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An exact solution of Einstein-Maxwell gravity coupled to a scalar field.

Slava G. Turyshev [1]

Jet Propulsion Laboratory MS 301-230,
California Institute of Technology
4800 *Oak Grove* Drive - *Pasadena, CA* 91109- *USA*

Abstract

The general solution to low-energy string theory representing static spherically symmetric solution of the Einstein-Maxwell gravity with a massless scalar field has been found. For each value of the coupling constant a, this solution is characterizes by set of two parameters, the physical mass μ_0 and electric charge Q. The presence of the interaction between the matter fields is found to have important consequences. In particular, the interaction puts a strict limitations on the scalar field parameter k, setting it to be $k = \pm 1/2$. In partial cases, obtained solution appears to coincide with corresponding well-known solutions and to describe black holes and naked singularities. One of the partial cases corresponds to gravity and electromagnetic fields coupled to a scalar field with a negative kinetic term, This particular solution has two regular horizons. We speculate the behavior of the general solution in the extreme regime. The structure of the scalar curvature singularities in a general case has also been examined. The final results are presented in a parametric form.

[1] On leave from Bogolyubov institute for Theoretical Microphysics, Moscow State University, Moscow, Russia, 119899; Electronic address: sgt@zeus.jpl.nasa.gov

I. INTROD1TCTION

The tensor-scalar theories of gravity where the tensor field which is usual for general relativity is coexisting together with one or several long-range scalar fields are the most natural extension of the theoretical basement of the gravitational theory. The superstring, many-dimensional Kaluza-Klein, and inflationary cosmology theories are responsible for reviving the interest to so-called " dilaton fields", i.e. neutral scalar fields whose background values determine the strength of the coupling constants in the effective four-dimensional theory. Although the scalar field naturally arises in theory, its existence from the point of view of the general relativity is quite problematic. It is well-known that the including, of the scalar field in the theory will lead to violation of the strong equivalence principle and modification of large-scale gravitational phenomena [1], [2]. The presence of the scalar fields will also affect the equations of motion of the other matter fields. "1'bus, for example, solutions which correspond to a pure electromagnetic field appear to be drastically modified by the scalar field. Such solutions were studied in [3]- [1 2], where it was shown that the scalar field generally destroys the horizons leading to the singularities in a scalar curvature on a finite radii. Special attention has been paid to the extreme case of the charged dilaton black hole solution obtained in [3], [5]. This solution has been used in [7] for the studies of the problems of black hole thermodynamics, causal structure and quantum phenomena in a strong fields. In particular, analysis of the perturbations around the extreme holes demonstrated the analogy of the behavior of the black holes and elementary particles in the sense that there exists an energy gap in the excitation spectrum of the black hole. The analysis of this analogy and the general description of the quantum-mechanical behavior of the black holes has been continued for extreme supersymmetric dilatonic black hole solutions with respect to cosmic censorship conjecture in [4]. It was shown, that supersymmetry plays the role of a cosmic censor in that it keeps the singularities hidden from an observer except one falling into a black hole. The exterme rotating charged black holes coupled to dilaton have been studied in [6]. As a result, it was shown there that an arbitrarily small amount of angular momentum can significantly change the properties of the solution.

In this paper we will focus our attention on the simplest extension of the standard matter i.e. gravity coupled to electromagnetic and scalar fields. The density of the Lagrangian function L_M for the massless scalar and electromagnetic fields is suggested by the low-energy limit of the string theory in the following form:

$$L_M = \frac{1}{16\pi}\sqrt{-g}\Big(- R + 2g^{mn}\nabla_m\phi\nabla_n\phi - e^{-2a\phi}g^{mn}g^{kl}F_{mk}F_{nl}\Big), \tag{1}$$

where F_{mn} is the tensor of the electromagnetic field which is given as usual: $F_{mn} = \nabla_m A_n -$ $\nabla_n A_m = \partial_m A_n - \partial_n A_m$. The geometrical units $c = \gamma = 1$ arc used throughout the paper and metric convention is accepted to be $(+ - -)$.

