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A New Solution for Dilaton-Maxwell Gravity.

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Abstract

An interesting static spherically symmetric solution corresponding to Einstein-Maxwell gravity coupled to a dilaton field with negative kinetic term has been obtained. This solution is characterized by the set of two arbitrary parameters, the physical mass μ_0 and electric charge Q. It has two horizons on which the metric, scalar curvature and both dilaton and electromagnetic fields are regular. Another feature of this solution is that, the physical mass is bounded by the electric charge as $\mu_0 \geq \sqrt{2}Q$ (unlike the Reisner-Nordström solution for which $\mu_0 \geq$ Q). The structure of the scalar curvature has been analyzed.

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The theoretical problems of modern cosmology and gravity have recently been attacked from the point of view of the tensor-scalar and many-dimensional theories of the Kaluza-Klein type. These approaches are reviving interest in the so called dilaton fields (i.e. neutral scalar fields). It is now believed that the scalar-tensor theories of gravity are the most promising extension of the theoretical basement of the gravitational theory [1], [2]. Although they naturally arise in theory, the existence of the scalar fields conflicts with the basic principles of the general relativity. Thus, the inclusion of the scalar field in the theory leads to a violation of the equivalence principle, modification of large scale gravitational phenomena, and casts doubt upon the constancy of the constants [2]. The behavior of the matter fields in the presence of the dilaton has been studied in many publications, notably [1]- [8]. It was shown in [3] that the extreme charged dilatonic black hole solutions obtained in [4] behave as elementary particles in a sense and that there exists a mass gap in their excitation spectrum. It was conjectured in [5], that the supersymmetry, co-existing with the scalar fields, is likely playing the role of the cosmic censor by hiding the singularities from external observers. These conclusions make obtaining a solution with regular horizons in the presence of a scalar field quite intriguing.

A generic class of the theories with an arbitrary number of the scalars coupled to one tensor field has been recently considered in [6]. By studying the tensor-bi-scalar case, the authors illustrated that although these theories might have the same post-Newtonian limits as the general theory of relativity, they predict non-Einsteinian behavior of the stellar objects in a strong gravitational field. This discrepancy leads to an observable effects which might be extracted from the binary pulsar data. The generalized bi-scalar Jordan-Brance-Dicke theory has been investigated in [7]. One of the results of these investigations is that some of the scalar fields in tensor-multi-scalar theories must have non-positively defined kinetic terms. This result comes from the necessity of meeting the constraints based on the post-Newtonian tests of the general relativity performed to date. Because of these results, it would be interesting to explore the behavior of the static gravity coupled a scalar field with the negative kinetic term in the presence of the other fields of matter.

In this paper we will focus our attention on the simplest extension of gravity coupled to interacting electromagnetic and massless scalar fields. The form of the action is suggested by the low-energy limit of the string theory [1], [5] and might be written as:

$$S = -\frac{1}{16\pi}\int \sqrt{-g}\Big(R + 2g^{mn}\nabla_m\phi\nabla_n\phi + e^{-2\phi}g^{mn}g^{kl}F_{mk}F_{nl}\Big)d^4x. \qquad (1)$$

We also would like to study how the interaction between the dilaton and electromagnetic fields (given by this expression) will manifest itself on the structure of the solution. Thus, extremizing the action (1) with respect to g_{mn}, ϕ and A_m it is easy to obtain the following fields equations:

$$R_{mn} + 2\nabla_m\phi\nabla_n\phi + e^{-2\phi}\Big(2g^{kl}F_{mk}F_{nl} - \frac{1}{2}g_{mn}g^{cd}g^{ef}F_{ce}F_{df}\Big) = 0, \qquad (2)$$

$$g^{mn}\nabla_m\nabla_n\phi + \frac{1}{2}e^{-2\phi}F_{mn}F^{mn} = 0, \qquad (3)$$

$$\nabla_m(\sqrt{-g}g^{mn}g^{kl}F_{nl}) = 0, \qquad \nabla_k F_{mn} + \nabla_n F_{km} + \nabla_m F_{nk} = 0. \qquad (4)$$

