# Unconditionally Stable Explicit Methods for Massively Parallel Solution of Acoustic Wave Equation

**Amir Fijany[1], Fred Aminzadeh[2], Paul C. Messina[3], and Jacob Barhen[4]**

[1]*Jet Propulsion Laboratory, California Institute of Technology*
[2]*Unocal Corporation*
[3]*California Institute of Technology*
[4]*Oak Ridge National Laboratory*

## ABSTRACT

Simulated seismic models are important tools for a more cost-effective oil exploration. Model building is a computationally intensive process since it requires repeated solution of the Acoustic Wave Equation (AWE) assuming different sound velocity distributions. For realistic 3D models, each solution of the AWE involves a number of spatial grids on the order of millions and a number of time steps on the order of thousands, resulting in massive CPU time and memory storage requirements. In fact, current estimates indicate that building such models requires a computation time of the order of several thousand GFLOPS hours. Emerging massively parallel MIMD architectures offer both the computing power and memory storage "required for tackling realistic 3D models. Consequently, there are extensive ongoing research efforts on efficient implementation of the existing algorithms for solution of the AWE on these architectures.

Most existing algorithms are based on the explicit methods for solution of the AWE. These algorithms are efficient for parallel computation since, at each time step, they mainly involve a highly sparse matrix-vector multiplication with an optimal computational complexity of the order of the number of spatial grid points. Also, further improvement in the computational efficiency of these algorithms have been achieved by increasing their spatial accuracy (from the conventional fourth order). However, due to their stability constraint, the explicit methods require a very large number of time steps, resulting in excessive computation and storage requirements. The implicit methods offer the advantage of unconditional stability, thus demanding a much smaller number of time steps. However, this superior numerical property is achieved at the cost of significant increase in the computation cost per time step and the loss of efficiency for parallel implementation since, at each time step, the solution of a sparse linear system is now required. It is therefore clear that a promising direction to achieve a greater overall computational efficiency in solution of the AWE is to seek novel methods which preserve the parallel computational efficiency of the explicit methods, offer the unconditional stability of the implicit methods, and achieve a higher accuracy in both spatial and temporal domains.

In this paper we present such a novel method. It is an explicit method in the sense that, at each time step, it mainly involves sparse matrix-vector multiplications with high efficiency for parallelization and with a computational cost proportional to the number of spatial grid points. It is also unconditionally stable and thus allows a much larger time step size. Furthermore, higher, order accuracy in both time and space can be achieved with a constant factor increase in the computational cost per time step, thus preserving the asymptotic computational cost. We discuss the mathematical foundation of this method by first deriving an unconditionally stable explicit algorithm with a second-order accuracy in both time and space. We then discuss the derivation of more accurate algorithms (fourth order accurate in time and fourth and higher order accurate in space). We also discuss some techniques for a more efficient implementation of resulting algorithms on massively parallel MIMD architectures. The incorporation of the first and higher order absorbing boundary conditions in the algorithms is also analyzed.

---

[1]Corresponding Author: MS 303-310, Jet Propulsion Laboratory, 4800 Oak Grove Drive, Pasadena, CA 91109. Tel: (818) 354-9552, Fax: (818) 393-5013, Email: fijany@nips.jpl.nasa.gov