One might note that the symmetries of this Lagrangian arc the general covariance and the gauge symmetry. Besides this, the expression (1) is invariant under the global scale transformations, namely: $\phi'(x) = \phi(x) + \nu$ and $A'_m(x) = e^{\nu}A_m(x)$. This freedom can be eliminated by specifying the value of the scalar field at the infinity ϕ_∞. The constant, a in (1) is a dimensionless, arbitrary parameter. To study the dependence of the solutions on the strength of interaction between the scalar and electromagnetic fields, an arbitrary coupling constant a was introduced in [5]. For a = O, Eq. (1) becomes the standard 13instein-Maxwell Lagrangian with the scalar field. In the case $a = 1$, it corresponds to the contribution in total action from the low-energy limit of the superstring theory, treated to the lowest order in work-sheet and string loop expansion. The arbitrariness of this constant makes it possible to have both weak ($a << 1$) and strong ($a >> 1$) coupling regimes.

We would like to find the static, spherically symmetric, harmonic solution to the equations of the general theory of relativity corresponding to the density of the Lagrangian function given by (1). In [8] the harmonic solution was obtained in the special case when the interaction between the matter fields is absent ($a = 0$). However, it is interesting to investigate how small changes in the matter fields in the theory will affect the general solution in the case of an arbitrary value of parameter a. Thus, the analysis of the solution in the Schwarzschild coordinates obtained in [5] shows, that in the case of $a = 0$ it reduces to Reisner-Nordström solution of the Einstein-Maxwell gravity. However, for $a \neq 0$ it represents qualitatively different physics. In particular, this solution has a regular outer event horizon, but for any non-zero value a, the inner horizon is singular. These results make it interesting to explore the possibility of the existance of the spherically symmetric harmonic solution for interacting scalar and electromagnetic fields which would be regular on both horizons.

The structure of the paper is as follows: In Section 1 I we will derive the main system of equations for the gravitational, scalar and electromagnetic fields. The possibility of the additional parametrization of the metric functions due to covariant de Donder's harmonic gauge will be discussed in Section III. In Section IV, we will construct the solution for the radial coordinate. The solution for the scalar field will be obtained in Section V. The general static spherically symmetric solution for interacting scalar and electromagnetic fields in general relativity will be presented in Section VI. Section VII will be devoted to analysis of the general solution in some special cases and will show its correspondence to well-known results. The structure of the singularities in scalar curvature and horizons will be examined in Section VIII. And at last, the final results in parametric form will be presented in Section IX. In Section X we will summarize and suggest future directions for studies of the behavior of the static spherically symmetric solution for scalar and electromagnetic fields in general relativity.

II. THE EQUATIONS OF MOTION.

The gravitational field equ ations for the general theory of relativity in the presence of fields of matter with a Lagrangian function L_M given by Eq. (1) takes the form:

$$R_{mn} = 8\pi(T_{mn} - \frac{1}{2}g_{mn}T), \tag{2.1}$$

where the symmetric energy-momentum tensor of the matter fields T_{mn} may be easily calculated to be:

$$T_{mn} = \frac{1}{4\pi}\left(\nabla_m\phi\nabla_n\phi - \frac{1}{2}g_{mn}g^{kl}\nabla_k\phi\nabla_l\phi\right) + \frac{e^{-2a\phi}}{4\pi}\left(-g^{kl}F_{mk}F_{nl} + \frac{1}{4}g_{mn}g^{cd}g^{ef}F_{ce}F_{df}\right). \tag{2.2}$$

We will be looking for solutions of these equations, which will admit the covariant de Donder's harmonica gauge condition [13] -[15], namely:

$$D_m\sqrt{-g}g^{mn} = 0, \tag{2.3}$$

where D_m is the covariant derivative with respect to Minkovsky metric γ_{mn}:

$$\gamma_{mn} = \mathrm{diag}(1, -1, -r^2, -r^2\sin^2\theta).$$

The equations of motion of the scalar and electromagnetic fields corresponding to L_M (1) might be written as follows:

$$g^{mn}\nabla_m\nabla_n\phi - \frac{a}{2}e^{-2a\phi}F_{mn}F^{mn} = 0, \qquad (2.4)$$

$$\nabla_m(\sqrt{-g}g^{mn}g^{kl}F_{nl}) = 0, \qquad \nabla_k F_{mn} + \nabla_n F_{km} + \nabla_m F_{nk} = 0. \qquad (2.5)$$

