Reliability Prediction Using Flight Experience

Weibull Adjusted Probability of Survival, WAPS Method

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The **Problem with the Constant Failure Rate (Standard) Reliability Prediction**

- **Standard Reliability Prediction**
  - Homogeneous Poisson Process
  - Failure Distribution: Exponential

- **Flight Experience**

  - **Voyager Spacecraft Reliability**
    - Crosstrapped SIC: \[ R(16 \text{ years}) > 0.5 \]
    - (2 out of 2 S/C Operational)

  - **9 Earth Orbiter S/C Reliability**
    - \[ R(5 \text{ years}) = 0.4 \]
    - *W. Bangs Study (Goddard)*

**Example**

- **Voyager 21 7-Predicted Failure Rate:** \[ \lambda = 225 \text{ E-6 Failures/hour} \]
- **Predicted Reliability of the Cross-strapped S/C:**
  - \[ R(15.5 \text{ years}) = 8 \text{ E-8} \]
  - \[ R(5 \text{ years}) = 0.06 \]

**Conclusion:**

- Reliability Predictions with MIL - HDBK - 217 Constant Failure Rates are UNREALISTIC
Introduction and Summary

Flight experience shows decreasing failure rate of Voyager spacecraft:

\[
MLEFR(t_i) = \frac{i}{\sum_{j=1}^{i} t_j}
\]
Cumulative Flight Average Failure Rate Compared to the MIL-HDBK-217-Predicted “Constant” Failure Rate; Data from 48 Orbiter S/C

Flight Average Failure Rate =
Number of Flight Failures/Cumulative Times to Failure
(Calculations Independent of Failure Distributions)

MIL-HDBK-215-Predicted
Assumed Average 500 E-6 failures/hour

Average Flight Failure Rate
“Constant” MIL-HDBK-217 Failure Rate

Flight Time (hours)
Facts Learned from the Flight Experience:

1. Hazard rates ("failure rates") \(\neq\) constant

2. Reliability predictions are:
   - Overly pessimistic for the case of decreasing hazard rates.
   - Overly optimistic for the case of increasing hazard rates.

3. Reliability Prediction models must match with actual flight histories.

4. Standard probability concepts for reliability modeling must apply.

5. It is desirable and convenient to use an existing part or assembly failure rate database.

6. Exponential reliability function cannot be used, unless modified to represent actual data.

7. Data from 132 orbiter and 9 interplanetary S/C fitted successfully with two-parameter Weibull distributions.

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"failure rate" = popular term for the hazard rate

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Weibull Adjusted Probability of Survival Method, WAPS:

1. A new method for realistic prediction of spacecraft or other hardware reliability developed at the Jet Propulsion Laboratory (JPL), California Institute of Technology.

2. Modifies classical reliability modeling utilizing actual S/C failure data histories and the valuable information on component constant failure rates of all failure modes of MIL-HDBK-217 or other data bases.

3. Is not a substitute for the Physics of Failure method, as it does not identify or predict particular failure modes.

4. Provides a simple way to modify the classical exponential reliability function to convert it to the Weibull reliability function.

5. Uses standard modeling techniques to model complex subsystems and systems.

6. Requires knowledge of the similar system (reference system) flight history.

7. Modifies reliability of low level assemblies (individual blocks) in reliability block diagrams as follows:

\[ R(t) = \exp(-\lambda \cdot t) \quad \Rightarrow \quad R(t) = \exp[-K(\beta) \cdot \lambda \cdot t^\beta] \]
Flight Failure Data Analysis

Actual spacecraft data is gathered:

- Each electrical part failure is recorded against its time of occurrence.
- This technique assumes that the analyzed spacecraft is in series configuration (any redundancies are disregarded).

The result: information on Weibull parameters of a series system.

\[
\beta_0, \eta_1, \beta_0, \eta_2, \beta_0, \eta_i, \beta_0, \eta_m, \beta_0, \eta_1, \beta_0, \eta_2, \beta_0, \eta_i, \beta_0, \eta_m
\]

Voyager 1 Voyager 2

\[
\eta_0 \quad \eta
\]

Two or more spacecraft of the same spacecraft type analyzed simultaneously: the spacecraft are modeled as being in series, which allows compilation of data from the entire group:

\[
\eta_{\text{SingleS/C}} = S^3 \cdot \eta_{\text{KS/C}}
\]

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Flight Failure Data Analysis

Flight failure data analysis or any other field data analysis of a system other than spacecraft, produces the following information:

a. Shape parameter, $\beta_0$, reference shape parameter:
   - of the series system configuration, and
   - of each individual low-level assembly.

b. Scale parameter, $\eta_0$, referencescaleparameter:
   - of the series system configuration.

