ALTERNATIVE COMPUTATIONAL APPROACHES FOR PROBABILISTIC FATIGUE ANALYSIS

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Abstract

The failure probabilities for specific failure modes of mechanical or electronic components can be estimated by Direct Monte Carlo simulation. The feasibility of alternative efficient methods for such failure probability computations is discussed. First and second order reliability methods are used for fatigue crack growth and low cycle fatigue structural failure modes to illustrate problems which can arise when realistic applications with complex nonanalytic failure models are considered.

Nomenclature

D	region of failure in driver space
$g(x_1, \ldots x_n)$	limit state or g-function which is the boundary between failure and nonfailure regions in driver space
$N(0, 1)$	normal distribution with mean 0 and variance 1
p_f	failure probability
$x_1, \ldots x_n$	parameters or drivers which define the space in which failure and nonfailure regions may be identified
β	reliability index
&	amplitude of periodic noise
ω	frequency of periodic noise

Introduction

A procedure developed by the Jet Propulsion Laboratory (JPL) for assessing failure risk of mechanical or electronic components by means of a quantitative methodology for estimating the probability of occurrence of specific failure modes is presented in [1] and [2]. This methodology, called Probabilistic Failure Assessment (PFA), is applicable to failure modes that can be characterized by analytical or empirical modeling of failure mechanisms, such as those of structural, electro optical, propulsion, power, and thermal control systems. It is especially useful when models or information used in analysis are significantly uncertain or approximate, which is typical in advanced systems development projects. That suitability is in contrast with pure sampling probabilistic methods which typically require infeasible amounts of test and/or operating data in order to estimate failure probabilities with acceptable confidence.

In the PFA methodology, analytical or empirical models that characterize specific failure modes are used in a prescribed probabilistic structure in which quantified uncertainty in the information used in the analysis and quantified uncertainty of the models themselves are both considered in estimating failure risk. These models are typically complex and the approach requires an accurate characterization of the left-hand tail of the failure probability distribution function warranted by the available information for each failure mode of interest. There are various computational methods which can be used in order to accurately generate those failure probability distribution functions.

Monte Carlo Simulation

The most straightforward computational procedure, in the sense that it can accommodate naturally the forms in which engineering modeling information is available, is Monte Carlo simulation. Monte Carlo simulation provides an information framework which mirrors the actual physical failure process, minimizing any problems associated with the modeling requirements of alternative computational methods.

The usual reasons advanced for preferring alternative computational methods to Monte Carlo simulation are

(1) that for very small failure probabilities Monte Carlo simulation is infeasible due to the large number of simulations required,

(2) that finite element analysis models and other procedures used in complex deterministic engineering analysis cannot feasibly be embedded in a Monte Carlo simulation since the computations for such procedures themselves are extremely time intensive, and

(3) that Monte Carlo simulation is inefficient when performing repeated computations for the failure models with small changes in the characterization of the driver variables, as might be done in the design process.

In the case of (1), for many practical problems the state of knowledge about the failure mode will be such that the failure probabilities of interest can be feasibly estimated by direct Monte Carlo simulation. For cases where it is necessary to estimate very small failure probabilities, Monte Carlo simulation may still be feasible by using variance reduction techniques such as importance sampling, a procedure which increases the number of failures in any simulation of fixed size. Variance reduction techniques are discussed in [3]. In the case of (2), it is quite true that it isusually infeasible to embed a finite element analysis model or perform a procedure like cycle-by-cycle crack growth integration within a Monte Carlo simulation. But response surface methods can be used to resolve both of these issues and in the case of crack growth integration there also exist block-by -block integration schemes which can be used. Response surface methods are discussed in [4] and [5] and block-by-block integration is discussed in [6]. In the case of (3), if a more efficient method can be validated using Monte Carlo methods, then it may be possible to achieve savings in repeated computations, although those savings may not be significant if more than one or two points on the failure probability distribution are required.