We will look for a solution to (2)-(4) together with the covariant de Donder gauge condition [8]:

$$D_m\sqrt{-g}g^{mn} = 0, \qquad (5)$$

where D_m is the covariant derivative with respect to Minkovsky metric:

$$\gamma_{mn} = \mathrm{diag}(1, -1, -r^2, -r^2 \sin^2\theta).$$

We accept that the static spherically symmetric fields are everywhere radial (meaning that $F_{10}(t, r, u, \varphi) = E(r)$ and $\phi(t, r, \theta, \varphi) = \phi(r)$). The effective metric then might be given by:

$$g_{mn} = \mathrm{diag}(u(r), -v(r), -w(r), -w(r)\sin^2\theta). \tag{6}$$

Note that if the scalar field is absent (i.e. $\phi = 0$), the solution resulting from the action (1) describes a charged black hole with mass μ_0 and electromagnetic charge Q, which, in the harmonic coordinates of the Minkovsky space-time, might be presented as follows:

$$ds^2 = \left(\frac{r-\mu_0}{r+\mu_0} + \frac{Q^2}{(r+\mu_0)^2}\right)dt^2 - \left(\frac{r-\mu_0}{r+\mu_0} + \frac{Q^2}{(r+\mu_0)^2}\right)^{-1} dr^2 - (r+\mu_0)^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{7}$$

This is the harmonic Reisner-Nordström solution. It describes a charged black hole with one horizon $\mu > 0$ which is defined by the relation $\mu = \sqrt{\mu_0^2 - Q^2}$.

The general static spherically symmetric asymptotically flat solution of the system of equations (2)-(4) together with the covariant harmonic condition (5) might be obtained as follows:

$$u(r) = \frac{1}{v(r)} = \frac{r^2-\mu^2}{w(r)}, \qquad E(r) = \frac{Q}{w(r)} e^{2\phi(r)}, \tag{8a}$$

$$\phi(r) = \frac{1-4k^2}{8k} \ln\frac{r+\mu}{r-\mu} + \frac{Q^2}{16\mu^2 k^2}\left[1 - \left(\frac{r-\mu}{r+\mu}\right)^{2k}\right] + \phi_\infty, \tag{8b}$$

$$w(r) = (r^2-\mu^2)\left(\frac{r+\mu}{r-\mu}\right)^{\frac{1+4k^2}{4k}} \exp\left[\frac{Q^2}{8\mu^2 k^2}\left(1 - \left(\frac{r-\mu}{r+\mu}\right)^{2k}\right)\right], \tag{8c}$$

where $\mu > 0$, Q and k are some parameters (arbitrary for the moment). It is easy to see that because of the first term in the expression for ϕ (8b), this solution is in general singular. Fortunately, by choosing the parameter $k = \pm 1/2$, one may eliminate the logarithmic term and hence can avoid the singularity [3]. For example for $k = 1/2$, from the solution (8) one will obtain a qualitatively different result:

$$\phi(r) = \frac{Q^2}{4\mu^2}\left(1 - \frac{r-\mu}{r+\mu}\right), \tag{9a}$$

$$w(r) = (r+\mu)^2 \exp\left[\frac{Q^2}{2\mu^2}\left(1 - \frac{r-\mu}{r+\mu}\right)\right], \tag{9b}$$

$$u(r) = \frac{1}{v(r)} = \left(\frac{r-\mu}{r+\mu}\right)\exp\left[\frac{Q^2}{2\mu^2}\left(\frac{r-\mu}{r+\mu} - 1\right)\right], \tag{9c}$$

where we have taken $\phi_\infty = 0$. Note that in the case ($Q \to 0$) this result correspond to harmonic Fock solution for a static spherically symmetric distribution of matter [10]:

$$ds^2 = \left(\frac{r-\mu}{r+\mu}\right)dt^2 - \left(\frac{r+\mu}{r-\mu}\right)dr^2 - (r+\mu)^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{10}$$

[3] Note that if the interaction between the dilaton and the electromagnetic fields is neglected (i.e. setting $e^{-2\phi} = 1$ in the action (1)), the choice $k = +1/2$ corresponds to extracting of the dilaton from the action. The solution in this case is presented by expression (7).

