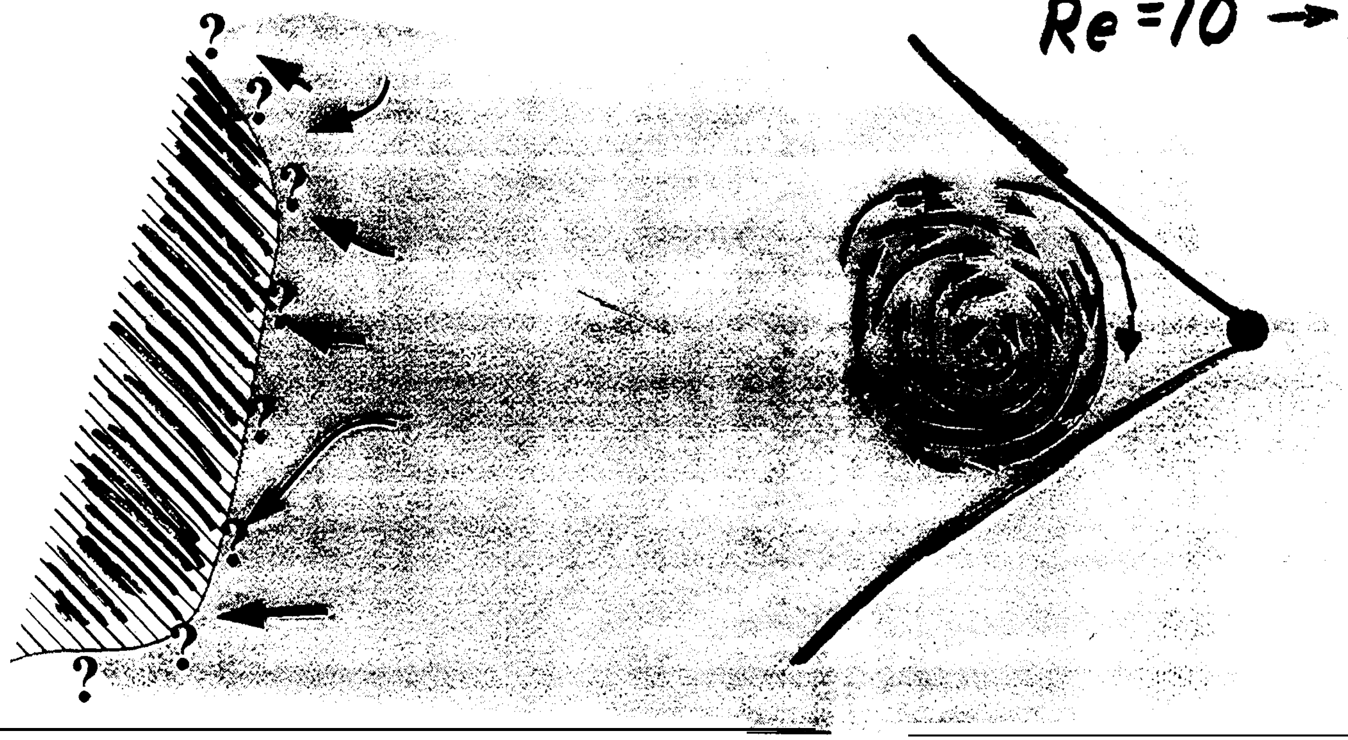
# MODEL FOR PREDICTION

$$\frac{\partial v}{\partial t} + v\nabla v = -\frac{1}{\rho}\nabla\rho + \upsilon\nabla^2 v$$

$$\nabla \bullet v = 0$$

$Re = 10^4 \rightarrow N = 10^8$

$Re = 10^6 \rightarrow N = 10^{13}$



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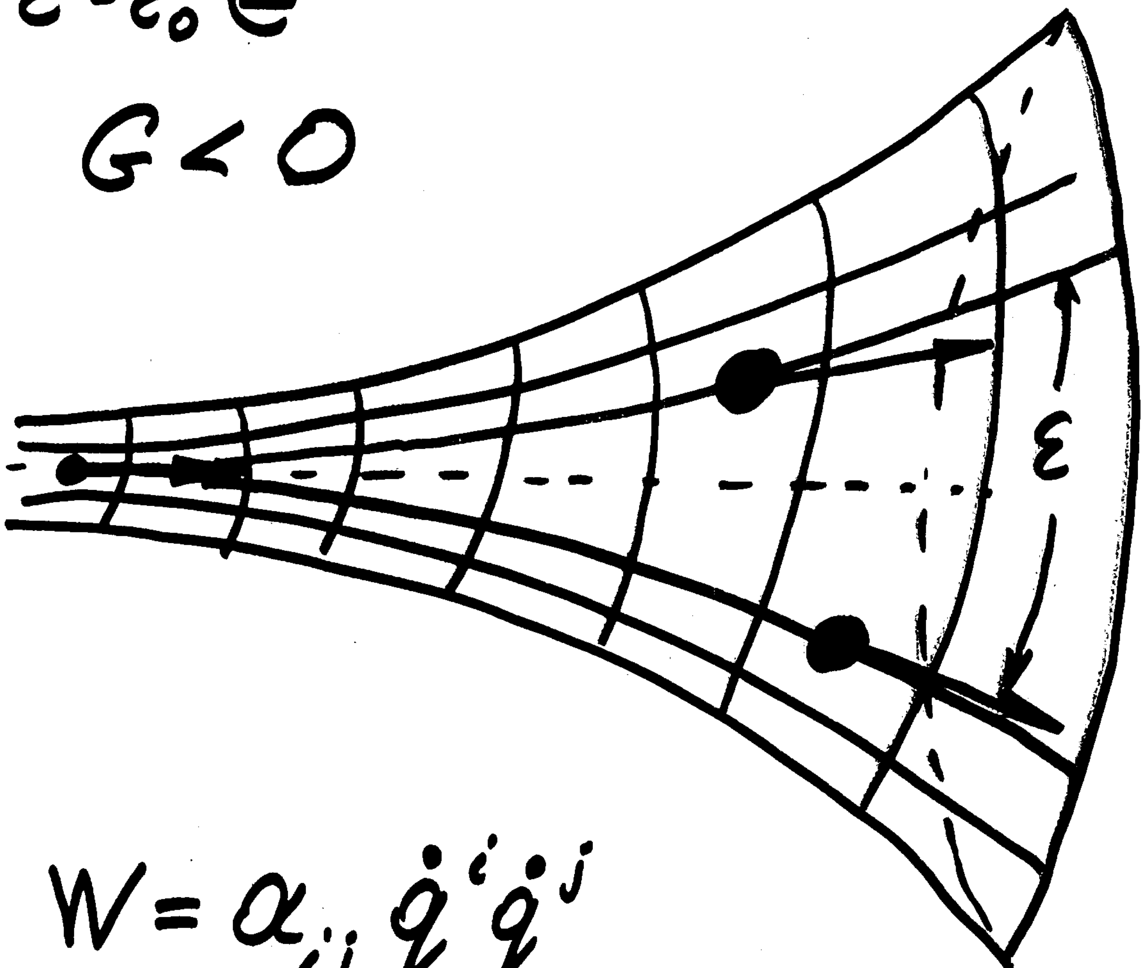
# Instability and Unpredictable Solutions