Other Computational Methods

Alternatives to Monte Carlo simulation often fail to give demonstrably accurate results or are not significantly faster for realistic problems in which complex failure models are employed. Alternative computational methods can be used in probabilistic analyses which employ simple well behaved failure models, particularity if the failure criterion is expressed explicitly in a closed form equation as opposed to a complex multi-step algorithm. However, in these simpler cases execution time on widely available modern computers usually is not an issue, even for the direct Monte Carlo method,

A comprehensive discussion of computational methods for generai probabilistic iife modeling is provided in [7]. The alternative procedures compared are (1) Direct Analytical integration, (2) Numerical integration, (3) Direct Monte Carlo simulation, (4) Efficient Monte Carlo simulation, (5) Propagation of errors, and (6) g-function methods. For the kinds of complex failure models in which we are interested, the only conceivable alternatives to direct or efficient Monte Carlo simulation are g-function methods. g-function methods are discussed in detail in [3], [8], [9], [10], [1 1], and [12].

Alternative Procedures For Failure Probability Evaluation

The basic problem can be couched in the evaluation of the following integrai:

$$p_f = \int \overset{\cdots}{_D} \int f(x_1, \ldots, x_n)\, dx_1 \ldots dx_n \qquad (1)$$

where p_f is the probability of failure, $f(x_1, \ldots x_n)$ is the joint probability density function of the drivers $x_1, \ldots x_n$ and $D = \{(x_1, \ldots x_n) : g(x_1, \ldots x_n) < 0\}$ is the region of failure. The boundary, $g(x_1, \ldots x_n) = 0$, which divides the multi-dimensional driver space into regions of failure and regions of safety is known as the limit state or g-function. This integral can be evaluated in closed form for a very limited set of driver distributions and limit state functions. The directnumerical evaluation of this integral is also only possible in simple cases of limited practical interest.

In the Monte Carlo method this integral is evaluated by a direct simulation of the failure process. If the failure analysis is structuredproperly and a particular parameter identified (e.g., life, in a fatigue analysis), then the entire cumulative probability of failure curve against that parameter is readily obtained. In the Monte Carlo simulation, the failure model analysis can follow the same procedures used in a state-of-the-art deterministic failure analysis. However, the failure analysis has to be performed a large number of times, typically ten to one hundred divided by the lowest probability of failure of interest. There are numerous techniques such as importance sampling, response surface methods, directional simulation, and others which are designed to decrease the simulation effort required.

In the first and second order reliability methods (FORM/SORM), the approach is to transform the integral of Equation 1 to an approximately equal integral which can be efficiently evaluated. This is done by: (1) transforming the driver variables to a space where the region of the main contribution to the probability integral can be located; *(2)* approximating the limit state function as a simply defined surface in that region (linear for FORM, quadratic for SORM); and (3) analytically or numerically computing p_f using the newly defined g-function and the transformed variables. The transformation used in step 1 is one which maps the probability distribution for each driver into a standard normal distribution. For a given limit state function, the main contribution top, comes from the region where g is closest to the origin in the transformed driver space. The closest point to the origin in the transformed driver space is called the *"design point"* or the β-point. The g-function is then approximated by a simply defined surface at that point and step 3 can readily be performed. Thus the problem has been changed from a complex integration problem to a mathematical optimization problem for finding this closest point using the gradients of the g-function. For analytic g-functions, numerous techniques for solving this problem exist; however, for complex nonanalytic g-functions, this optimization problem has often been found to be unstable.

There have been numerous algorithms developed which fall under the broad classification of Mean Value First Order (MVFO) techniques [10]. All of the MVFO methods presented in the literature usc a linear g-function approximation obtained by expanding the limit state function about the mean value of the basic variables. Thus, these methods do not require formal optimization schemes to locate the design point as is needed when one uses the FORM/SORM algorithm. Note that the dependence on linearization is a serious drawback if the g-function is a product of some of the drivers. The linear g-function approximation cannot be expected to properfy represent the actual g-function under that circumstance. By taking logarithms one could transform a product function into a summation; however this scheme (or any similar scheme) breaks down when the driver dependence is mixed or is not explicitly known, as is the case for the class of engineering problems with which we are concerned. Some of the current MVFO methods can be applied repeatedly in an attempt to improve the initial reliability y estimates. These methods involve more calculations than the MVFO methods and there is no guarantee that the final estimates are superior to the initial ones. In contrast, FORM/SORM algorithms involve a similar level of calculations but approach the exact reliability for a unique β-point provided the optimization algorithm is stable.