To avoid a confusion, let us note, that words "static spherically symmetrical" here imply, that not only the electromagnetic field F_{mn}, but also the electromagnetic potential A_m is spherically symmetrical and doesn't depend on time:

$$\phi(t,r,\theta,\varphi) = \phi(r), \qquad A(t,r,\theta,\varphi) = (A_0(r), A_1(r), 0, 0).$$

Imposing the same conditions on the scalar and gravitational fields, one might write the general form of the effective metric for the static spherical symmetric case as follows:

$$g_{mn} = \mathrm{diag}(\,u(r), -v(r), -w(r), -w(r)\sin^2\theta). \qquad (2.6)$$

Then, having taken into account the definitions above, the system of the gravitational field equations (2. 1) might be written as follows:

$$R_{00} = \frac{u''}{2v} + \frac{u'}{2v}(\frac{w'}{w} - \frac{v'}{2v} - \frac{u'}{2u}) = \frac{1}{v}(A_0')^2e^{-2a\phi}, \qquad (2.7a)$$

$$R_{11} = -\frac{u''}{2u} - \frac{w''}{w} + \frac{u'}{2u}(\frac{u'}{2u} + \frac{v'}{2v}) + \frac{w'}{w}(\frac{w'}{2w} + \frac{v'}{2v}) = 2(\phi')^2 - \frac{1}{u}(A_0')^2e^{-2a\phi}, \qquad (2.7b)$$

$$R_{22} = -\frac{w''}{2v} + \frac{w'}{2v}(\frac{v'}{2v} - \frac{u'}{2u}) + 1 = \frac{w}{uv}(A_0')^2e^{-2a\phi}. \qquad (2.7c)$$

The equation for the component R_{33} coincides with the one for R_{22}, and the other equations become exact equalities. And finally, the equations of the scalar and electromagnetic fields from Eqs. (2.4), (2.5) take the form:

$$\phi'' + \phi'(\frac{u'}{2u} - \frac{v'}{2v} + \frac{w'}{w}) = \frac{a}{u}(A_0')^2e^{-2a\phi}, \qquad (2.8)$$

$$(\frac{w}{\sqrt{uv}}A_0'e^{-2a\phi})' = 0. \qquad (2.9)$$

111. PARAMETERIZATION OF THE METRIC FIJNCTIONS AND THE GENERAL SYSTEM OF THE EQUATIONS.

In order to solve the system of the equations Eqs. (2.7)-(2.9) we will make a linear combination of the first and third equations from the system (2.7) with the coefficients $1/u$ and – l/w respectively. The right hand side of the obtained relation becomes equal to zero, because the matter fields fall out:

$$\frac{u''}{2v} + \frac{u'}{2v}(\frac{w'}{w} - \frac{v'}{2v} - \frac{u'}{2u}) + \frac{w''}{2w} + \frac{w'}{2v}(\frac{u'}{2u} - \frac{v'}{2v}) - \frac{v}{u} = 0. \qquad (3.1)$$

From the gauge condition Eq. (2.3) one might get another pure gravitational equation, namely:

$$(\sqrt{\frac{u}{v}}w)' = 2r\sqrt{uv}. \qquad (3.2)$$

Let us define new functions $\alpha(r)$ and $\beta(r)$ as: $\alpha = \sqrt{uv}$, $\beta = w\sqrt{u/v}$. Then one might get from the Eqs. (3.1) and (3,2) the following system of equations:

$$\beta' = 2r\alpha, \qquad (\alpha'\beta^2/\alpha)' = 0. \tag{3.3}$$

The general solution of that system might be written in a parametric form. Indeed, let us present the functions $\alpha(p)$ and $\beta(p)$ in a following way:

$$\alpha(p) = A \cdot r_p^{-1}, \qquad \beta(p) = A\,(p^2 - \mu^2)\, r_p, \qquad r_p = \frac{dr}{dp}, \tag{3.4}$$

where A, p and μ are constants (arbitrary for the moment). This substitution will enable us to eliminate the functions α and β from both equations (3.3) and, as a result, one will obtain two equations for the same function r(p), namely:

$$(p^2 - \mu^2)\, r_{pp} + 2p\, r_p - 2r = 0, \tag{3.5a}$$

$$(p^2 - \mu^2)^2\, r_{pp} + \frac{B}{A^2} = 0, \tag{3.5b}$$

where B is another arbitrary integrating constant. Equations (3.5) are easy to integrate and the general solutions for both of them may correspondingly be presented as follows:

$$r(p) = q\left[p + Z\left(p \ln\frac{p-\mu}{p+\mu} + 2\mu\right)\right], \tag{3.6a}$$

$$r(p) = \left[q\,p + Z\left(p\ln\frac{p-\mu}{p+\mu}\right)\right] + 2\mu, \quad \frac{B}{A^2} = 4\mu^3 qZ, \tag{3.6b}$$

with arbitrary integrating constants q, h, Z. By choosing $h = 2\mu$ and $B = 4\mu^3 qZA^2$ and with the help of expression (3.4), one might write the general solution for the system of Eqs. (3.3) in the following parametric form:

$$\alpha(p) = A\left[1 + Z\left(\ln\frac{p-\mu}{p+\mu} + \frac{2\mu p}{p^2-\mu^2}\right)\right]^{-1}, \tag{3.7a}$$

$$\beta(p) = A(p^2 - \mu^2)\left[1 + Z\left(\ln\frac{p-\mu}{p+\mu} + \frac{2\mu p}{p^2-\mu^2}\right)\right], \tag{3.7b}$$

$$r(p) = q\left[p + Z\left(p\ln\frac{p-\mu}{p+\mu} + 2\mu\right)\right]. \tag{3.7c}$$

Note that although we have solved the problem for the partial case with $Z = 0$, it is easy to expand the obtained results on more general case with $Z \neq 0$. Because of this we will take $Z = 0$ from now on and will reconstruct a non-zero value of the constant Z in the final results only. The constants A and q are the multipliers which define the scale of measurements of the coordinate. Without losing generality, we may set these constants to be equal to unity.

The relations (3.7) enable us to express the variables u and v in such a way that:

$$u(r) = \frac{1}{v(r)} = \frac{r^2 - \mu^2}{w(r)}. \tag{3.8}$$

After this substitution, the system of equations Eqs. (2.7) and Eq. (2.8) might be written as:

$$[-\frac{w'}{w}(r^2-\mu^2)+2r]'=\frac{2Q^2}{w}e^{2a\phi}, \tag{3.9a}$$

$$-\frac{w''}{w}+\frac{1}{2}\left(\frac{w'}{w}\right)^2=2(\phi')^2, \tag{3.9b}$$

$$[\phi'(r^2-\mu^2)]'=\frac{aQ^2}{w}e^{2a\phi}, \tag{3.9c}$$

where the electric charge Q is the integral of the Maxwell equations (2.9). This allow Gauss' law to be generalized for curved space-time in the following way:

$$E=\frac{Q}{w}e^{2a\phi}, \tag{3.10}$$

where $E=A_0'$ is the intensity of the electromagnetic field.

In order to find the solution for the function $w(r)$, let us define a new function $f(r)$ as follows:

$$w(r)=f(r)e^{2a\phi(r)}, \tag{3.11}$$

One might note that in the limit $(a\to 0)$, the function $f(r)$ and solution for $w(r)$ obtained due to substitution *(3.11)* will correspond to the unperturbed function $w(r)_0$ obtained in [4]. Then, in terms of the function $f(r)$, the system of the equations (3.9) might be rewritten as:

$$[-\frac{f'}{f}(r^2-\mu^2)+2r]'f=2(1+a^2)Q^2, \tag{3.12a}$$

$$a\phi''+a\phi'\frac{f'}{f}-(1+a^2)(\phi')^2=-\frac{1}{2}\left[\left(\frac{f'}{f}\right)'+\frac{1}{2}\left(\frac{f'}{f}\right)^2\right], \tag{3.12b}$$

$$[\phi'(r^2-\mu^2)]'f=aQ^2. \tag{3.12c}$$

Our future strategy will be the following: first, we will solve the equation (3.12a) for the function f. Second, the obtained function f will then be used in the equation $(3.12b)$ which is considered here as determining of the scalar field ϕ. Solutions for the functions f and ϕ, obtained this way, should satisfy the equation of motion of the scalar field, which is presented by the equation (3.12 c).