Thus, the obtained result (9) is a conformal modification of the Fock solution in the presence of the dilaton and electromagnetic fields. This solution is labeled by the means of two parameters μ and Q and has two regular events horizons ($r_\pm$), which correspond to the physical mass μ_0 and the electric charge Q of the hole as follows:

$$r_\pm = \mu_\pm = \frac{1}{2}\left(\mu_0 \pm \sqrt{\mu_0^2 - 2Q^2}\right). \tag{11}$$

It is easy to see that the determinant of the metric g_{mn} corresponding to the solution (9) is also regular on the horizons (11):

$$g = w(r)^2 \sin^2\theta = -(r+\mu)^4 \exp\left[\frac{Q^2}{\mu^2}\left(1 - \frac{r-\mu}{r+\mu}\right)\right] \sin^2\theta. \tag{12}$$

The solution, analogous to (9), might be obtained in the Schwarzschild coordinates as well. Indeed, by changing the gauge conditions from ones given by (5) to:

$$u_S(r) = \frac{1}{v_S(r)} = p(r)e^{2\phi_S(r)}, \qquad w_S(r) = r^2 e^{-2\phi_S(r)}, \tag{13a}$$

from the Eqs.(2)-(3) one will get the following result:

$$p(r) = 1 - \frac{2\mu}{r}, \qquad \phi_S(r) = \frac{Q^2}{2\mu r}, \qquad E_S(r) = \frac{Q}{r^2}. \tag{13b}$$

Note that the metric and both fields in this case have a singularity at the point $r = 0$. However, this singularity is hidden by the horizons ($r_\pm$) Eq. (11) on which the metric and both fields are regular.

One might expect that the scalar curvature for both solutions (9) and (13) will be also regular on the surfaces (11). It is well-known that as far as the electromagnetic part of the energy-momentum tensor corresponding to action (1) is traceless, the only contribution to the Riemann curvature R comes from the dilaton field. By taking the trace of the equations (3), we might present R for the solution (9) as follows:

$$R(r) = -2g^{mn}\nabla_m\phi\nabla_n\phi = 2\frac{\phi'^2}{w}(r^2-\mu^2) = \frac{Q^4}{2\mu^2}\frac{r^2-\mu^2}{(r+\mu)^6}\exp\left(\frac{Q^2}{2\mu^2}\left[\frac{r-\mu}{r+\mu}-1\right]\right). \tag{14a}$$

Analogously, the scalar curvature R_S might be obtained with the help of solution (13):

$$R_S(r) = \frac{Q^4}{2\mu^2 r^4}\left(1 - \frac{2\mu}{r}\right)\exp\left(-\frac{Q^2}{2\mu r}\right).$$

An interesting case arises when $\mu_0^2 = 2Q^2$ (the extreme black hole). Thus, from the expression (12) we will obtain $\mu = \mu_\pm = \mu_0/2$ and, as a result, the scalar curvature R_0 in the extreme regime for the result ($14a$) can be given by:

$$R_0(r) = 8\mu_0^2\frac{4r^2-\mu_0^2}{(2r+\mu_0)^6}\exp\left(-\frac{2\mu_0}{2r+\mu_0}\right). \tag{14b}$$

The scalar curvature ($14b$) tends to be zero at a large distance ($R_0(r) \to 0$, $r \to \infty$). For any non-zero value of the physical mass μ_0, the curvature R_0 behaves near the horizon as:

$$R_0(r) \approx \frac{2r-\mu_0}{4e\mu_0^3} + O(r^2).$$

One might easily see that the solutions (7) and (9) coincide in the post-Newtonian limit (p/r $\to$ 0). By expressing these results in terms of the physical mass μ_0 and electromagnetic charge Q, we will obtain the same result for the linear element ds^2 in the weak field approximation:

$$ds^2 = \Big(- 2\frac{\mu_0}{r} + \frac{2\mu_0^2}{r^2} + \frac{Q^2}{r^2}\Big)dt - \Big(1 + 2\frac{\mu_0}{r}\Big)dr^2 - (r + \mu_0)^2(d\theta^2 + \sin^2\theta d\varphi^2). \qquad (15)$$

It is quite interesting that up to the post-Newtonian level of accuracy, these results, which were obtained with and without the scalar field, produce the same contribution of the electrostatic energy in the effective metric. This means that they can not be distinguished by any experiments in the solar system performed to date [10]. Moreover, this contribution in the effective metric is unlikely to be detected even in the next generation of experiments [2]. Indeed, the constraints imposed on new weak forces from the behavior of the astrophysical objects [12] gives as the maximum possible electric charge Q_{max} carried by celestial bodies the following estimation: $Q_{max} \leq 10^{36}e$. Using this, the parameterized post-Newtonian parameter β in the case of the celestial body with the physical mass $\mu_0 = nM_\odot$ might be obtained as:

$$\beta - 1 = \frac{Q_{max}^2}{2\mu_0^2} \leq \frac{4.35}{n^2} \times 10^{-7},$$

where $M_\odot$ is the mass of the Sun. However, even with $n = 1$, this result gives a practically unmeasurable value for the contribution of the electrostatic energy in the gravitational field generated by the charged astrophysical body.