The instability problems associated with the use of FORM/SORM for complex nonanalytic problems would not be apparent in the evaluation of reliability using MVFO methods, However, we may clearly conclude that the use of MVFO methods for these problems would also not give correct answers, since it is highly unlikely that the approximations associated with MVFO methods will allow a correct answer to be generated when a FORM/SORM evaluation would fail to do so. There have been published validation examples for MVFO methods which used nonanalytic g-functions [10] and [11]. However, in these validation examples the g-function dependency on the drivers was generally monotonic, the failure models were relatively simple, the number of drivers that need not be treated deterministically was small, and the coefficient of variation of the stochastic drivers was never large. Thusthe robustness of MVFO methods has not been verified for engineering problems where the g-functions are complex nonlinear, non-monotonic, nonanalytic multi-step algorithmic functions of products of highly non-normally distributed drivers and where the combination of incomplete knowledge and intrinsic variation often causes the driver distributions to have large coefficients of variation (e.g., greater than 1).

There is a computation time advantage to the MVFO methods as compared to the FORM/SORM methods. However the FORM/SORM methods are sufficiently fast to make the need for increased speed superfluous. The lack of a theoretical means for establishing the level of error due to the

Table 1 Commonly Used Methods for the Computation of Probability of Failure in Engineering

Method	Formulation	Attributes
Analytical	Direct Integral	Only possible for extremely limited set of driver distributions and failure models. Academic interest only.
Numerical Direct	Direct Integral	Only possible for limited set of driver distributions and failure models of limited practical interest.
Monte Carlo Direct	Simulation	General, accurate solution for all driver distributions and failure criteria. Many simulations required.
Monte Carlo Efficient	Selected Simulation	Generai, faster than direct Monte Carlo. Not yet demonstrated as effective for our very complex, non-analytic multi-step failure models.
FORM/SORM	Limit State Function	Approximates exact solution for low probabilities of failure. Significantly faster than Monte Carlo methods. Maybe unstable for complex, non-analytic failure models.

approximations within the MVFO approaches in contrast to the sound mathematical basis of the FORM/SORM approaches leads us to conclude that for those problems where approaches other than Monte Carlo simulation are desirable and tractable, the method of choice would be a FORM/SORM approach. A summary of the computational methods discussed above is given in Table 1.

In addition to the approaches listed above there are many other methods for estimating reliability, e.g., the optimal distribution estimator, propagation of errors, and the Markov process model [13]. The use of these methods can significantly reduce computation time relative to direct Monte Carlo simulation. However, these methods, like all the other approximate techniques for reliability analysis, (i) are not general; (ii) can yield significantly inaccurate reliability estimates; and (iii) may not converge when based on iteration procedures.

Convergence Problems Using FORM/SORM Computation

One problem that can arise with FORM/SORM methods is that SORM may fail to converge for some modeling parameters. Consider the following simple analytical problem:

Let the limit state be defined by

$$x_2 = -.05X_1^2 + 3 + \varepsilon \sin(\omega x_1) \quad (2)$$

where $X_1, x_2 \sim N(0,1)$. The failure region is indicated in Figure 1 fore = O and the convergence region for SORM in Figure 2 as a function of ω and ε. The SORM algorithm fails to converge when the noise in the limit state, $\varepsilon \sin(\omega x_1)$, fluctuates rapidly and its amplitude is not small. Such noise can occur in practical problems when the limit state is not available in closed-form.

FORM/SORM results were computed using the STRUREL program documented in [12]. That program also generates efficient Monte Carlo simulation results using an importance sampling algorithm, at the cost of additional computation time.

