

# The topographic torque on a bounding surface of a rotating gravitating fluid and the excitation by core motions of decadal fluctuations in the Earth's rotation

Raymond Hide<sup>1</sup>

Space Geodetic Science and Applications Group, Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, MS 238-332, Pasadena, California 91109

**Abstract.** General expressions (with potential applications in several areas of geophysical fluid dynamics) are derived for all three components of the contribution made by the geostrophic part of the pressure field associated with flow in a rotating gravitating fluid to the topographic torque exerted by the fluid on a rigid impermeable bounding surface of any shape. When applied to the Earth's liquid metallic core, which is bounded by nearly spherical surfaces and can be divided into two main regions, the "torosphere" and "polosphere," the expressions reduce 10 formulae given previously by the author, thereby providing further support for his work and that of others on the rôle of topographic coupling at the core-mantle boundary in the excitation by core motions of Earth rotation fluctuations on decadal time scales. They also show that recent criticisms of that work are vitiated by mathematical and physical errors. Contrary to these criticisms, the author's scheme for exploiting Earth rotation and other geophysical data (either real or simulated in computer models) in quantitative studies of the topography of the core-mantle boundary (CMB) by intercomparing various models of (a) motions in the core based on geomagnetic secular variation data and (b) CMB topography based on seismological and gravity data has a sound theoretical basis. The practical scope of the scheme is of course limited by the accuracy of real data, but this is a matter for investigation, not *a priori* assessment.

## Int reduction

The local gradient  $\nabla p$  of the instantaneous pressure  $p(r, t)$  at a general point  $l'$  in a gravitating fluid which rotates with angular velocity  $\Omega(t)$  relative to an inertial frame of reference can be expressed formally as the sum of "gravitational," "geostrophic" and "ageostrophic" parts,  $\nabla p^{(V)}$ ,  $\nabla p^{(G)}$  and  $\nabla p^{(A)}$  (see (3.5) below), each satisfying a precise diagnostic relationship with the density  $\rho(r, t)$ , Eulerian relative flow velocity  $u(r, t)$ , magnetic field  $B(r, t)$ , etc. The relationship satisfied by  $\nabla p^{(G)}$  is

$$\nabla p^{(G)}(r, t) \equiv -2\rho(r, t) \Omega(t) \times u(r, t). \quad (1.1)$$

General expressions are derived below (see (4.1) to (4.5)) for all three components of the contribution  $\mathbf{I}_S^{(G)}(t)$  made by  $\nabla p^{(G)}$  to the instantaneous topographic torque  $\Gamma_S(t)$  (where  $t$  denotes time) exerted by the moving fluid on a rigid impermeable bounding surface  $S$  of any shape through the action of normal pressure stresses  $p$  on  $S$ . The expressions are then applied to the Earth's liquid metallic core, which has nearly spherical bounding surfaces, extends from the solid inner core to the overlying solid mantle, and for certain dynamical purposes can be divided into two broad regions, the "pol(oid)osphere" and the "tor(oid)osphere" (see Fig. 1). The results provide further support for: (a) the author's proposal that in the excitation of fluctuations in the rotation of the mantle on decadal time scales by motions in the underlying core, topographic coupling is an important and possibly the predominant mechanism, (b) his ideas concerning the subtle but crucial role of Lorentz forces  $\mathbf{j} \times \mathbf{B}$  (per unit volume, where  $\mathbf{j}(r, t)$  is the electric current density) in the dynamical processes involved, which depend *inter alia* on the dimensionless measure  $R \equiv |\nabla p^{(A)}|/|\nabla p^{(G)}|$  (see (3.7)) of ageostrophic effects, and (c) the theoretical basis and preliminary findings of his scheme for exploiting Earth rotation data in quantitative investigations of the topography of the CMB, by intercomparing various models of motions in the polosphere (where  $R \ll 1$  nearly everywhere) based

on geomagnetic secular variation data, and various models of CMB topography based on gravity and seismological data. However certain geophysicists, whilst accepting (a) explicitly and (b) implicitly in their recent work on core dynamics, claim (see (2.2) below) that (c) “must be rejected,” asserting that the scheme is “theoretically flawed” and “could provide no quantitative-information about the CMB even with perfectly accurate geophysical data.” The claim is thoroughly refuted by the present study as being inaccurate owing to demonstrable mathematical and physical errors. It is based on arguments that (i) not only fail to recognise the essentially diagnostic (but nonetheless dynamic) nature of the geostrophic relationship, but (ii) also suppose incorrectly that the axial component of  $\Gamma_s^{(G)}(t)$  must be identically equal to zero (cf. (4.5)) even when quasi-geostrophic flow is largely confined to the polosphere (i.e., when  $R \neq 1$  nearly everywhere in the torosphere), and (iii) overlook advection of angular momentum in an inaccurate attempt to provide a physical interpretation of the dynamical processes involved.