Thus, we have obtained the static spherically symmetric solution of the Einstein-Maxwell gravity coupled to the dilaton field with the negative kinetic term. It has has two regular horizons which coincide for the extreme black hole. The distinctive property of this solution is that the physical mass μ_0 is always bounded by the electric charge as $\mu_0 \geq \sqrt{2}Q$ (unlike the Reisner-Nordström solution (7) for which the extreme value of for the mass defined as $\mu_0 = Q$). For Q = 0 the obtained result coincides with the Fock (9) or Schwarzschild (13) solutions of Einstein's equations. Another interesting feature of that type of solution is that it satisfies the no-hair theorem, which states [13] that no parameters other than mass, electric charge and angular momentum may be associated with a black hole. It is easy to see that the dilaton charge D in this case is not an independent variable. By analyzing the behavior of the dilaton field at a large distance, one might get the following value of dilaton charge $D = Q^2/2\mu$. This result states that the solution is characterized by the set of two independent parameters only (physical mass μ_0 and electric charge Q).

Unfortunately, the negative kinetic term $-g^{mn}\nabla_m\phi\nabla_n\phi$ in (1) generally leads to a theory without any stable states, This means that when the system is quantized, any flat wave will have negative energy and the more negative the energy, the bigger the amplitude or the frequency of the waves. The authors of [2] note the need for scalar fields with the negative kinetic terms in multi-scalar-tensor field theories, and point out the potential of such fields for approaching the problems with the stellar and quantum stability. However, it was shown in [7] that for the theory with the negative kinetic terms, the cosmological solutions which are suitable for experimental tests in the solar system are unstable. Because of these results, we believe that although the obtained solution (9) is unlikely to be stable, it might provide an interesting possibility for investigating the behavior of the fields of matter in the tensor-multi-scalar theories.

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REFERENCES

1. Gibbons, G. W. (1982). *Nucl. Phys.* **B207**, 337;
   Gibbons, G. W. and Maeda, K. (1988). *Nucl. Phys.* **B298**, 741;
   Damour, T., Gibbons, G. W., and Gundlach, G. (1990). *Phys. Rev. Lett.* 64, 123.
2. Damour, T., and Polyakov, A. M. (1994). *GRG* 26, 1171.
3. Holzhey, C. F. E., and Wilczek, F. (1992). *Nucl. Phys.* **B380**, 447.
4. Garfinke, D., Horowitz, G. T., and Strominger, A. (1991). *Phys.* Rev. *D***43** 3140;
   Horne, J. H., and Horowitz, G. T. (1992). *Phys. Rev. D***46**, 1340.
5. Kallosh, R., Linde, A., Ortin, T., Peet, A., and Van Proen, A. (1992). *Phys.* Rev. *D***46**, 5278.
6. Damour, T., and Esposito-Farese, G. (1992). *Class. Quantum Grav.* **9**, 2093.
   Damour, T., and Taylor, J. H. (1992). *Phys. Rev. D***45**, 1840
7. Berkin, A. L., and Hellings, R. W. (1994). *Phys. Rev. D***49**, 6442.
8. Silaev, P. (1989). *Theor.* Math. *Fiz.* 91, 418;
   Hardell, A., and Delmen, H. (1 993). *GRG* 25, 1165.
9. Logunov, A. A., Mestvirishvili, M. A. (1985). *Prog. Theor. Phys.* 74, 31.
   Duan, Y., Zhang, S., and Jiang, L. (1992). *GRG* 24, 1033.
10. Fock, V. A. (1959). *The theory of Space, Time and Gravitation* (Pergamon, Oxford).
11. Will, C. M. (1993). *Theory and Experiment in Gravitational Physics*, (Cambridge Univ. Press, Cambridge).
12. Krause, D. E., Kloor, H. T., and Fischbach, E. (1994). *Phys. Rev. D***49**, 6892.
13. Koikawa, T., and Yoshimura, M. 0987). *Phys. Lett. B***1** 89, 29.