The convergence problem illustrated in Figure 2 is not just of academic interest. Probabilistic

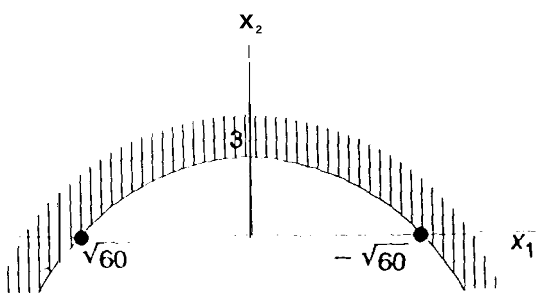

Figure 1 Failure Region For ε = O

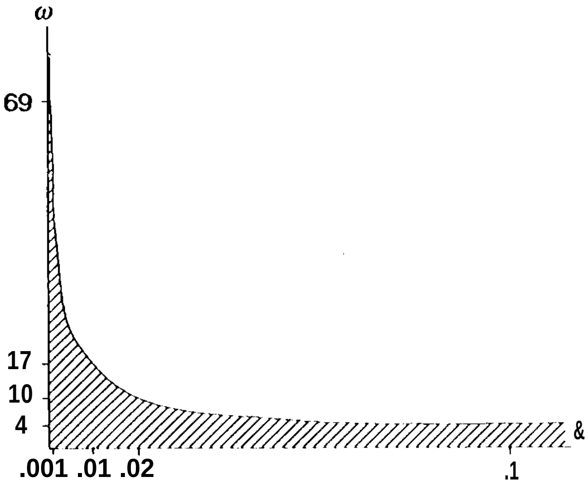

Figure 2 SORM Convergence Region

Failure Assessment for the fatigue crack growth failure mode in a Heat Exchanger (HEX) tube using direct Monte Carlo simulation has been described in detail in [2].

Results using a simplified version of the HEX tube assessment were presented in [1 4]. A simplified stochastic driver specification was used, a shorter load history with only three stochastic loads was used, aerodynamic load factors were fixed, and a generic stochastic stress analysis accuracy factor was used rather than separate dynamic and static factors. Results were calculated for five points in the tail of the failure probability curve. Convergence was achieved for all but the smallest failure probability SORM value.

A granularity problem induced by removing those simplifying assumptions prevented STRUREL SORM algorithm convergence and FORM failed to converge for the smallest failure probability. Those results are summarized in Table 2 where the Direct Monte Carlo results were based on 10,000 simulations.

Multiple β-point Problems Using FORM/SORM Computation

Another problem that can arise with FORM/SORM methods is that there may be more than one β-point. Consider the following simple analytical problem:

Let the limit state be defined by

Table 2 HEX Tube Flaw Propagation Failure Probabilities

Direct Monte Carlo	FORM	Efficient Monte Carlo	SORM
.0001	–		–
.0002	.0007	.00009	–
.0004	.0019	.0003	-
.0006	.0027	.0005	-
.0008	.0032	.0007	-
.0010	.0048	.0010	-
.0020	.0087	.0026	–
.0040	.0143	.0050	–
.0060	.0186	.0064	–
.0080	.0232	.0086	–
.0100	.0268	.0106	-

$$X^* = 3 - (x, + .01)^2 + 2(x, + .01)^4 \qquad (3)$$

where $X_1, x_2 \sim N(0, 1)$. The failure region is indicated in Figure 3.

The FORM/SORM results are presented in Table 3 where Direct Monte Carlo simulation with 1,000,000 simulations yields a failure probability, p_f = .000914.

The multiple β-point problem illustrated by this example is also not just of academic interest. Probabilistic Failure Assessment for the Low Cycle Fatigue (LCF) crack initiation failure mode in a turbopump turbine disk using direct Monte Carlo simulation has been described in detail in Moore, et al. (1992a). Two /?-points were found and the corresponding computational results are presented