### Decadal fluctuations in the Earth’s rotation

The principal manifestation of time-varying fluid motions in the Earth’s liquid metallic core is the main geomagnetic field, generated by the self-exciting magnetohydrodynamic dynamo process. The fluid motions also produce forces on the mantle which give rise *inter alia* to a fluctuating net torque  $\Gamma^*(t)$ . Geophysicists now accept that tiny but detectable irregular fluctuations in the magnitude of  $\Omega$ , the vector rotation of the mantle with respect to an inertial frame, on decadal time scales are largely produced by the action of axial component  $\Gamma^* \cdot \hat{z}$  (if  $\hat{z}$  is a unit vector along the mean polar axis). It is also possible that the equatorial components of  $\Gamma^*$ , namely  $\Gamma^* \cdot \hat{x}$  and  $\Gamma^* \cdot \hat{y}$ , contribute significantly to observed polar motion on decadal time scales. Here  $\hat{x}$  and  $\hat{y}$  are unit vectors along the x and y axes of a Cartesian frame of reference fixed in the mantle with its origin  $O$  located at the Earth’s center of mass, with the x-axis pointing towards the Greenwich meridian where the longitude angle  $\phi = 0$ , and the y-axis in the direction  $\phi = \pi/2$ , see Fig. 1. The vector distance  $r = r \hat{r}$  from  $O$  to a general point  $P$  satisfies  $r = x\hat{x} + y\hat{y} + z\hat{z}$ . (For general discussions of fluctuations in the Earth’s rotation on all time scales and extensive lists of references, see *Munk and MacDonald* [1960], *Lambeck* [1980], *Melchior* [1986], *Moritz and Mueller* [1987], *Wahr* [1988], *Hide and Dickey* [1991] and *LeMouél* [1993].)

The forces that contribute to  $\Gamma^*(t)$  are of four types, namely (a) tangential viscous stresses in the thin frictional (Ekman-Hartmann) boundary layer at the CMB, (b) Lorentz forces associated with electric currents induced in the weakly-conducting mantle by core processes, (c) non-radial buoyancy forces associated with gravitational interactions between the heterogeneous core and mantle (*Jault and LeMouél* [1990]), and (d) normal pressure forces acting on topographic features (“bumps”) on the CMB, which are able to exert a net torque on a non-spherical surface (see *Hide* [1969, 1977(a), 1986]; *Hide et al.* [1993], *Hinderer et al.* [1990], *Jault and LeMouél* [1991], *Voorhies* [1991]). Denote by  $\Gamma(t)$  the contribution to  $\Gamma^*(t)$  associated with topographic coupling, a mechanism first proposed by *Hide* [1969] who (a) argued that bumps no higher than about a kilometre might suffice to account for the magnitude of  $\Gamma^* \cdot \hat{z}$  implied by observations of decadal variations in the length of the day even with a high degree of instantaneous cancelling between positive and negative couples (see *Hide et al.* [1993, especially (2.1)]), (b) outlined a strategy for research on the essential fluid dynamical processes involved (see *Hide* [1977(a)], *Anufriev and Braginsky* [1977], *Eltayeb and Hassan* [1979], *Moffatt* [1978],

Kuang and Bloxham [ 1993]), and (c) proposed and implemented a method (independently put forward by Professor J-L LeMouél) for determining  $\Gamma(t)$  from geomagnetic and other geophysical data (Hide [ 1986, 1989, cited as H89], Hide et al. [1993], Hinderer et al. [ 1990], Jault and LeMouél [ 1991 ]).