## 1. Orbital Instability

$$\varepsilon = \varepsilon_0 e^{\sqrt{-G}\, t}$$

$$G < 0$$



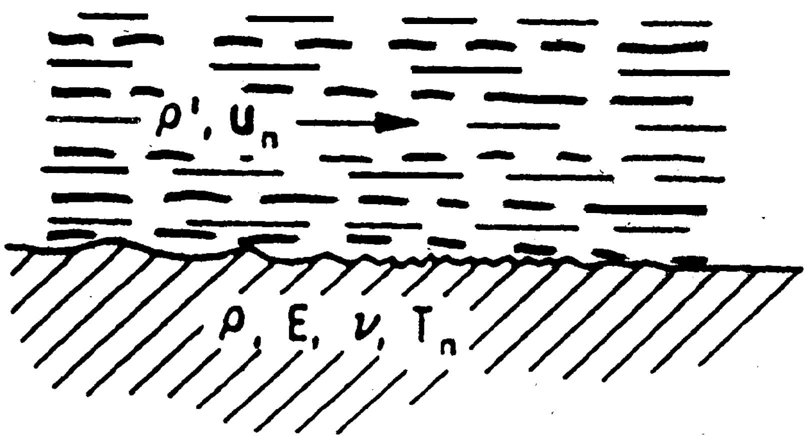

$$W = \alpha_{ij} \dot{q}^i \dot{q}^j$$

$$g_{ij} = \alpha_{ij}$$

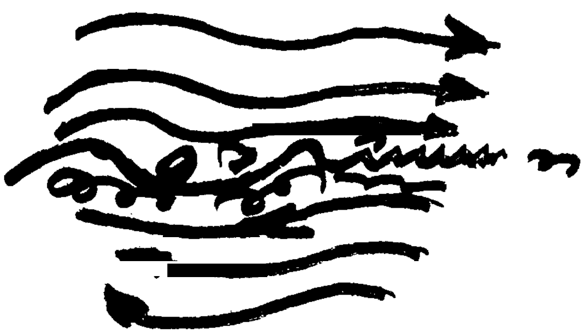
$$\boxed{\begin{aligned} \frac{\partial L}{\partial \varepsilon} &\equiv 0 \\ Q_\varepsilon &\equiv 0 \end{aligned}}$$

# Warping of Boundaries

$$T_{nn} < \frac{\rho_1 \rho_2}{(\rho_1 + \rho_2)^2} v_n^2 - \frac{E}{2(1+\nu)}$$