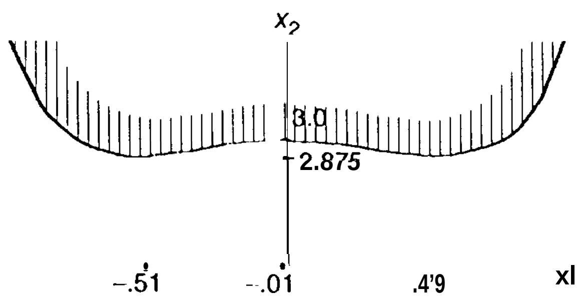

Figure 3 Failure Region

Table 3 FORM/SORM Computations

		FORM	Efficient Monte Carlo	SORM	Efficient Monte Carlo
β-Point #1	β	2.913	2.966	3.259	3.244
	p_f	.00179	.00151	.00056	.00059
β-Point #2	β	2.914	2.952	3.258	3.206
	p_f	.00178	.03158	.00056	.00067

in Table 4 where the Direct Monte Carlo results were based on 400,000 simulations. The approximate probabilities of failure based on a single/?-point can differ significantly from the "true" failure probability determined by direct simulation. FORM/SORM approximations of the failure probability when there is more than one β-point can be obtained by using concepts of system reliability. However, one generally does not know in a complex problem how many #?-points exist. Therefore, the use of system reliability concepts encounters serious difficulties.

Computation Time Issues

For the HEX tube fatigue crack growth application the following three alternative computer systems were used to perform computations.

(1) a 33 MHz 80386/80387 PC using the Lahey FORTRAN F77L-EM/32 Ver. 5.01 compiler

(2) a 33 MHz 80486DX PC using the Lahey FORTRAN F77L-EM/32 Ver. 5.01 compiler

(3) an Intel Touchstone Gamma parallel processing computer system with hypercube architecture using the Pacific Group's pgi compiler (8 of the 16 available nodes were used)

Monte Carlo results were computed for each of these three systems and STRUREL results were derived using system (1). The computation times for 10,000 Monte Carlo simulations were 17.8 hours, 12.1 hours, and 1.1 hours for systems (l), (2), and (3), respectively. These results illustrate the ability to take advantage of a parallel processing computer system for Monte Carlo simulation. STRUREL results in Table 2 required 37.4 hours of system (1) computation time. Approximately 50°A of that time was required to generate the FORM solutions, with the remaining time required to generate Efficient Monte Carlo results using 100 samples in the FORM-based importance sampling procedure available in STRUREL.

For the turbine disk LCF application both Monte Carlo results and STRUREL results were computed on system (1). The computation time for 400,000 Monte Carlo simulations was 3.8 hours and the STRUREL results presented in Table 4 required 4.5 hours. Less than 1% of that time was required to generate the FORM and SORM solutions. Almost all

Table 4 Turbine Disk LCF Failure Probabilities

	β-Point #1			β-Point #2		
Direct Monte Carlo	FORM	Efficient Monte Carlo	SORM	FORM	Efficient Monte Carlo	SORM
.0001	.0001	.00009	.00005	.00008	.00006	.00003
.0002	.0003	.0002	.0001	.0002	.0002	.00007
.0004	.0006	.0004	.0003	.0004	.0003	.0001
.0006	.0008	.0006	.0004	.0005	.0005	.0002
.0008	.0012	.0009	.0005	.0007	.0007	.0003
.0010	.0013	.0010	.0006	.0008	.0008	.0003
.0020	.0025	.0020	.0012	.0016	.0017	.0006
.0040	.0047	.0040	.0023	.0030	.0034	.0013
.0060	.0067	.0059	.0033	.0043	.0051	.0019
.0080	.0087	.0079	.0045	.0056	.0069	.0026
.0100	.0106	.0098	.0056	.0069	.0086	.0033

of that time was spent generating Efficient Monte Carlo results using 2000 samples in the FORM-based Importance sampling procedure available in STRUREL.

These computational results illustrate that to derive accurate numerical results for complex problems computation time constraints relevant for Monte Carlo simulation are not a serious issue with available computer systems.

Conclusions

There are numerous conceptual alternatives to Direct Monte Carlo simulation for the evaluation of small failure probabilities. For relatively simple failure models with closed-form g-functions, FORM/SORM methods can be a useful alternative. However, for complex nonanalytical problems, such as the crack initiation and crack growth analyses discussed in this paper, Direct Monte Carlo simulation is necessary to validate FORM/SORM results. In fact, for the HEX tube fatigue crack growth failure mode model, it was found that SORM did not converge. And, for the turbine disk LCF failure mode model it was found that there were two /?-points, so that FORM/SORM results could not be used.

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