ABSTRACT Performance of the second order digital Data-Aided Loop (DAI) is evaluated using the current available analog results. To utilize the analog results, both Impulse Invariant Transformation (IIT) and Linear Interpolation Transformation (LIT) techniques are used in the approximation of the digital loop filter. The analytical results obtained from these transformation methods will be compared with the computer simulation results. The results obtained by LIT method are in good agreement with computer simulation results. In addition, the paper will also investigate the impact of the DAI tracking phase jitter on the Bit Error Rate (BER) performance, and the results are then compared with the commonly used Costas loops, namely, Costas loop with matched filter and clock feed back in the arm filter and Costas loop with second order Butterworth filter in the arm filter. The analytical results demonstrate the superior performance of the digital DAI.

1.0 Introduction

In the past, the analog Data-Aided Loop (DAI) has been proposed for space applications [1, 2]. The DAI can be employed by both suppressed carrier or residual carrier communication systems. For space applications, the residual carrier systems usually use the subcarrier to separate the data from the residual carrier [3]. Basically, the DAI uses the power in the composite received signal sidebands to enhance the Signal-to-Noise Ratio (SNR) in the bandwidth of the carrier (or subcarrier) tracking. The composite received signal used in the DAI can consist of the carrier and data, or carrier and data modulated subcarrier for suppressed or residual carrier system, respectively [2]. Furthermore, for residual carrier systems, the DAI can also be used for subcarrier tracking where the composite received signal is subcarrier and data [3, Chapters 5-6]. Because of the advance in digital signal processing technology, the DAI can be easily implemented in a single Digital Signal Processing (DSP) chip. This has motivated the use of the digital DAI for space applications where subcarrier tracking is required to be done with great accuracy. Figure 1 shows a simplified block diagram of the digital DAI. The digital loop filter, F(s), shown in this figure is of the second order type, hence the name second order digital DAI.

The performance of the first and second order analog DAI, has been analyzed thoroughly by Simon and Springett [2]. However, the results for the second order loop are only applicable to the second order analog loop filter. This paper attempts to use the current available results provided in [2] to derive the tracking rms phase error (or tracking phase jitter) for the second order digital DAI, and assesses the impact of this phase jitter on the Bit Error Rate (BER) performance. The results of the phase jitter obtained by the computer simulation for the digital DAI, will also be represented and compared with the theoretical results. Furthermore, the BER performance for systems using DAI will be compared against those employing Costas loops.

2.0 Derivation of Tracking Phase Jitter

2.1 Current Results for the Analog DAI

A simplified block diagram for the analog DAI is shown in Figure 2. This loop has been analyzed in [1-2]. For the second order DAI, the loop filter F(s) is given by

\[
F(s) = \frac{1 + s \tau_2}{1 + s \tau_1}
\]

where \( \tau_1 \) and \( \tau_2 \) are the time constants of the second order loop filter. When the Loop Signal-to-Noise Ratio (LSNR) is large and the bit SNR is greater than 4 dB with the tracking phase jitter less than 15° (or π/12), the variance of the tracking phase jitter can be shown to have the following form [2]

\[
\alpha^2 = \left[1 + \frac{1 - F_1}{r}\right]\left[1 + \frac{1 - r_1}{r}\right] \beta^2
\]

where \( F_1 \) is defined as the time constant \( \tau_2 \)-to-\( \tau_1 \) ratio and the parameter \( r \) is given by

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1. The work described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.
missions, the nominal sampling period is \((8 \times 10^4)\) s c c. Let \(a_1\) and \(a_2\) be defined as
\[
\begin{align*}
    a_1 & = \frac{2 \tau_1}{T}, \\
    a_2 & = \frac{2 \tau_2}{T},
\end{align*}
\]
then the approximated digital loop filter \(F_{AP}(z)\) of \(F(z)\) using LTI transformation technique can be written in terms of \(a_1\) and \(a_2\) as
\[
    F_{AP}(z) = \left( \frac{a_2 z^{-1}}{a_1 z^{-1}} \right) \left( \frac{1 - a_1 z^{-1}}{1 - \frac{a_1}{a_2} z^{-1}} \right) \left( \frac{1 - \frac{a_2}{a_1} z^{-1}}{1} \right).
\]
Next, one would want to map Eq. (10) into Eq. (6). By equaling the coefficients between Eqs. (10) and (6) one obtains the appropriate time constants \(\tau_1\) and \(\tau_2\) for the corresponding analog filter that represents \(F(z)\) in the s-domain, namely,
\[
\begin{align*}
    \tau_1 & = \frac{1}{K_y (Y_2 - 1) / 2}, \\
    \tau_2 & = \frac{1}{K_y (Y_2 - 1) / 2}.
\end{align*}
\]
Since one wants the approximated second order digital loop filter \(F_{AP}(z)\) to be the same as \(F(z)\) described in Eq. (6), the values \(\tau_1\) and \(\tau_2\) calculated from Eq. (11) must be selected in such a way that the following condition is satisfied
\[
y_1 \left( \frac{a_1 z^{-1}}{a_2 z^{-1}} \right) = 1
\]
As an example, for typical deep space missions, one obtains:
\[
\begin{align*}
    \tau_1 & = 4.0585 \times 10^3 \text{ s c c}, \\
    \tau_2 & = 9.366 \times 10^3 \text{ s c c}.
\end{align*}
\]
Using the calculated values for \(\tau_1\) and \(\tau_2\) in Eq. (13), one wishes to verify the condition in Eq. (12). By substituting these values into Eq. (12) one gets \(y_1 = 1.03\), which is approximately equal to 1. Therefore, for deep space applications, using the time constants found in Eq. (13), one can approximate the discrete loop filter described by Eq. (1) in the s-domain. Thus, using the LTI1' method, the approximated analog loop filter for the digital filter described by Eq. (6) is found to be:
\[
    F(s) = \frac{1}{\frac{1}{T} s + 1} \left( \frac{9.366 \times 10^3 s}{4.0585 \times 10^3 s} \right).
\]
To preserve the transient response of the analog loop filter \(F(s)\) in the discrete domain, the actual representation of Eq. (1) in the \(z\)-domain can be
derived by using ITT method. This is derived as follows. First, the impulse response \( f(t) \) of the analog loop filter is found by taking the inverse Laplace transform of \( F(s) \). The desired impulse response \( f(n) \) for the digital loop filter then can be found by sampling \( f(t) \) at each sampling interval \( T \), i.e., \( f(n) = f(t=nT) \). The actual loop filter, \( V_{\text{Ac}}(s) \), in the discrete domain is found by taking the Z-transform of \( f(n) \), namely