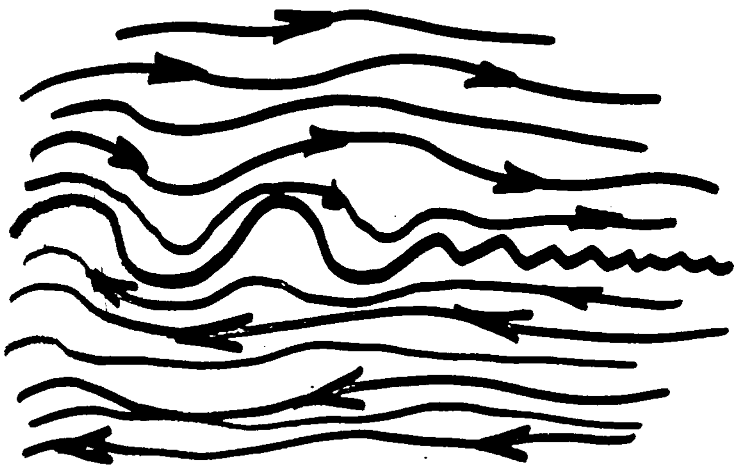


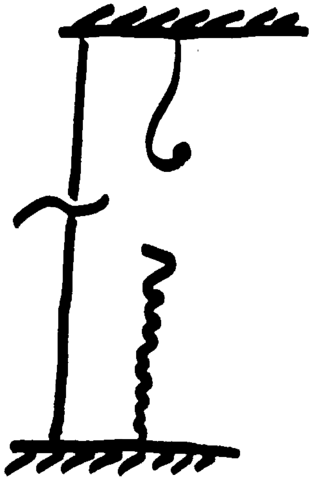

# Eulerian Turbulence if $E \longrightarrow 0$

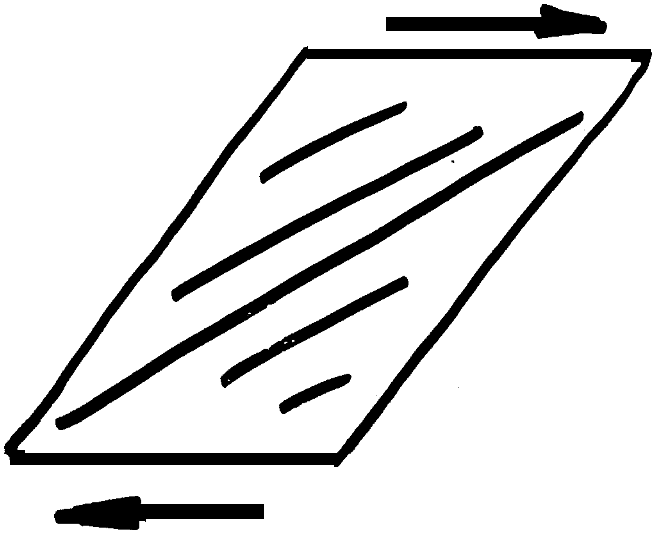


$$\lambda = \frac{1}{2}(u_2 - u_1)(1 \pm i)$$

## 2. Hadamard's Instability (Failure of Hyperbolicity)







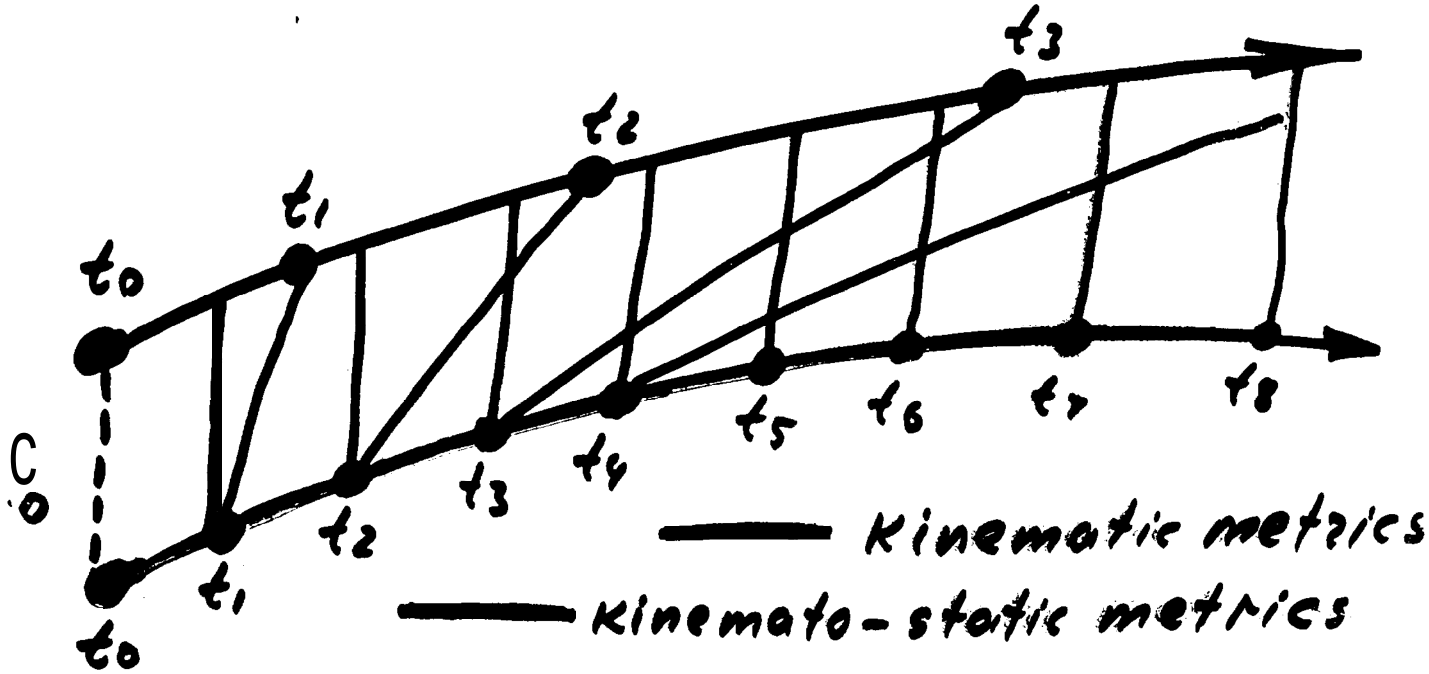
$$\frac{\partial^2 u}{\partial t^2} \quad \lambda \frac{\partial^2 u}{\partial x^2} = 0$$

$$\lambda < 0$$

$$\boxed{\begin{gathered} \frac{\partial L}{\partial \Omega} \equiv 0 \\ \mathcal{Q}_{\Omega} \equiv 0 \end{gathered}}$$