IV. THE CONSTRUCTION OF THE SOLUTION FOR THE FUNCTION $f(r)$.

To find the solution for the function $f(r)$ from Eqs. (3.12a), we will be using the following form:

$$f(r)=2(1+a^2)Q^2\cdot(r^2-\mu^2)\nu^2(r), \tag{4.1}$$

where we have introduce a new function v(r). Then, the equation Eq. (3.12a) might be rewritten in terms of the function ν:

$$[-(r^2-\mu^2)\frac{2\nu'}{\nu}]'(r^2-\mu^2)\nu^2=1. \tag{4.2}$$

To integrate this equation we first introduce a new radial coordinate ρ by the relation:

$$\rho = \left(\frac{r-\mu}{r+\mu}\right)^{s}, \tag{4.3}$$

where s is an arbitrary parameter. Then the equation (4.2) may be rewritten as:

$$\nu\nu_{\rho\rho} - \nu_{\rho}^{2} + \frac{1}{\rho}\nu\nu_{\rho} + \frac{1}{8s^{2}\mu^{2}}\frac{1}{\rho^{2}} = 0. \tag{4.4}$$

After some algebra, one might obtain two solutions for this equation, namely:

$$\nu_1(\rho) = \pm\frac{1}{4\sqrt{2}s\mu h}\left(B\rho^{h} - \frac{1}{B}\rho^{-h}\right), \tag{4.5a}$$

$$\nu_2(\rho) = b \pm \frac{1}{2\sqrt{2}s\mu}\ln\rho, \tag{4.5b}$$

where h, B, b are again arbitrary constants. It is easy to see that the result (4.5b) is the limiting case of the solution (4.5a) with parameter $h = 0$. So, the expression for $\nu_1(\rho)$ from Eq. (4.5a) is the general solution for the equation (4.4) where parameters b, B and h might arbitrarily take both real and imaginary values as well.

The function $f_1(r)$ which corresponds to the general solution of ν_1 **(4.5a)** might then be presented in the following form:

$$f_1(r) = (1+a^{2})\frac{Q^{2}}{16\mu^{2}s^{2}h^{2}}(r^{2}-\mu^{2})\left[B\left(\frac{r-\mu}{r+\mu}\right)^{s+h} - \frac{1}{B}\left(\frac{r+\mu}{r-\mu}\right)^{s+h}\right]^{2}. \tag{4.6}$$

Substituting that expression into Eq. (3.12a), one can see that function f_1 becomes the solution of this equation if the following condition is satisfied: $s^{2}h^{2} = (s+h)^{2} = k^{2}$, where k is some new arbitrary parameter. After this, the general solution for the function $f(r)$ might finally be presented as follows:

$$f(r) = (1+a^{2})\frac{Q^{2}}{16\mu^{2}k^{2}}(r^{2}-\mu^{2})\left[B\left(\frac{r-\mu}{r+\mu}\right)^{k} - \frac{1}{B}\left(\frac{r+\mu}{r-\mu}\right)^{k}\right]^{2}. \tag{4.7a}$$

This result in the limiting case of $k = 0$, might be written as:

$$f_0(r) = 2(1+a^{2})Q^{2}(r^{2}-\mu^{2})\left(b \pm \frac{1}{2\sqrt{2}\mu}\ln\frac{r-\mu}{r+\mu}\right)^{2}. \tag{4.7b}$$

In the following sections, we will find the solution for the function $\phi(r)$ from the equation (3.12b) with the general solution for the function $f(r)$ given by the expression *(4.7a)*.

V. SOLUTION FOR THE SCALAR FIELD $\phi(r)$.

To *find* the solution for the function ϕ' from the equation (3.1213) we will use the following substitution:

$$\phi'(r) = \frac{\xi(r)}{r^{2}-\mu^{2}}, \tag{5.1}$$

where $\xi(r)$ is a new function to be determined. Then, with the help of the expressions *(4.5a)* and (4.7a), the equation Eq. (3.12b) becomes:

$$a(r^2-\mu^2)\left(\xi' + 2\xi\frac{\nu_1'}{\nu_1}\right) + (1+a^2)\xi^2 = \mu^2(1-4k^2). \tag{5.2}$$

Let us define a new radial coordinate z as follows:

$$z = \frac{B^2\rho^2-1}{B^2\rho^2+1}, \qquad \rho = \left(\frac{r-\mu}{r+\mu}\right)^k. \tag{5.3}$$

Using this new coordinate z, the equation (5.2) might be rewritten as:

$$\xi_z(1-z^2) + \frac{2}{z}\xi + \frac{1+a^2}{2\mu k a}\xi^2 = \frac{\mu}{2ka}(1-4k^2). \tag{5.4}$$