\[
V_{\text{Ac}}(z) = \frac{\alpha_0 \alpha_4 z^{-4}}{1 - z^{-1} \alpha_1} \tag{15}
\]

where the parameters \( \alpha_0, \alpha_1, \alpha_2, \) and \( \alpha_4 \) are given by

\[
\alpha_0 = \frac{\tau_4 \tau_3}{\tau_1 \tau_4} \quad \alpha_1 = 1 - \frac{\tau_4}{\tau_1} \quad \alpha_2 = \frac{\tau_2}{\tau_1} \tag{16}
\]

To approximate the digital loop filter expressed by Eq. (5) in the analog domain one rewrites Eq. (5) as follows

\[
F(z) = \left[ \begin{array}{c} \gamma_2 z^{-2} \\ 1 - z^{-4} \end{array} \right] \tag{17}
\]

where \( K_f \) and \( \gamma_2 \) are given by Eq. (7) and \( \gamma_2 = 1 \). By comparing the coefficients of Eqs. (15) and (17), one can determine the time constants \( \tau_{1Ac} \) and \( \tau_{2Ac} \) for the corresponding, analog loop filter, \( V_{\text{Ac}}(s) \), that represents an approximation using ITT transformation for the digital loop filter described by Eq. (6). Comparing the coefficients between Eqs. (15) and (17), one gets

\[
\left( \begin{array}{cc} \tau_2 \tau_4 \\ \tau_1 \tau_4 \end{array} \right) = \gamma_2, \quad \text{and} \quad \gamma_2 = \tau_1 \tau_4 \tag{18}
\]

The solutions to Eq. (18) should satisfy the constraint

\[
\tau_{2Ac} \tau_{1Ac} = 1 \tag{19}
\]

As an example, the calculated time constants using ITT for typical deep space applications are, from Eqs. (18) and (19),

\[
\tau_{1Ac} = 0.1389 \text{ sec}, \quad \tau_{2Ac} = 0.1357 \text{ sec} \tag{20}
\]

Plots of the variance tracking jitter using ITT and 1.1 are shown in Figure 3. The computer simulation results are also shown for comparison and verification purpose. ITT technique was found to be the best and will be used later for BER calculation.

3. BER Performance

In this section one will assume that the bit tracking is perfect in the bit synchronizer. When the two-sided loop bandwidth of the digital DAII (or the analog Costas loop) is small relative to the incoming channel rate, the phase error \( \phi \) can be considered to be constant for many bit periods. Under these conditions, the conditional error probability is given by [2]

\[
P(c/\phi) = \frac{1}{2} \text{erfc}(\sqrt{\frac{R}{\alpha}} \cos(\phi)) \tag{21}
\]

Again, \( R \) denotes the bit SNR. The average error probability is then obtained by averaging Eq. (21) over the probability density function (pdf) \( P(\phi) \) of the phase error [2]

\[
P(\phi) = c \cos(\phi) \left[ \int_{-\pi/2}^{\pi/2} P(c/\phi) P(\phi) d\phi \right] \tag{22}
\]

For digital DAII, operates at high bit SNR (\( R > 4 \) dB) and small phase error (\( \phi < 15^\circ \)), using Reference [2], the pdf for the phase error can be shown to have the following form

\[
P(\phi) = c \cos(\phi) \left[ \int_{-\pi/2}^{\pi/2} P(c/\phi) P(\phi) d\phi \right] \tag{23}
\]

where

\[
\alpha = \sigma_\phi^2 \tag{24}
\]

and \( \sigma_\phi \) is given by Eq. (2). The average BER performance for the digital DAII loop and the results are calculated and plotted in Figure 4. For Costas loop with matched filter and clock feedback in the arm filter, and second order Butterworth in the arm filter, the results can be found in [3, Chapter 3]. The results are also plotted in Figure 4 for comparison purpose.

4.0 Conclusion

The analytical model employing the 1.1'1' method can be used to predict the tracking jitter of the second order digital DAII for all data rates. Using the derived tracking phase jitter, one can determine the BER performance of the second order digital suppressed-carrier DAII. Numerical results show that the digital DAII outperforms the commonly used Costas loops.

Acknowledgement

The authors are grateful to D. D Lanson for providing the computer simulation results, Drs. V. Vilhron and S. Hinoj for their invaluable comments and
suggestions during the review of this work, and A. Kermode and B. Charny for their support.

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