Failure rate prediction (classical) will produce:

c. Failure rate, $\lambda_0$ of the reference system in series configuration.
Derivation of the Weibull-Adjusted Probability of Survival (WAPS) Method

Based on flight failure data history, the system reliability follows the Weibull distribution. Weibull reliability function is modified to contain the database-predicted failure rate, \(\lambda\).

\[
R(t) = \exp \left[ -\left( \frac{t}{\eta} \right)^\beta \right]
\]

can be transformed as follows:

\[
\exp \left[ -\left( \frac{1}{\eta^\beta \lambda} \right) \cdot \lambda \cdot t^\beta \right] = \exp[-K(\eta, \beta, \lambda) \lambda \cdot t^\beta],
\]

Where:

\[
K(\eta, \beta, \lambda) = \left( \frac{1}{\eta^\beta \lambda} \right) \rightarrow \text{conversion function}
\]

Also: \(\eta = f(\beta, \lambda)\)
Derivation of the WAPS Method, cont.

Abbreviated: \( R(t) = \exp[-K(\beta) \cdot \lambda \cdot t^\beta] \)

Reliability of the reference series spacecraft \( \beta_0 \) and \( \eta_0 \), and the 217-predicted total series system failure rate, \( \lambda_0 \):

\[
R_0(t) = \exp[-K_0(\beta_0) \cdot \lambda_0 \cdot t^{\beta_0}]
\]

and

\[
K_0(\beta_0) = \frac{1}{\eta_0^{\beta_0} \cdot \lambda_0}
\]

Reliability of one single lower level assembly of the same or different spacecraft, in case of equal shape parameters is written as:

\[
R_i(t) = \exp[-K_i(\beta_0) \cdot \lambda_i \cdot t^{\beta_0}]
\]

and

\[
K_i(\beta_0) = \frac{1}{\eta_i^{\beta_0} \cdot \lambda_i} = K_0(\beta_0) \text{ as it will be shown below.}
\]
Derivation of the WAPS Method, cont.

Assume a system:

\[ \eta_1, \lambda_1 \rightarrow \eta_2, \lambda_2 \rightarrow \eta_3, \lambda_3 \rightarrow \ldots \rightarrow \eta_i, \lambda_i \rightarrow \ldots \rightarrow \eta_n, \lambda_n \]

\[ \lambda_0 = \Sigma \lambda_i \quad (i = 1 \text{ through } n) \]

Reliability of the above system:

\[ R_o(t) = R_1(t) \cdot R_2(t) \cdot R_3(t) \cdot \ldots \cdot R_i(t) \cdot \ldots \cdot R_n(t) = \prod_{i=1}^{n} R_i(t) \]

\[ \left( \frac{t}{h_0} \right)^{b_0} = \left( \frac{t}{h_1} \right)^{b_0} + \left( \frac{t}{h_2} \right)^{b_0} + \ldots + \left( \frac{t}{h_i} \right)^{b_0} + \ldots + \left( \frac{t}{h_n} \right)^{b_0} = t^{b_0} \cdot \sum_{i=1}^{n} \frac{1}{h_i^{b_0}} \]

Therefore:

\[ \frac{1}{h_0^{b_0}} = \sum_{i=1}^{n} \frac{1}{h_i^{b_0}} \]

The predicted failure rate for the total system made out of \( n \) "boxes" would produce a well-known expression:

\[ \lambda_0 = \sum_{i=1}^{n} \lambda_i \]
Derivation of the WAPS Method, cont.

