

APPLICATION OF HIGHER ORDER VECTOR ELEMENTS TO THE COUPLED FINITE ELEMENT-COMBINED FIELD INTEGRAL EQUATION (FE/CFIE) TECHNIQUE

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In the original formulation of the coupled finite element-integral equation method, first order vector tangential edge elements (Whitney elements) were used (T. Cwik, V. Jamnejad, and C. Zuffada, IEEE-APS Symposium, June 1993). Although excellent accuracy with relatively fast computation time is achieved, it is possible to obtain even better accuracy for the same mesh size and/or a smaller mesh size but with similar accuracy, by using higher order basis elements. These elements have been rigorously formulated by the French researcher Nedelec. The elements are formulated in terms of barycentric coordinates in a tetrahedron, which provides the simplest approach to solving the problem. The field inside the tetrahedron is then written as a sum over these basis functions. We will discuss three types of elements in a comparative investigation of these elements.

i) The first order edge-based elements are composed of six (6) basis functions related to the six edges of a tetrahedral finite element. These elements have a constant value along the edges. This is the approach taken by most finite element researchers (e.g., Bossavit, et al). These basis functions preserve the continuity of tangential components of the fields across element interfaces, and are superior to node-based elements in a majority of cases, by avoiding spurious modes and easily handling material boundary edges, corners, etc.

ii) The incomplete second order elements are composed of twelve (12) basis functions related to the six edges of a tetrahedral finite element, two per edge. These elements have a linear variation along the edges as opposed to the first elements which provide only a zero order along the edges. These twelve basis functions are only a part of a complete second order representation as proposed by Nedelec. This approach is similar to the one proposed by G. Mur and T. De Hoop. The total number of unknowns in this approach is twice that of the first but provides a higher accuracy for the same number of finite elements.

iii) The complete second order Nedelec elements are composed of twelve (12) basis functions related to the six edges of a tetrahedral finite element, two per edge, as above, and eight additional basis functions related to the four facets of the tetrahedron, two per facet, for a total of 20 basis functions. The edge elements have a linear variation along the edges, while the face elements are constant vectors on their respective facets. The number of unknowns in this formulation is about six times that of the first order elements, but even higher accuracies can be achieved for the same number of elements.

Application of the above basis functions to the finite element equation for the EM field reduces to the evaluation of two element matrices for each tetrahedron element:

$$[S]_{mn} = \int_{tet} (\nabla \times \bar{\alpha}_m) \cdot (\nabla \times \bar{\alpha}_n) dv \text{ and } [T]_{mn} = \int_{tet} \bar{\alpha}_m \cdot \bar{\alpha}_n dv$$

These matrices involve two types of integrals. Both types can be integrated analytically in terms of the barycentric coordinate system in the tetrahedron element, and reduce to terms involving only edge vectors and normal vectors to the facets. Application of all three methods to typical three-dimensional field problems and their pros and cons will be discussed. Some examples will be demonstrated and comparative results will be presented.