# Instability is not an invariant of motion

## 1. Instability and metrics of configuration space




$$\Pi = -x + \frac{1}{2}y^2$$

$$\begin{cases} x = \frac{1}{2}t^2 + At + B \\ y = C\sin(t + D) \end{cases}$$

Unperturbed motion

$x = \frac{1}{2}t^2 + t, \quad y = 0$

is stable in ——

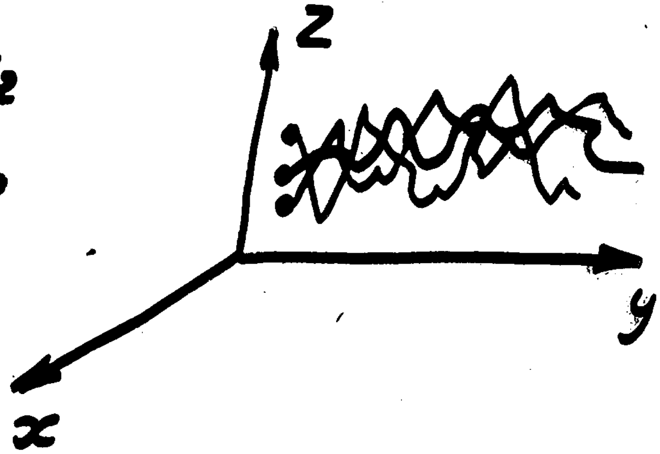
and unstable in ——

## 2. Instability and Frame of Reference

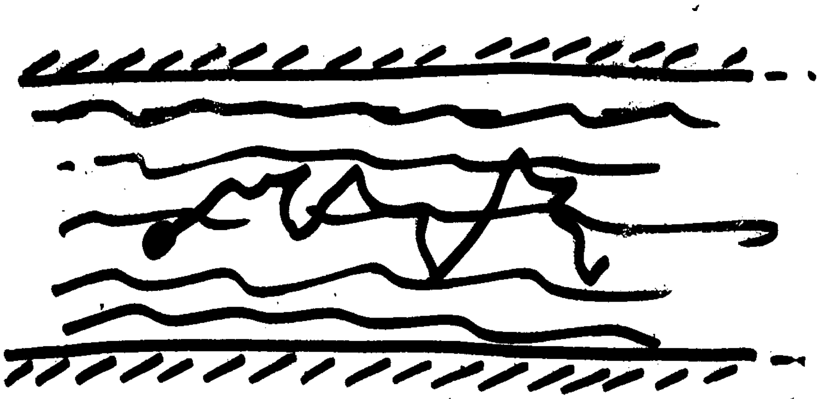
$$v_1 = A \sin x_3 + C \cos x_2$$

$$v_2 = B \sin x_1 + A \cos x_3$$

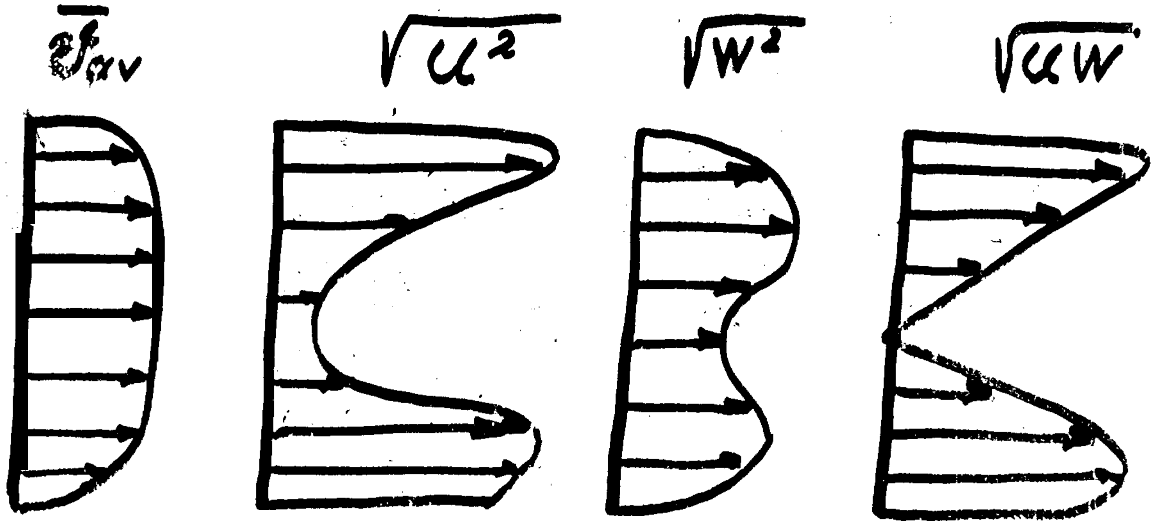
$$v_3 = C \sin x_2 + B \cos x_1$$




## 3. Instability and Class of Functions



$R > R_{cr}$



# STABILIZATION PRINCIPLE

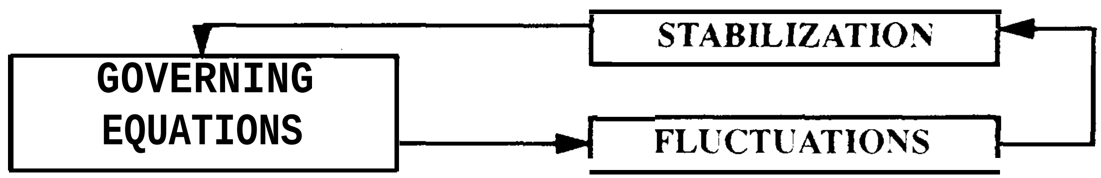
"The flows that occur in Nature must not only obey the equations of (fluid) dynamics. but also be stable." — Landau

$$\frac{\partial v}{\partial t} + v\nabla v = -\frac{1}{\rho}\nabla p + \upsilon\nabla^2 v + \boxed{v\nabla v}$$

$$\nabla \bullet v = 0$$

$$\nabla \bullet v = 0$$

Choose $v\nabla v$ in such a way that the model is stable!



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# Stabilization Principle

If

$$\lambda^+ > 0 \quad \text{at} \quad \overline{x^i x^j} \equiv 0$$

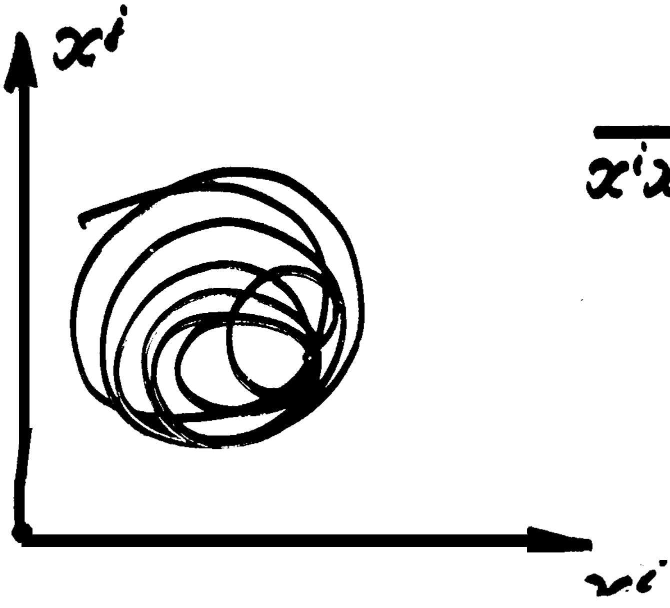
find $\overline{x^i x^j}$ from the co#wlL'4'-