Suppose that the CMB is the locus of points where  $r = c + h(\theta, \phi)$ , where  $c = 3486\text{km}$  is the mean radius of the CMB, and introduce the dimensionless parameter

$$\delta \equiv \bar{h} / c \quad (2.1)$$

where  $\bar{h}$  is the rms value of  $h(\theta, \phi)$ . It is unlikely that  $\bar{h}$  exceeds the height of the equatorial bulge of the CMB, about  $10^4\text{m}$ , (see Gwinn et al. [ 1986]), so that  $\delta \leq 3 \times 10^{-3}$ . Define a spherical surface  $r = c - A$  in the free stream near the top of the polosphere and suppose that the Eulerian flow velocity  $\mathbf{u} = u$ , on that surface, with components  $(u, v, w)$  in the  $(r, \theta, \phi)$  directions equal to  $(u_1, v_1, w_1)$ . Here  $u_1$  is typically much less than  $v_1$  and  $w_1$  in magnitude, by a factor  $\delta$ , and can be neglected for some purposes, but not all (see e.g. (4.5) below). Expressions for all three components of  $\Gamma(t)$  to leading order in  $R$  in the polosphere and in  $\delta$  are given in terms of  $(u_1, v_1, w_1)$  by equations (2. 1, 2.8, 2.9 H89), where  $R$  is a measure of ageostrophic contributions to the momentum equation (see (3.7) below).

Many geophysicists now accept that topographic torques may be important and possibly even dominant, but unsubstantiated criticisms of the validity of the author's expression for  $\Gamma \cdot \hat{\mathbf{z}}$  (see (2.9 H89)) have been voiced in well-publicized abstracts of papers presented at various recent scientific meetings. Thus:

(a) "... although such calculations (based on the  $z$  component of (2.9 H89)) are essentially kinematic in nature, dynamical considerations show that they are flawed, since geostrophic flow cannot in itself result in topographic coupling"; (b) "( Hide's) scheme (would be) inapplicable even if we had perfect knowledge of the core flow and topography (for) geostrophic balance precludes the transfer of angular momentum, so, for a core in geostrophic balance, the (instantaneous axial component) of the topographical couple is (identically) zero"; (c) "maps of the fluid flow at the core surface and length of day observations have been used (by Hide et al., [ 1993]) to place a constraint on the amplitude of topography at the core surface. We argue . . . that such constraints have been wrongly applied..."; and (d) "the geostrophic pressure yields no information on the topographical torque". (2,?)

(see e.g. J. Bloxham et al., *EOS Trans. Am Geophys. Un. Supplement*, April 20, 1993, p. 51; November 1, 1994, pages 58 and 84). These statements are vitiated by demonstrable mathematical and physical errors in the arguments upon which they are based. They stem from misconceptions concerning the dynamical processes involved, and from an expression for  $\Gamma \cdot \hat{\mathbf{z}}$ , namely  $\Gamma \cdot \hat{\mathbf{z}} \equiv 0$ , which is incorrect owing apparently to an elementary but crucial mathematical error in its derivation.  $\Gamma \cdot \hat{\mathbf{z}}$  is certainly *not* identically equal to zero in the situation envisaged by the critics, namely when quasi-geostrophic balance is confined to the polosphere (see (4.5) below). And the statements fail to take into account of the essentially diagnostic (as opposed to prognostic, see Hide [ 1977, 1982]) nature of the geostrophic relationship, and would in the author's view be unacceptable on that basis alone. Bernoulli's celebrated equation (see e.g. Birkhoff [1960]), is another useful diagnostic expression in fluid dynamics whose poor prognostic properties can lead to highly erroneous conclusions, as in the well-known d'Alembert paradox concerning the drag on a moving body in a fluid of low viscosity.

## Expressions for the topographic torque

Denote by  $p(\mathbf{r}, t)$  the pressure at a general point P and by S the closed surface (see Fig. 1) that coincides with the "CMB," the shape of which in the following general analysis need not be nearly spherical. The topographic torque  $\Gamma_S(t)$  exerted by the fluid "core" on the overlying "solid mantle" is

$$\Gamma_S(t) = \iint_S p(\mathbf{r}, t) \mathbf{r} \times d\mathbf{S} \quad (3.1)$$

where the integral is taken over the whole of S, the vector element of area  $d\mathbf{S}$  of which is directed generally away from O. Clearly  $\Gamma_S(t) = 0$  for a sphere  $r = \text{constant}$ , for then  $\mathbf{r} \times d\mathbf{S} = 0$  everywhere. Also equal to zero would any component of  $\Gamma_S(t)$  in a direction about which S were a figure of revolution.

