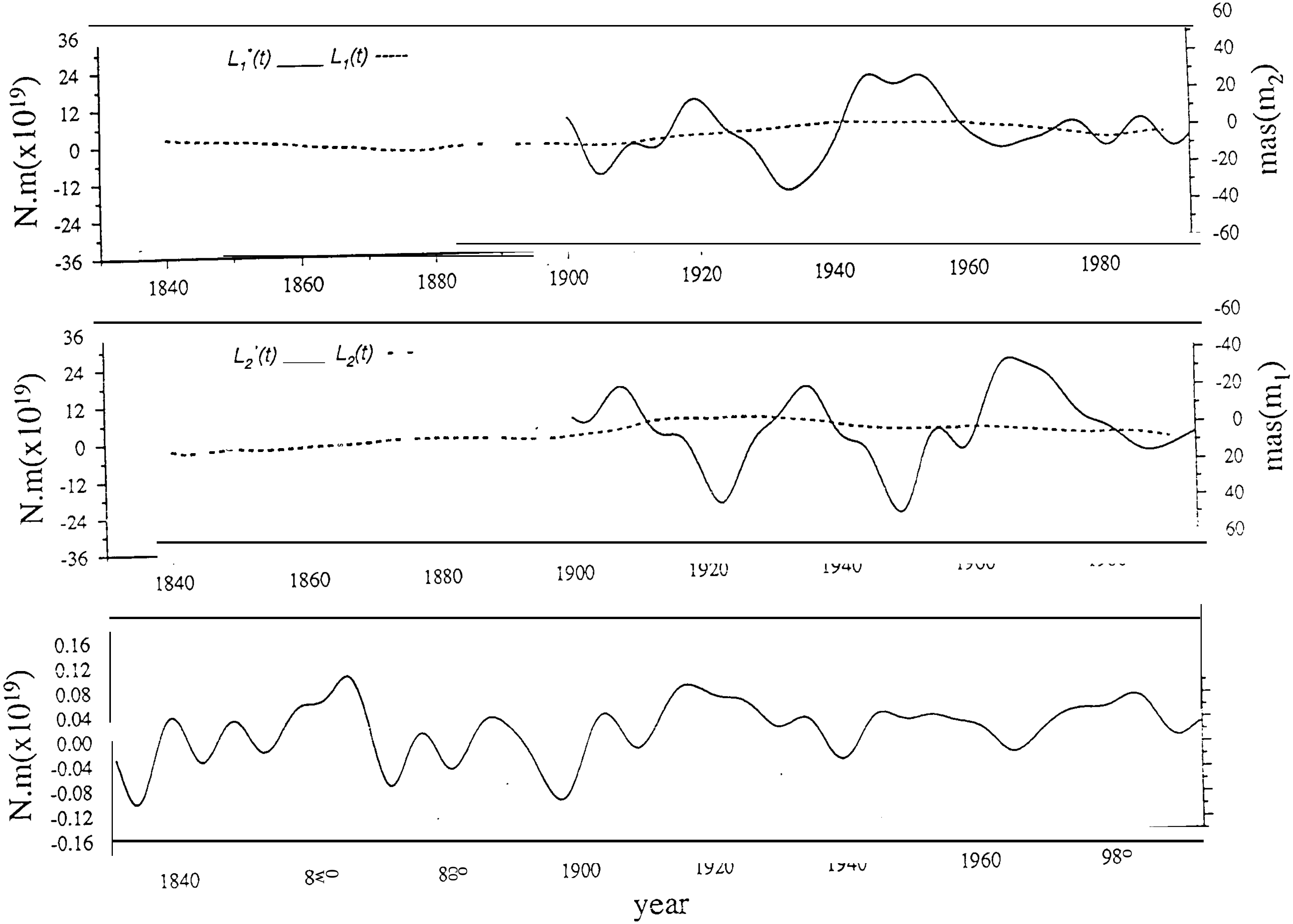

Fig. 2

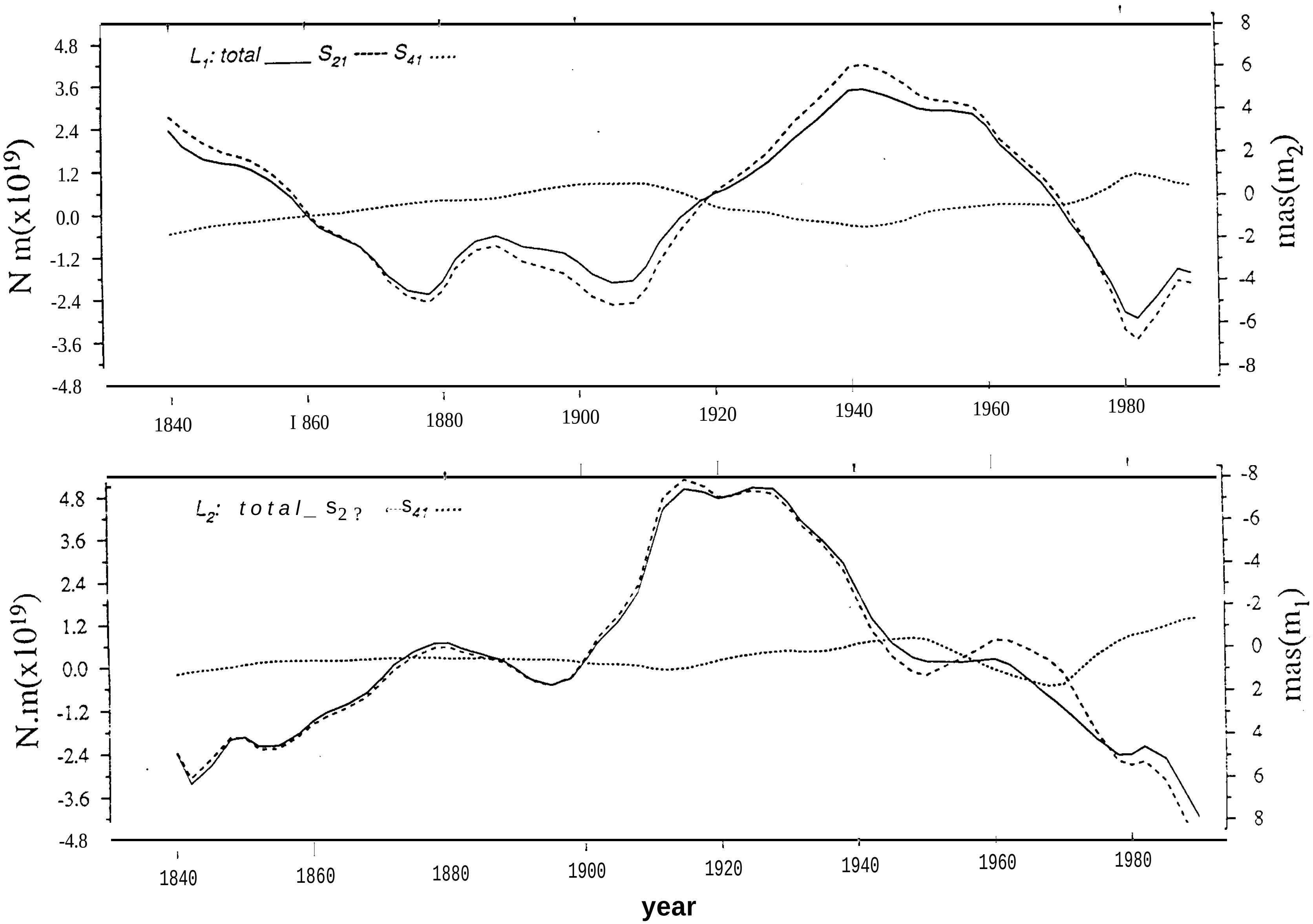
L1: total ____ S21 ----- S41 .....
N m(x10^19)
mas(m2)
4.8
3.6
2.4
1.2
0.0
-1.2
-2.4
-3.6
-4.8
8
6
4
2
0
-2
-4
-6
-8
1840
I 860
1880
1900
1920
1940
1960
1980
L2: total_ S2? S41 .....
N.m(x10^19)
mas(m1)
1840
1860
1880
1900
1920
1940
1960
1980
year

Fig 3.