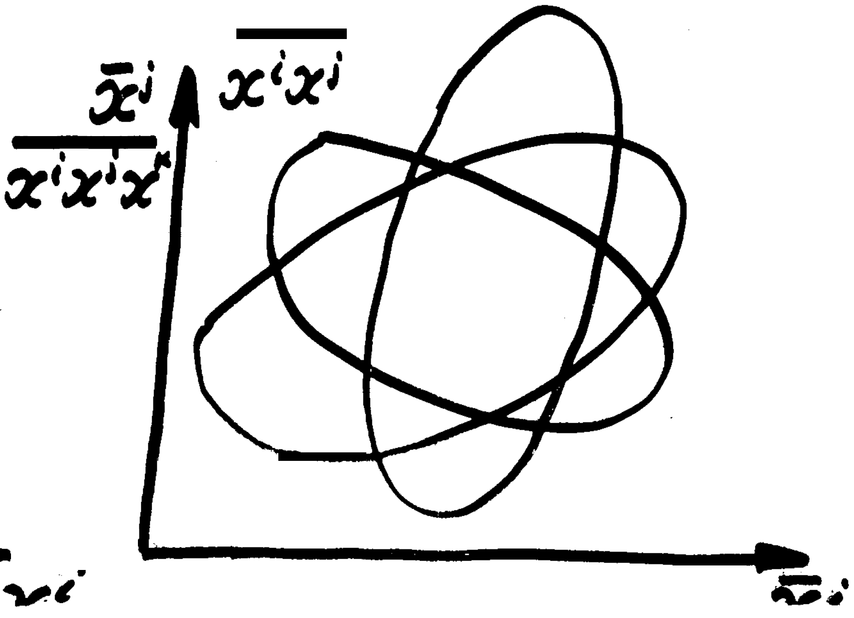
$$\lambda^+ = 0$$

$$\lambda^0 = \lambda^0$$

$$\lambda^- = \lambda^-$$

$$\overline{X^i X^j} = \varphi(x^k, \dot{x}^k, \ldots \text{etc})$$



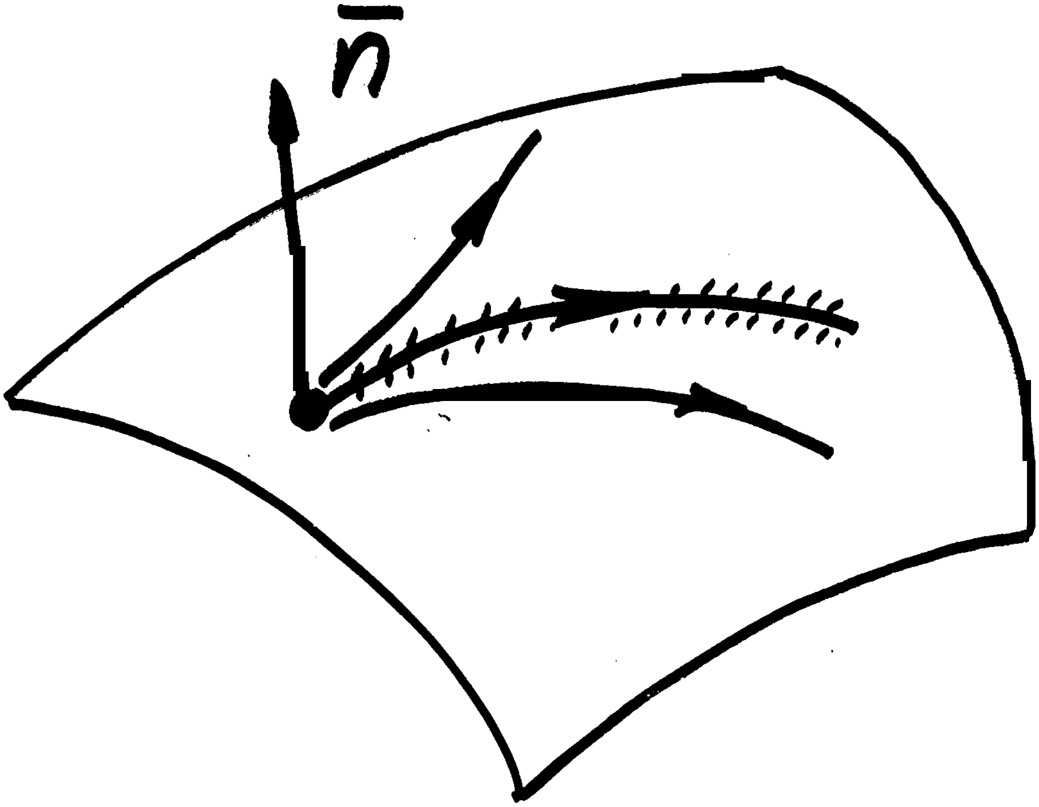



# Example 2: Potential Motions

$$\ddot{q}^{\alpha} + \Gamma^{\alpha}_{\beta\gamma}\,\dot{q}^{\beta}\dot{q}^{\gamma} = -\frac{\partial \Pi}{\partial q^{\alpha}} + Q^{\alpha}_{(i)}, \quad Q^{\alpha}_{(i)} = -\frac{\partial \Pi_{(i)}}{\partial q^{\alpha}}$$

$$G + 3\left[\frac{\nabla(\Pi + \Pi_{(i)})\cdot \bar{n}}{2W}\right]^2 +$$

$$\frac{1}{W}\left[\frac{\partial^2(\Pi+\Pi_{(i)})}{\partial q^i \partial q^j} - \Gamma_{ij}\,\frac{\partial(\Pi+\Pi_{(i)})}{\partial q^k}\right] n^i n^j = 0$$