If the system is made of \( m \) assemblies with \( \lambda_i \):

\[
\lambda_0 = \sum \lambda_i = m\lambda_i \quad \rightarrow \quad m = \frac{\lambda_0}{\lambda_i}
\]

System reliability will then be:

\[
R_o(t) = R_1(t) \cdot R_2(t) \cdot R_3(t) \ldots R_i(t) \ldots \cdot R_n(t) = \prod_{i=1}^{m} R_i(t)
\]

\[
\left( \frac{t}{\eta_0} \right)^{\beta_0} = \left( \frac{t}{\eta_1} \right)^{\beta_0} + \left( \frac{t}{\eta_i} \right)^{\beta_0} + \ldots + \left( \frac{t}{\eta_i} \right)^{\beta_0} = t^{\beta_0} \cdot \frac{m}{\eta_i^{\beta_0}}
\]

Therefore:

\[
\frac{1}{\eta_0^{\beta_0}} = \frac{m}{\eta_i^{\beta_0}}
\]
Derivation of the WAPS Method, cont.

\[ m = \frac{\lambda_0}{\lambda_i}, \]

therefore:

\[ \frac{1}{\eta_0^{\beta_0}} = \frac{\lambda_0}{\lambda_i} \cdot \frac{1}{\eta_i^{\beta_0}}, \]

and

\[ \frac{1}{\eta_0^{\beta_0} \cdot \lambda_0} = \frac{1}{\eta_i^{\beta_0} \cdot \lambda_i}. \]

\( R_i(t) = \exp[-K_i(\beta_0) \cdot \lambda_i \cdot t^{\beta_0}] \)

From above:

\[ K_i(\beta_0) = \frac{1}{\eta_i^{\beta_0} \lambda_i} = \frac{1}{\eta_0^{\beta_0} \lambda_0} \]

Therefore:

\[ K_i(\beta_0) = K_0(\beta_0) \]

and

\[ R_i(t) = \exp[-K_0(\beta_0) \cdot \lambda_i \cdot t^{\beta_0}] \]

It can be demonstrated that:

\[ K(\beta) = K_0(\beta) \left( \frac{\beta - 1}{\beta_0 - 1} \right) \]

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Derivation of the WAPS Method, cont.

**Derivation of **$K(\beta)$**: Consider two different systems with: $\lambda_i = \lambda_0$, and $\beta \neq \beta_0$.

a. A reference system: $\beta_0$, $\eta_0(\beta_0, \lambda_0)$, and $\lambda_0$, $K_0(\beta_0)$ or $K_0(\eta, \beta_0, \lambda_0)$;

b. A different system: $\beta$, $\eta(\beta, \lambda_0)$, and the same failure rate, $\lambda_0$, having a conversion function $K(\beta)$.
Derivation of the WAPS Method, cont.

The average failure rates: \( \frac{X^{\beta-1}}{\eta^\beta} = q \cdot \lambda_0 \), and \( \frac{X^{\beta_0-1}}{\eta_0^\beta} = q \cdot \lambda_0 \)

Knowing: \( K_0(\beta_0) = \frac{1}{\lambda_0 \cdot \eta_0^{\beta_0}} \) and \( K(\beta) = \frac{1}{\lambda_0 \cdot \eta^\beta} \)

\( X^{\beta-1} = \frac{q}{K(\beta)} \) or \( (\beta - 1) \ln(X) = \ln(q) - \ln[K(\beta)] \), and

\( X^{\beta_0-1} = \frac{q}{K_0(\beta_0)} \), or \( (\beta_0 - 1) \ln(X) = \ln(q) - \ln[K_0(\beta_0)] \)

From the same figure: \( \frac{Y^{\beta_0-1}}{\eta_0^{\beta_0}} = \lambda_0 \), and \( \frac{Z^{\beta-1}}{\eta^\beta} = \lambda_0 \). \( Y^{\beta_0-1} = \frac{1}{K_0(\beta_0)} \), and \( Z^{\beta-1} = \frac{1}{K(\beta)} \)

Expressing the slopes of the lines in terms of the respective failure rates and times, we arrive at:

\[ \beta_0 - 1 = \frac{\ln(q \cdot \lambda_0) - \ln(\lambda_0)}{\ln(Y) - \ln(X)} \Rightarrow \beta_0 - 1 = \frac{\ln(q)}{\ln(Y) - \ln(X)} \]

\[ \beta - 1 = \frac{\ln(q \cdot \lambda_0) - \ln(\lambda_0)}{\ln(Z) - \ln(X)} \Rightarrow \beta - 1 = \frac{\ln(q)}{\ln(Z) - \ln(X)} \]

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Derivation of the WAPS Method, cont.