Introduce mathematical control surfaces C within the fluid "core" where by definition C-surfaces are spherical and centered on O (i.e.  $r = \text{constant}$  on C), see Fig. 1; the pseudo-torque

$$\Gamma_C(t) \equiv \iint_C p(\mathbf{r}, t) \mathbf{r} \times d\mathbf{S} \quad (3.2)$$

is therefore equal to zero. It follows from (3.1) and (3.2) and a well-known vector identity that

$$\Gamma_S(t) = \iiint_{S-C} \mathbf{r} \times \nabla p(\mathbf{r}, t) d\tau \quad (3.3)$$

where  $d\tau$  is the volume element and the integral is taken over the whole volume of fluid lying between the surface S (the "CMB") and the spherical mathematical control surface C lying at or below  $r = c - \Delta$  (see (2.1)).

An expression for  $\nabla p$  can be obtained from the equations of fluid dynamics. For our purposes it is sufficient to consider the equations of mass continuity and momentum. The first of these is  $\partial\rho/\partial t + \nabla \cdot \rho\mathbf{u} = 0$ , which can be written

$$\nabla \cdot \mathbf{U} = 0 \quad (3.4)$$

(if  $\mathbf{U} \equiv \rho\mathbf{u}$ ) without fear of serious error when dealing with highly subsonic motions. The second is conveniently written as follows:

$$\nabla p + 2\hat{\boldsymbol{\Omega}} \times \mathbf{U} + \rho \nabla V = \mathbf{A} \quad (3.5)$$

where  $V$  is the potential due to gravity and centripetal effects and  $\mathbf{A}$  comprises all the "ageostrophic" terms. Thus

$$\mathbf{A} = \mathbf{A}(\mathbf{r}, t) \equiv \mathbf{j} \times \mathbf{B} + \mathbf{F}_{\text{visc}} - \rho \hat{\boldsymbol{\Omega}} \times \mathbf{r} - \rho [\partial\mathbf{u}/\partial t + (\mathbf{u} \cdot \nabla) \mathbf{u}], \quad (3.6)$$

where the Lorentz force (per unit volume),  $\mathbf{j} \times \mathbf{B}$ , is the largest contribution to  $\mathbf{A}$  in the core outside the thin Ekman-Hartmann viscous boundary layers (see Hide [1977(a)]).  $\mathbf{F}_{\text{visc}}$  is the viscous force,  $\rho \hat{\boldsymbol{\Omega}} \times \mathbf{r}$  is a "fictitious" force associated with time-variations in  $\hat{\boldsymbol{\Omega}}$  (since  $\dot{\boldsymbol{\Omega}} \equiv d\hat{\boldsymbol{\Omega}}/dt$ ), and  $\rho [\partial\mathbf{u}/\partial t + (\mathbf{u} \cdot \nabla) \mathbf{u}]$  is the acceleration of a moving fluid element relative to the rotating reference frame. The flow would be *strictly geostrophic* in regions where the dimensionless parameter

$$R \equiv |\mathbf{A}| / |\hat{\boldsymbol{\Omega}} \times \mathbf{U}| \quad (3.7)$$

is equal to zero, *quasi-geostrophic* in regions where  $0 < R \ll 1$  and *non-geostrophic* where  $R \gg 1$ .

Since  $\nabla p = \mathbf{A} - \rho \nabla V - 2\hat{\boldsymbol{\Omega}} \times \mathbf{U}$ , it is convenient to write

$$\Gamma_S(t) = \Gamma_S^{(A)}(t) + \Gamma_S^{(V)}(t) + \Gamma_S^{(G)}(t) \quad (3.8)$$

where (see (3.3))

$$\Gamma_S^{(A)} \equiv \iiint_{S-C} \mathbf{r} \times \mathbf{A} d\tau \quad (3.9)$$

is the contribution to  $\Gamma_S$  associated with the ageostrophic terms in the momentum equation,

$$\Gamma_S^{(V)} \equiv - \iiint_{S-C} \mathbf{r} \times \rho \nabla V d\tau \quad (3.10)$$

is the contribution associated with the non-radial components of  $\mathbf{V}\mathbf{V}$ , and

$$\Gamma_s^{(G)} \equiv -2\Omega \iiint_{s.C} \mathbf{r} \times (\hat{\mathbf{z}} \times \mathbf{U}) d\tau \quad (3.11)$$

is the contribution associated with what we here term the "geostrophic pressure field"  $p^{(G)}$ .