$$W = \frac{1}{2} g_{ij}\,\dot{q}^i \dot{q}^j$$

# Smoothing out of velocity discontinuity


$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial \tilde{\tau}}{\partial y}, \qquad \tau = -\rho\,\overline{u'v'}$$

$$\bar{u}_1 = \overset{\circ}{u}_1 - \frac{\sin[k(y-H)]}{\sinh(kH)}\left(\overset{\circ}{u}_- - \frac{1}{[(8ak^2/\pi)\coth(kH)]t + 1/\overset{\circ}{u}_-}\right)$$

$$\bar{u}_2 = \overset{\circ}{u}_2 + \frac{\sinh[k(y+H)]}{\sinh(kH)}\left(\overset{\circ}{u}_- - \frac{1}{[(8ak^2/\pi)\coth(kH)t + 1/\overset{\circ}{u}_-}\right)$$

$$\overline{{u'_1}^2 + {v'_1}^2} = \frac{2ak\sinh[k(y+H)]}{\pi\sinh(kH)\tanh(kH)}\left(\frac{1}{[(8ak^2/\pi)\coth(kH)]t + 1/\overset{\circ}{u}_-}\right)$$

$$\overline{{u'_2}^2 + {v'_2}^2} = \frac{2ak\sinh[k(y-H)]}{\pi\sinh(kH)\tanh(kH)}\left(\frac{1}{[(8ak^2/\pi)\coth(kH)]t + 1/\overset{\circ}{u}_-}\right)$$

# Higher Order Approximations

$$\boxed{\dot{x}_i = \alpha_{ij}\, x_j + b_{jmi}\, x_j\, x_m}$$

$$\overline{AB} = \bar{A}\,\bar{B} + \overline{A'B'}$$
$$A = \bar{A} + A'$$
$$B = \bar{B} + B'$$

$$\dot{\bar{x}}_i = \alpha_{ij}\,\bar{x}_j + b_{jmi}\left(\bar{x}_j\,\bar{x}_m + \overline{x_j' x_m'}\right)$$

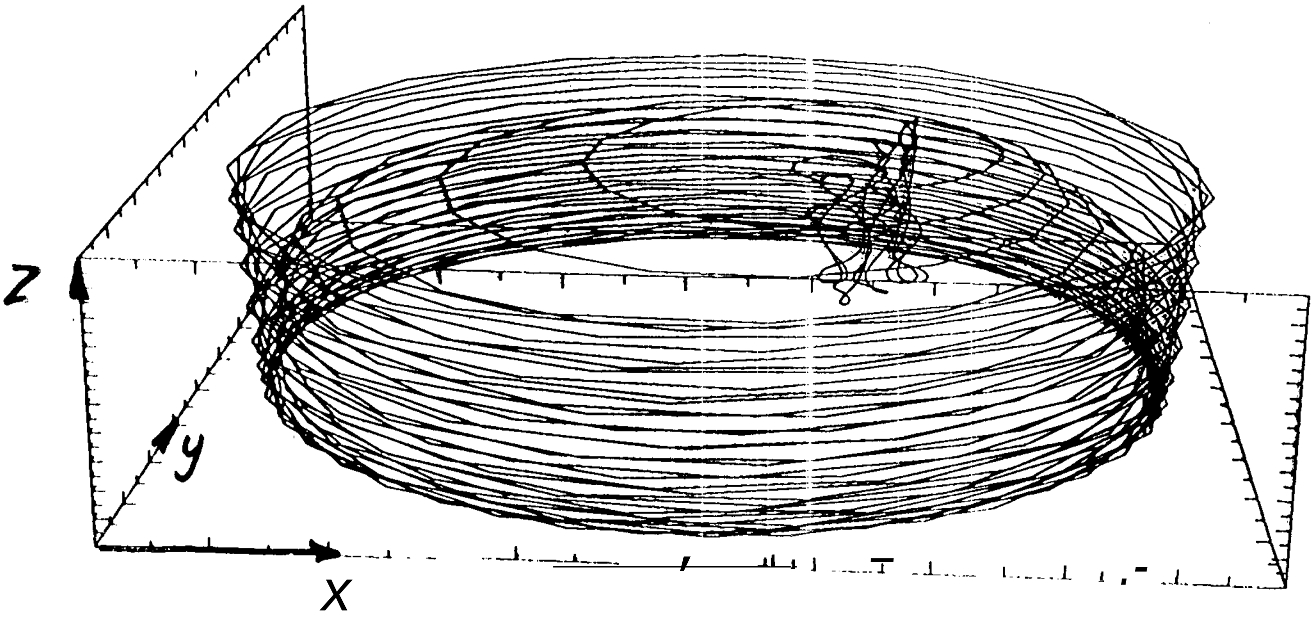
$$\dot{\overline{x_i' x_k'}} = \alpha_{ij}\,\overline{x_j' x_k'} + \alpha_{jk}\,\overline{x_j' x_i'} +$$

$$+ b_{jmi}\left(\bar{x}_j\,\overline{x_m' x_k'} + \bar{x}_m\,\overline{x_k' x_j'} + \bar{x}_k\,\overline{x_j' x_m'}\right) +$$