The general solution of this differential equation is easy to find and it may be presented as a function of coordinate ρ in the following form:

$$\xi(\rho) = \frac{2\mu k a}{1+a^2}\left(\frac{\delta}{k}\frac{C_0^2(B\rho)^{2\delta/k}+1}{C_0^2(B\rho)^{2\delta/k}-1} - \frac{(B\rho)^2+1}{(B\rho)^2-1}\right), \tag{5.5}$$

where C_0 is arbitrary integrating constant and δ is defined as the expression:

$$\delta = \pm\frac{1}{2a}\sqrt{1+a^2-4k^2}. \tag{5.6}$$

This finally gives the following general solution for the function $\phi(\rho)$:

$$\phi(\rho) = \phi_0 + \frac{a}{1+a^2}\ln\left[\left(C_0(B\rho)^{\delta/k} - \frac{1}{C_0(B\rho)^{\delta/k}}\right)\left(B\rho - \frac{1}{B\rho}\right)^{-1}\right], \tag{5.7}$$

where ϕ_0 is an arbitrary integrating constant. Note that the obtained result for the function $\phi(\rho)$ omits the homogeneous non-trivial limit $a \to 0$.

VI. THE GENERAL SOLUTION.

Now we are in the position to write down the general solution for the function $w(r)$. By substituting the expression (4.7a) into Eq. **(3.11)** and expressing $e^{2a\phi(r)}$ with the help of the relation (5.7) one might write the result for $w(r)$ as follows:

$$w(r) = (1+a^2)\frac{Q^2}{16\mu^2k^2}e^{2a\phi_0}(r^2-\mu^2)\left[B\left(\frac{r-\mu}{r+\mu}\right)^k - \frac{1}{B}\left(\frac{r+\mu}{r-\mu}\right)^k\right]^{\frac{2}{1+a^2}} \times$$

$$\times\left[C_0B^{\delta/k}\left(\frac{r-\mu}{r+\mu}\right)^\delta - \frac{1}{C_0B^{\delta/k}}\left(\frac{r+\mu}{r-\mu}\right)^\delta\right]^{\frac{2a^2}{1+a^2}}, \tag{6.1}$$

where the constants μ, Q, k, B, C_0 and ϕ_0 are arbitrary for the moment. In order to limit the number of arbitrary constants in this solution, we will impose two asymptotical conditions on functions $\phi(r)$ and $w(r)$, namely:

$$\phi(r\to\infty) = \phi_\infty = 0, \qquad w(r\to\infty) = r^2. \tag{6.2}$$

Then from the relation (5.7) we will have:

$$e^{2a\phi_0} = \left(C_0B^{\delta/k} - \frac{1}{C_0B^{\delta/k}}\right)^{-\frac{2a^2}{1+a^2}}\left(B - \frac{1}{B}\right)^{\frac{2a^2}{1+a^2}}. \tag{6.3}$$

Making of use the second condition from Eq. (6.2) and taking into account the expression for $e^{2a\phi_0}$ above, one might write another constraint as:

$$\left(B-\frac{1}{B}\right)^{-2} = (1+a^2)\frac{Q^2}{16\mu^2k^2}. \tag{6.4}$$

Eliminating parameter B with the help of relation (6.4), we can rewrite the expression for the function $w(r)$ given by Eq. (6.1) as follows:

$$w(r) = (r^2-\mu^2)\left[\frac{B^2}{B^2-1}\left(\frac{r-\mu}{r+\mu}\right)^k - \frac{1}{B^2-1}\left(\frac{r+\mu}{r-\mu}\right)^k\right]^{\frac{2}{1+a^2}} \times$$

$$\times\left[\frac{C_0^2B^{2\delta/k}}{C_0^2B^{2\delta/k}-1}\left(\frac{r-\mu}{r+\mu}\right)^\delta - \frac{1}{C_0^2B^{2\delta/k}-1}\left(\frac{r+\mu}{r-\mu}\right)^\delta\right]^{\frac{2a^2}{1+a^2}}, \tag{6.5}$$

where constant B defined by Eq. (6.4) becomes:

$$B = \pm\frac{1}{A}\left(1\pm\sqrt{1+A^2}\right), \qquad A^2 = (1+a^2)\frac{Q^2}{4\mu^2k^2}. \tag{6.6}$$