### The geostrophic contribution to the topographic torque

Making use of a well-known vector identity and the facts that  $2(\mathbf{r} \cdot \mathbf{U}) = \mathbf{V} \cdot (\mathbf{r} \times \mathbf{U})$  and  $\mathbf{V} \cdot \mathbf{U} = 0$  (see (3.4)), (3.11) gives

$$\Gamma_s^{(G)} = 2\Omega \iiint_{s.C} z (U_x \hat{\mathbf{x}} + U_y \hat{\mathbf{y}} + U_z \hat{\mathbf{z}}) d\tau \quad (4.1)$$

where  $\mathbf{U} = (U_x \hat{\mathbf{x}} + U_y \hat{\mathbf{y}} + U_z \hat{\mathbf{z}}) = \rho(r, t)(u_x \hat{\mathbf{x}} + u_y \hat{\mathbf{y}} + u_z \hat{\mathbf{z}})$ . The validity of (2.8.1189) can be demonstrated by noting that the exact equation (4.1) reduces to (2.8.1189) when terms of second order and higher in the small quantity  $\delta$  (see (2.1)) are neglected. In carrying out the comparison, the volume of integration is taken to extend outward from that spherical control surface  $C$  where  $r = c - A$ , in the "free stream" (see (2.1)), to the CMB where  $r = c + h(\theta, \phi)$ , and expressing  $(u_x, u_y, u_z)$  in terms of  $(u_r, u_\theta, u_\phi)$ .

A more revealing comparison can be made by noting first that in virtue of (3.4) and the vanishing of  $\mathbf{U} \cdot d\mathbf{S}$  everywhere on the impermeable and rigid surface  $S$  (but not on  $C$ ):

$$\Gamma_s^{(G)} \hat{\mathbf{x}} = -2\Omega \left[ \iint_C xz \mathbf{U} \cdot d\mathbf{S} + \iiint_{s.C} xU_z d\tau \right], \quad (4.2)$$

$$\Gamma_s^{(G)} \hat{\mathbf{y}} = -2\Omega \left[ \iint_C yz \mathbf{U} \cdot d\mathbf{S} + \iiint_{s.C} yU_z d\tau \right], \quad (4.3)$$

$$\Gamma_s^{(G)} \hat{\mathbf{z}} = -\Omega \iint_C z^2 \mathbf{U} \cdot d\mathbf{S}. \quad (4.4)$$

In general, none of these components of  $\Gamma_s^{(G)}$  is identically equal to zero. We note here in passing that (4.4) refutes the assertion in (2.2b).

The physical interpretation of (4.4) becomes evident when it is re-written as

$$\Gamma_s^{(G)} \hat{\mathbf{z}} = \iint_C \left[ \rho \Omega (x^2 + y^2) \right] \mathbf{u} \cdot d\mathbf{S} \equiv Q_C \text{ (say)} \quad (4.5)$$

(remembering that  $x^2 + y^2 + z^2 = r^2$  and  $r = \text{constant}$  on  $C$ ). The quantity in the square bracket in the integrand is the axial component of the angular momentum of a fluid element of unit volume at a distance  $(x^2 + y^2)^{1/2}$  from the  $z$ -axis associated with its rotation about that axis, and  $Q_C$  is therefore the rate of advection of that quantity across the surface  $C$ . The equivalence to first order in  $\delta$  of (2.8.1189) to the more general equation (4.4) (or (4.5)) is readily demonstrated along the same lines as those followed when comparing (4.1) with (2.8.1189). Thus, we take  $C$  to be the surface where  $r = c - A$  and infer the radial motion on  $C$  by applying the boundary condition that  $\mathbf{u} \cdot d\mathbf{S} = 0$  on the CMB, where  $r = c + h(\theta, \phi)$ . It is important to note that whilst this radial motion can safely be neglected to leading order in the equation of motional induction when deducing core motions from geomagnetic secular variation data (see (4.1.1189), also *Bloxham and Jackson [1991, equation (38)]* and *Backus and Le Mouél [1987]*), it is certainly non-zero in general and plays a crucial role in angular momentum transfer and torque balance!

Possibly the simplest model of the Earth's liquid core that one could imagine for the purpose of an exercise in estimating  $\Gamma(t)$  from first principles would comprise a largely quasi-geostrophic polosphere within which  $R_p$ , the average value of  $R$ , satisfies  $R_p \ll 1$  (see (3.7)), and a possibly non-geostrophic torosphere where  $R_t$  (the average value of  $h$  there) may be significantly larger than  $R_p$ , even of order unity or greater.  $\Gamma_s^{(G)}$  would then provide a good

leading approximation to  $\Gamma$  (with errors no more than  $R_p$ ), and only in very special circumstances would  $\Gamma_S^{(G)} \cdot \hat{z}$  vanish, namely when  $R_T \ll 1$  and the shape of the interface between the liquid core and the underlying solid inner core is a figure of revolution about the z-axis!