Rewritten and combined, the above equations produce a set:

\[ \ln(q) = (\beta_0 - 1) \cdot \ln(X) - \ln[K_0(\beta_0)] \]

\[ \ln(q) = (\beta_0 - 1) \cdot \left[ \frac{-\ln[K_0(\beta_0)]}{(\beta_0 - 1)} - \ln(X) \right] \]

\[ \ln(q) = (\beta - 1) \cdot \left[ \frac{-\ln[K(\beta)]}{(\beta - 1)} - \ln(X) \right] \]

From the above, the solution for \( K_i(\beta) \) is:

Since: \( K_i(\beta) = K(\beta) \),

If needed: \( \eta_i(\beta, \lambda_i) = \left( \frac{\beta_i}{\beta_0-1} \right)^{1/b} \)

The Weibull-Adjusted Probability of Survival of a single assembly of a new spacecraft is:

\[ R_i(t) = \exp\left(-[K_0(\beta_0)]^{\beta_0-1} \lambda_i \right) \]
Reference Spacecraft Data Analysis

General Considerations and Assumptions

1. Regular time-to-failure data recording and analysis.

2. Every failure that could cause a system malfunction or loss (if not mitigated by redundancy, fault protection, workarounds, and other measures) is counted as a loss of one series (single-string) “non-repairable” system.

   - The total number of failures, \(r\)
   - The total number of the spacecraft in the group, \(n\).

3. The information on the number of the total failure count up to a 100% failed is not available, and therefore the total number of the spacecraft in the “group”, \(n\), is also not known in case of a living S/C.

4. The general assumption: the very next failure could be the last possible, the fatal (100% loss) failure so that the number of the units in the “group”, \(n\), is equal to the number of failures, \(r\), plus one. Result: a conservative parameter estimate.

5. Failure terminated yield less conservative Weibull parameters.

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Graphical Method for Data Analysis

\[ f(x) = -5.686355 \times 10^{-1} x + -3.904842 \times 10^0 \]
\[ R^2 = 9.013835 \times 10^{-1} \]

\[ \beta = 0.4303645 \]
\[ \eta = 102,775 \text{ hours} \]
\[ 0.316 < \beta < 0.585 \]
\[ 75,089 < \eta < 174,044 \text{ hours} \]

\[ \text{Slope} = \beta - 1 \Rightarrow \beta = \text{Slope} - 1; \eta = \left( \frac{1}{\beta} \sum_{i=1}^{r} t_i^\beta + T^\beta \right)^{\frac{1}{\beta}}; T = \text{end of observation time.} \]

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Assumptions in Data Analysis

- All relevant failures of the electronic and electromechanical hardware are counted, regardless of redundancy: all subsystems, assemblies, and parts are modeled in series.

- Every failure constitutes terminal failure of a series system. Redundancies, work-arounds, allow use of one system multiple times: multiple series systems.

One Redundant and/or Fault-Protected System

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Assumptions in Data Analysis, cont.

- Values of Weibull parameters are determined through the end of the observation period.

- All assemblies of the same spacecraft are assumed to belong to the same Weibull distribution, thus all have the same value of the Weibull shape parameter, $\beta$.

- Scale parameters determined for the series configuration are related to that configuration only.

- From $\lambda_0, \beta_0,$ and $\eta_0$: determined the factor function $K_0(\beta_0)$. 
Failure Scoring Criteria

Scored Failures:

- Failures attributable to electrical assemblies and/or parts;
- Failures of electromechanical devices not clearly attributed to mechanical effects.
- Failures not directly recognizable as failures caused by design deficiency.

Excluded (not scored) are the following flight failures:

- Failures attributed to computer software;
- Failures of mechanical parts, regardless of cause;
- Failures diagnosed to be caused by environmental effects;
- Failures attributed to the operator error;
- Failures attributed to design deficiency (common cause failures);
- Failures attributed to the workmanship defects.
Other Analysis Methods

Calculation of the cumulative hazard. Can be also used for verification of graphical analysis results.

Analytical Maximum Likelihood Estimate of Weibull parameters: Yields conservative values of shape parameters.

Hazard plotting methods: Also conservative shape parameter values.

The choice of the analysis method depends upon the analyst preference. However, plotting of the average failure rate is found the most convenient for later comparisons.
Reliability Modeling for Assessment, Prediction, or Trade-off Studies

From the equipment (spacecraft) functional block diagrams and schematics, prepare reliability block diagrams (RBD’s) following standard reliability practices.

Calculate the “constant” failure rates of each individual block, $\lambda_i$, using MIL-HDBK-217 or other database. The same release level/letter, E or F, of the MIL-HDBK-217 prediction must be used for both the reference and the new S/C system.