Making of use the results for the constants ϕ_0 and B obtained above, one might present the final solution for the function $\phi(r)$ given by the expression (5.7) as follows:

$$\phi(r) = \frac{a}{1+a^2}\left(\ln\left[\frac{C_0^2B^{2\delta/k}}{C_0^2B^{2\delta/k}-1}\left(\frac{r-\mu}{r+\mu}\right)^\delta - \frac{1}{C_0^2B^{2\delta/k}-1}\left(\frac{r+\mu}{r-\mu}\right)^\delta\right] -\right.$$

$$\left. -\ln\left[\frac{B^2}{B^2-1}\left(\frac{r-\mu}{r+\mu}\right)^k - \frac{1}{B^2-1}\left(\frac{r+\mu}{r-\mu}\right)^k\right]\right), \tag{6.7}$$

Thus, we have obtained a general solution for the system of equations Eqs. **(3.9).** For arbitrary values of the coupling constant *a*, this solution is labeled by four arbitrary parameters μ, Q, k and C_0.

VII. THE SPECIAL CASES OF THE GENERAL SOLUTION.

In this section we analyze the various special cases of the general solution by setting some of the parameters equal to zero, while the others remain unchanged.

(i). *a = O. in* this case the solution represents non-interacting scalar and electromagnetic fields. The similar solution in harmonica coordinates was previously obtained in [8]. Note that the dependence on the constant Co in (6.5)-(6.7) drops out and the solution in this case may be labeled by the set of threw parameters $(\mu, k$ and $Q)$:

$$\phi(r)|_{a=0} = \pm\frac{1}{2}\sqrt{1-4k^2}\ln\frac{r-\mu}{r+\mu}, \tag{7.1a}$$

$$w(r)|_{a=0} = (r^2-\mu^2)\frac{1}{4}\left[\left(1\pm\sqrt{1+A_0^2}\right)\left(\frac{r-\mu}{r+\mu}\right)^k + \left(1\mp\sqrt{1+A_0^2}\right)\left(\frac{r+\mu}{r-\mu}\right)^k\right]^2, \tag{7.1b}$$

where the constant, *A.* is given by the expression for *A* **(6.6)** with *a* = O. One might notice that the scalar field is real for $|k|\leq 1/2$, but becomes complex when $|k|>1/2$. For the $k=\pm1/2$ expressions (7.1) corresponds to the Reisner-Nordström solution with pure electromagnetic field in harmonic coordinates of the Minkovsky space-time. To get the actual form of the interval ds^2 corresponding to obtained solution, one should use the relations for the metric functions $u(r)$ and $v(r)$ presented by (3.8) and substitute them into (2.6). For example, for $k = -1/2$ one might get the following interval:

$$ds^2 = \left(\frac{r-\mu_0}{r+\mu_0} + \frac{Q_0^2}{(r+\mu_0)^2}\right) dt^2 - \left(\frac{r-\mu_0}{r+\mu_0} + \frac{Q_0^2}{(r+\mu_0)^2}\right)^{-1} dr^2 - (r+\mu_0)^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{7.2a}$$

where parameters μ and Q are connected to the physical mass μ_0 and the electromagnetic charge Q_0 as:

$$\mu = \pm\sqrt{\mu_0^2 - Q_0^2}, \qquad Q = Q_0, \tag{7.2b}$$

(ii). An interesting case is arises in the strong interaction regime when $a >> 1$. Examins Eqs. (6.5)-(6.6) in the extreme regime of $a \to \infty$ and $k = \pm 1/2$ one might get the following expressions for $\phi(r)$ and $w(r)$:

$$\phi(r) = 0, \tag{7.3a}$$

$$w(r) = (r^2 - \mu^2)\left[\frac{C_0^2}{C_0^2-1}\left(\frac{r-\mu}{r+\mu}\right)^{\pm 1/2} - \frac{1}{C_0^2-1}\left(\frac{r+\mu}{r-\mu}\right)^{\pm 1/2}\right]^2. \tag{7.3b}$$