Beyond the scope of the present short article is (a) the inclusion of all the details of the analysis leading to the general expressions deduced in §§ 3 and 4, and (b) full discussions of their implications for realistic models of the core and also for theoretical studies of the dynamics of oceans and atmospheres of the Earth and other planets. These matters will have to be treated elsewhere.

**Acknowledgment.** The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. It is a pleasure to acknowledge the scientific hospitality and helpful discussions with members of the Space Geodetic Science and Applications Group of Jet Propulsion Laboratory during a visit when this paper was written.

## References

- Anufriev, A. P. and S. I. Braginsky, Influences of irregularities in the boundary of the Earth's core on fluid velocity and magnetic field, *Geomagn. Aeron.*, **17**, 492-496, 1977.
- Backus, G. E. and J.-L. LeMouél, The region of the core-mantle boundary where the geostrophic velocity field can be determined from frozen flux data, addendum, *Geophys. J. Roy. Astron. Soc.*, **88**, 321-322, 1987.
- Birkhoff, G., *Hydrodynamics; a study in logic, fact and similitude*, Princeton University Press, 1960.
- Bloxham, J., and A. Jackson, Fluid flow near the surface of the Earth's core, *Rev. Geophys.*, **29**, 97-120, 1991.
- Eltayeb, I. A., and M. H. A. Hassan, On the effects of a bumpy core-mantle interface, *Phys. Earth Planet. Int.* **19**, 239-254, 1979.
- Eubanks, T. M., Variations in the orientation of the Earth, pp. 1-54 in *Contributions of space geodesy to geodynamics* (eds. D. E. Smith and D. L. Turcotte), Geodynamics Series Vol. 24, American Geophysical Union, Washington, D.C., 1993.
- Girt, C., and J.-L. LeMouél, Tangentially-geostrophic flow at the core-mantle boundary compatible with observed geomagnetic secular variation: the large-scale component of the flow, *Phys. Earth Planet. Int.*, **259-287**, 1990.
- Gwinn, C. R., I. A. Herring, and I. I. Shapiro, Geodesy by radio interferometry: Studies of the forced nutations of the Earth, 2. Interpretation, *J. Geophys. Res.*, **91**, 4755-4765, 1986.
- Hide, R., Interaction between the Earth's liquid core and solid mantle, *Nature*, **222**, 1055-1056, 1969.
- Hide, R., Towards a theory of irregular variations in the length of the day and core-mantle coupling, *Phil. Trans. Roy. Soc. Lond.*, **A284**, S47-554, 1977(a).
- Hide, R., Experiments with rotating fluids (Presidential address), *Quart. J. Roy. Meteorol. Soc.*, **103**, 1-28, 1977(b).
- Hide, R., "On the rôle of rotation in the generation of magnetic fields by fluid motions," *Philos. Trans. Roy. Soc. Lond. A* **306**, 223-234, 19X2.
- Hide, R., Presidential Address: The Earth's differential rotation, *Quart. J. Roy. Astron. Soc.*, **278**, 3-14, 1986.
- Hide, R., Fluctuations in the Earth's rotation and the topography of the core-mantle interface, *Phil. Trans. Roy. Soc. Lond.*, **A328**, 351-363, 1989; cited as H89.
- Hide, R., R. W. Clayton, B. H. Hager, M. A. Spieth, and C. V. Voorhies, Topographic core-mantle coupling and fluctuations in the Earth's rotation, in *Relating geophysical structures and processes, The Jeffreys volume* (edited by K. Aki and R. Dmowska), *Geophys. Monog. Am. Geophys. Un.*, **76**, 107-120, 1993.
- Hide, R., and J. O. Dickey, Earth's variable rotation, *Science*, **253**, 629-637, 1991.
- Hinderer, J., D. Jault, and J.-L. LeMouél, Core-mantle topographic torque: A spherical harmonic approach and implications for the excitation of the Earth's rotation by core motions, *Phys. Earth Planet. Int.*, **59**, 329-341, 1990.