Calculate reliability of an individual block as Weibull-adjusted:

$$R_i(t) = \exp\left\{-\left[K_0(\beta_0)\right]^{\beta_0-1}\lambda_i \cdot t^\beta\right\}$$

Calculate system reliability. Standby redundancy replaced by the active (integrals with the Weibull distribution non-converging)

**Need:** $K_0(\beta_0)$, and an estimate of $\beta$.

$\beta$ estimated based on design and manufacturing similarity, and type of WC.
Distribution of Shape Parameters

Orbiter S/C Shape Parameters, $\beta$: 92 Orbiter S/C

Interplanetary S/C $\beta$:

Mariner69: $\beta = 0.546$
Mariner71: $\beta = 0.504$
Mariner73: $\beta = 0.434$
Viking: $\beta = 0.424$
Voyager-: $\beta = 0.43$
Mars Observer: $\beta = 0.552$
Magellan: $\beta = 0.451$
WAPS Method Applied to Magellan and Orbiter S/C

From Magellan functional block diagrams and the respective parts lists, the reliability block diagrams were prepared for each of the subassemblies, and for the overall spacecraft. Propulsion omitted (mechanical parts)

Predictions:
Space Flight environment, 35 °C chassis temperature, and 50 % default electrical stresses.

Reliability block diagram for the overall Magellan electrical system:

Flight experience, Magellan Weibull parameters were found to be:

\[ \beta_F = 0.4489 \quad \eta_F = 27132 \text{ hour} \]

Flight single-string Magellan Reliability is calculated then from:

\[ R_{FSS}(t) = \exp\left(-\frac{t}{\eta_F}\right) \]
Reliability comparison for the Magellan Single String (Series) Configuration

Assumed Shape Parameter $\beta = 0.52$
Percent Error Between Magellan WAPS and the Actual Flight Reliability

The percent difference in reliability prediction and flight experience of the single-string configuration:

\[ \Delta R(t) \]

\[ \beta = 0.52 \]

![Graph showing the percent difference over flight time.](image)
Comparison of the Actual Flight to the WAPS-Calculated Average Failure Rate
Data from 48 Orbiter S/C

- Flight Average Failure Rate, AFR
- Flight AFR, 95% UCL
- Flight AFR, 95% LCL
- WAPS-Calculated AFR, $b = 0.53$
- Constant Failure Rate
- WAPS, 95% UCL on $b$
- WAPS, 95% LCL on $b$

95% A UCL on the Flight AFR

WAPS 95% UCL on beta

Flight AFR

WAPS, 95% UCL on b

WAPS-Calculated, AFR, (MIL-HDBK-217)

Constant Failure Rate

WAPS, 95% LCL on b

WAPS, 95% LCL on $b$

95% LCL on beta

Flight AFR

WAPS, 95% UCL on b

WAPS-Calculated, AFR, $\beta = 0.53$

95% LCL on $b$

Flight AFR

WAPS, 95% LCL on $b$

WAPS-Calculated, AFR, $\beta = 0.53$

95% LCL on $b$

Flight AFR

WAPS, 95% UCL on $b$

WAPS-Calculated, AFR, $\beta = 0.53$

95% LCL on $b$

Flight AFR

WAPS, 95% LCL on $b$

WAPS-Calculated, AFR, $\beta = 0.53$

95% LCL on $b$
WAPS-Calculated Reliability Compared to the Flight Experience and the Exponential Reliability
95% Confidence Limits on the WAPS-Calculated Reliability Included

- $R(t)_{\text{Flight}}$
- $R(t)_{\text{Const. Failure Rate}}$
- $R(t)_{\text{WAPS-Calculated}}$
- $R(t)_{\text{WAPS-Calculated, 95% Upper}}$
- $R(t)_{\text{WAPS-Calculated, 95% Lower}}$
- $R(t)_{\text{Constant Failure Rate, Cross-strapped}}$
- $R(t)_{\text{WAPS-Calculated, 95% Upper, Cross-strapped}}$
- $R(t)_{\text{WAPS-Calculated, 95% Lower, Cross-strapped}}$
- $R(t)_{\text{Flight Cross-Strapped}}$

○ ● Private Data; Orbiter Reliability Calculations at 3 and 5 years, Respectively

Flight Time (hours)