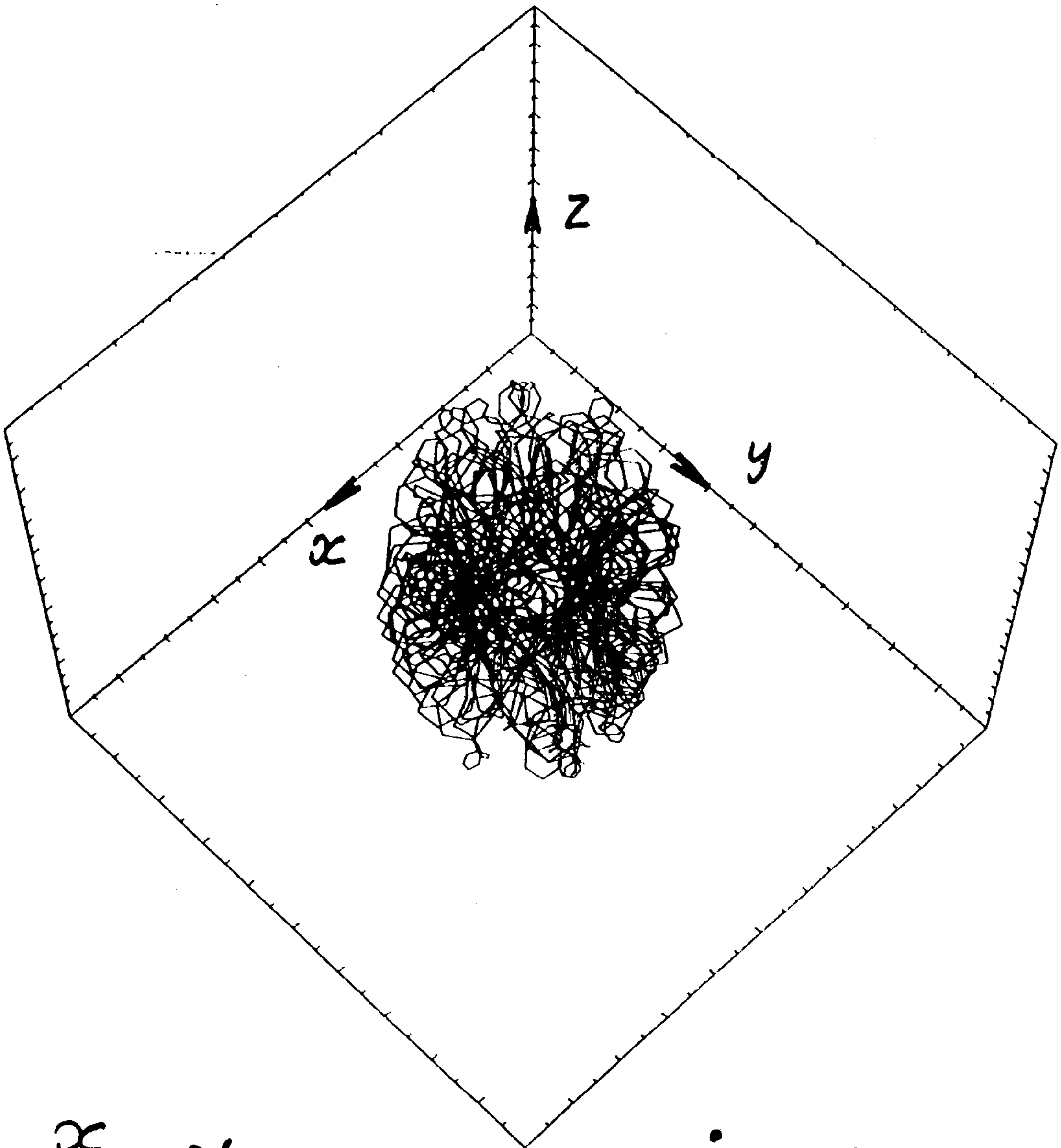
$$+ b_{jmk}\left(\bar{x}_j\,\overline{x_m' x_i} + \bar{x}_m\,\overline{x_i' x_j'} + \bar{x}_i\,\overline{x_j' x_m'}\right) +$$