Jault, D., and J-L. LeMouél, "Core-nwm[ic boundary shape: Constraints inferred from the pressure torque acting between the core and mantle," *Geophys. J. ht.*, 101,233-241, 1990.

Jault, D., and J-L, LeMouél, "Exchange of angular momentum between the core and mantle, *J. Geomag. Geoelect.*, 43, 111-129, 1991.

Kuang, W., and J. Bloxham, The effect of boundary topography on motions in the Earth's core, *Geophys. Astrophys. Fluid Dynam.*, 72, 161-195, 1993.

Lambeck, K., *The Earth's variable rotation*, Cambridge Univ. Press, London and New York, 1980.

Moffatt, H.K., Topographic coupling at the core-mantle interface, *Geophys. Astrophys. Fluid Dyn.*, 9,279-288, 1978.

Moritz, J.L., and I.L. Mueller, *Earth rotation: Theory and observation*, The Ungar Publishing Co., New York, 1987.

Musk, W. 11., and G. J. F. MacDonald, *The rotation of the Earth*, Cambridge University Press, 1960.

Voorhies, C.V., "Coupling an inviscid core to an ccc[ricrilly-insulating mantle," *J. Geomag. Geoelect.*, 43, 131-156, 1991.

Wahr, J. M., The Earth's rotation, *Ann. Rev. Earth Planet Sci.*, 16, 231-249, 1988.

---

<sup>1</sup>Raymond Hide, 65 Charlbury Road, Oxford OX2 6UX, England, U. K.; Department of Physics, Clarendon laboratory, University of oxford, Parks Road, Oxford OX 13PU, England, U.K.

Copyright 1995 by the American Geophysical Union.

Paper number 94-L-7228,

Received: August 9, 1994

Revised: February 1, 1995

Accepted: February 6, 1995

HIDE: TOPOGRAPHIC TORQUE

**Figure 1.** Schematic diagram illustrating the general system analyzed in this paper and the proposed division of the Earth's liquid metallic core into two main regions, the "pol(oid)osphere" and the "tor(oid)osphere" (which merge at the (non-spherical) "pol(oid)opause"), on the basis of the comparative strengths of the toroidal magnetic field  $B_T$  (which has no radial component and is largely confined to the core owing to the low electrical conductivity of the mantle) and the poloidal magnetic field  $B_P$ . In the polosphere  $|B_T|$  is typically no larger than  $|B_P|$  and in consequence polospheric flow is expected to be quasi-geostrophic nearly everywhere (see (3.7)). But in the torosphere  $|B_T|$  by definition has its highest values which may be at least comparable with  $|B_P|$  and may even exceed  $|B_P|$  by as much as an order of magnitude, in which case torospheric flow would be highly non-geostrophic.  $\Omega$  is the rotation vector of the mantle,  $S$  is the core-mantle boundary and  $C$  a general mathematical control surface used in the mathematical analysis presented in § 3 (see (1.1), (3.1) and (3.2)). Vertical scales of the bounding surfaces are greatly exaggerated. The position of the polopause is a matter for investigation, but its typical depth below the CMB may be much greater than  $h$  but much less than  $c$  (see (2.1)).

**Figure 1.** Schematic diagram illustrating the general system analyzed in this paper and the proposed division of the Earth's liquid metallic core into two main regions, the "pol(oid)osphere" and the "tor(oid)osphere" (which merge at the (non-spherical) "pol(oid)opause"), on the basis of the comparative strengths of the toroidal magnetic field  $B_T$  (which has no radial component and is largely confined to the core owing to the low electrical conductivity of the mantle) and the poloidal magnetic field  $B_P$ . In the polosphere  $|B_T|$  is typically no larger than  $|B_P|$  and in consequence polospheric flow is expected to be quasi-geostrophic nearly everywhere (see (3.7)). But in the torosphere  $|B_T|$  by definition has its highest values which may be at least comparable with  $|B_P|$  and may even exceed  $|B_P|$  by as much as an order of magnitude, in which case torospheric flow would be highly non-geostrophic.  $\Omega$  is the rotation vector of the mantle,  $S$  is the core-mantle boundary and  $C$  a general mathematical control surface used in the mathematical analysis presented in § 3 (see (1.1), (3.1) and (3.2)). Vertical scales of the bounding surfaces are greatly exaggerated. The position of the polopause is a matter for investigation, but its typical depth below the CMB may be much greater than  $h$  but much less than  $c$  (see (2.1)).