$$+ b_{jmi}\,\overline{x_k' x_j' x_m'} + b_{jmk}\,\overline{x_i' x_j' x_m'}$$

# The Mean Motion



# Charged Particle in a Uniform Magnetic Field




$$\dot{v}_x = -\frac{x}{r^3} - v_y$$

$$\dot{v}_y = \frac{y}{r^3} + v_x$$

$$\dot{v}_z = -\frac{z}{r^3}$$

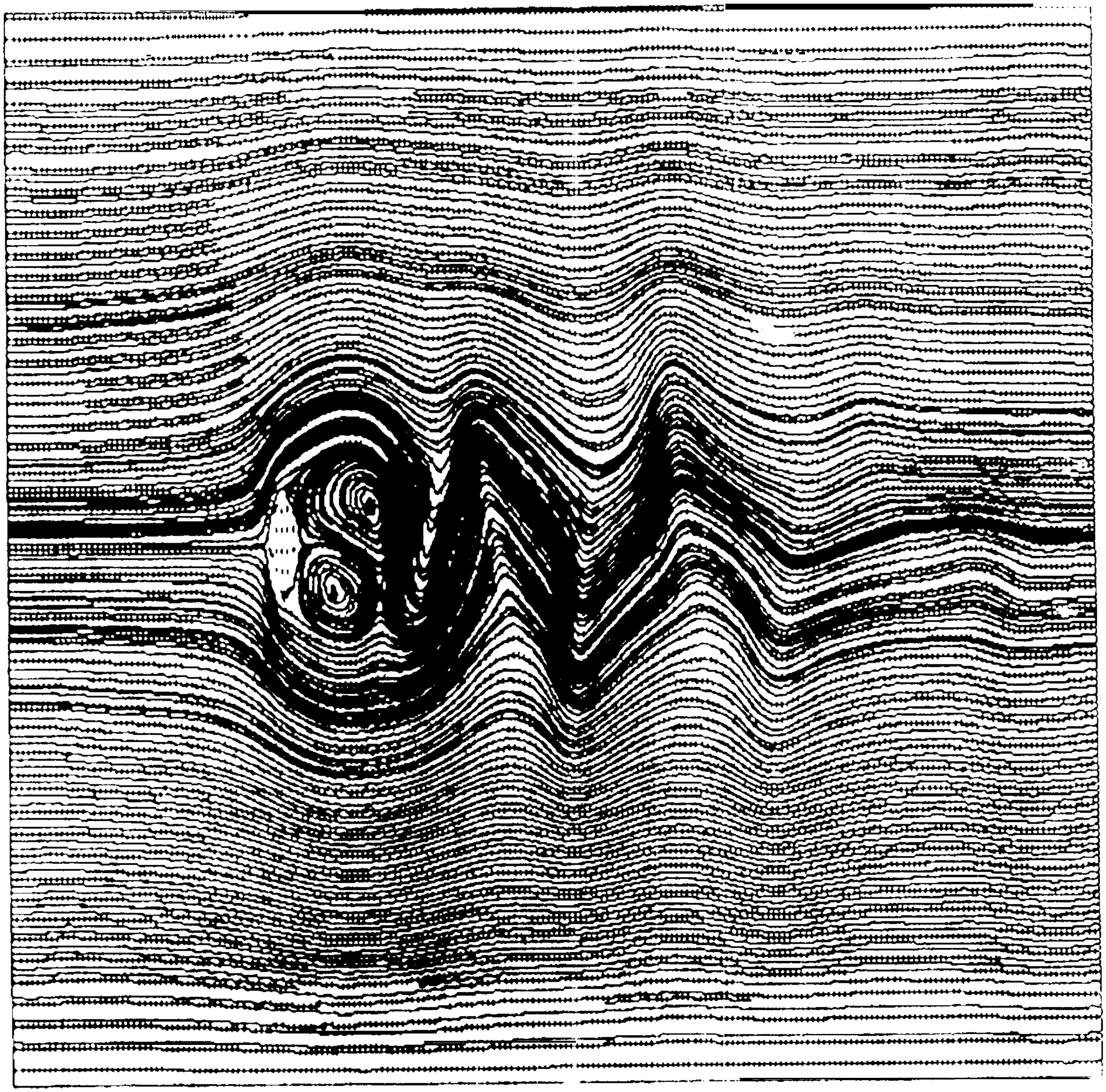
$$\dot{x} = v_x$$

$$\dot{y} = v_y$$

$$\dot{z} = v_z$$

$$r^2 = x^2 + y^2 + z^2$$

STABILIZED FLOW PAST CYLINDER Re = 500000



T = 52

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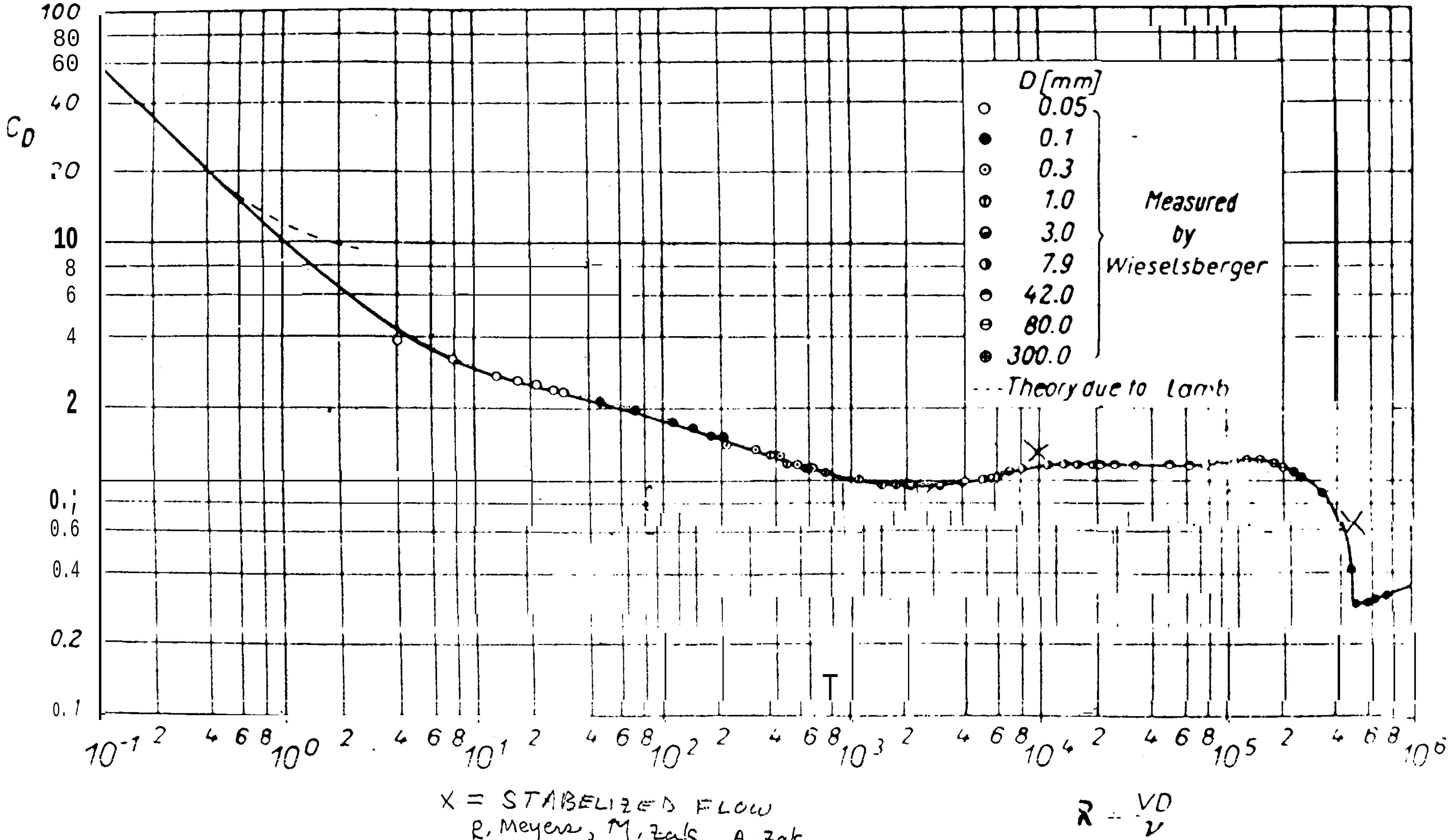


X = STABELIZED FLOW
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Fig. 1.4. Drag coefficient for circular cylinders as a function of the Reynolds number