This solution corresponds to that of the Reisner-Nordström type with the "induced charge" J generated by the constant C_0. Taking, for example, the minus sign in the powers of expression in (7.3b), one might get:

$$ds^2 = \left(\frac{r-\hat{\mu}}{r+\hat{\mu}} + \frac{J^2}{(r+\hat{\mu})^2}\right) dt^2 - \left(\frac{r-\hat{\mu}}{r+\hat{\mu}} + \frac{J^2}{(r+\hat{\mu})^2}\right)^{-1} dr^2 - (r+\hat{\mu})^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{7.4}$$

where parameters μ and C_0 are connected to physical mass $\hat{\mu}$ and "charge" J as follows:

$$\hat{\mu} = \mu\frac{C_0^2+1}{C_0^2-1} \qquad J = \frac{2\mu C_0}{C_0^2-1}.$$

This result is quite surmising. Indeed, taking the limit $a \to \infty$ is equivalent to cutting off the electromagnetic term in the Lagrangian density $L_{A4}(1)$. From the additional condition $k = \pm 1/2$ one might notice that the scalar field also tends to be zero. Then, because of no matter fields are present, this solution should be one for a pure static spherically symmetric gravity. Instead, as a result, one obtains the solution (7.4) of the Reisner-Nordström type with the effective metric similar to that in (7.2a). And since the scalar field is responsible for appearance of the constant C_0, then the "induced charge" J is caused by the scalar field, which is absent! In order to resolve this apparent paradox, one should require $C_0 = 0$. Implementation of this condition simply corresponds to the renormalization of the constant ϕ_0 in Eq. **(6.1)**. Then, the expression (7.4) becomes the usual Fock solution in harmonic coordinates of the Minkovsky space-time.

(iii). $Q = 0$. When the electric charge vanishes, the solution reduces to the one of the pure scalar gravity with interval ds^2 written as:

$$ds^2 = \left(\frac{r-\mu}{r+\mu}\right)^q dt^2 - \left(\frac{r+\mu}{r-\mu}\right)^q dr^2 - (r^2-\mu^2)\left(\frac{r+\mu}{r-\mu}\right)^q\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{7.5a}$$

where the constant q given by the relation

$$q = \frac{1}{1+a^2}\left(2k \pm a\sqrt{1+a^2-4k^2}\right). \tag{7.5b}$$

With the scalar field $\phi(r)$ for this case presented by:

$$\phi(r) = \pm\frac{\sqrt{1+a^2-4k^2}}{2(1+a^2)}\ln\left(\frac{r-\mu}{r+\mu}\right) - \frac{a}{1+a^2}\ln\left[\frac{1\pm 1}{2}\left(\frac{r-\mu}{r+\mu}\right)^k + \frac{1\mp 1}{2}\left(\frac{r+\mu}{r-\mu}\right)^k\right], \tag{7.5c}$$

The parameter $\mu \geq 0$ defines the locations of two horizons $(\mu_\pm)$ which, in the case (7.5), related to the physical mass μ_0 as $\mu_\pm = \mu_0/q$. For any $q \neq 1$ these horizons are singular.

Note that taking Q = O is equivalent to dropping the electromagnetic term from the Lagrangian density L_M (1). However, one might find it quite unexpecting that even after taking Q + O, our results still depend on the arbitrary parameter a which characterizes the intensity of the interaction between the matter fields. This contradiction might be resolved by choosing the parameter k to be $k = \pm 1/2$ and the signs in (7.5) in such a way that these expressions will not depends on a. This fact suggests that not just the scalar field affects the solutions for the gravitational and electromagnetic fields, but also the interaction between the matter fields puts the constraints on the scalar field itself. The usual Fock solution in harmonic coordinates [13], [14] might be obtained from the expressions (7.5) by setting $q = 1$ (or $k = \pm 1/2$) and choosing the same signs for both terms in (7.5b).

(iv). One might expect that all the expressions for the genera] solution should omit the homogeneous non-trivial limit in case where constant a becomes imaginary: $a \rightarrow \pm i$. Indeed, one might obtain the following result in that limit:

$$\phi(r)\big|_{a\rightarrow\pm i} = \mp i\left(\frac{1-4k^2}{8k}\ln\frac{r+\mu}{r-\mu} + \frac{Q^2}{16\mu^2k^2}\left(1-\left(\frac{r-\mu}{r+\mu}\right)^{2k}\right)\right), \tag{7.6a}$$

$$w(r)\big|_{a\rightarrow\pm i} = (r^2-\mu^2)\left(\frac{r+\mu}{r-\mu}\right)^{\frac{1+4k^2}{4k}}\exp\left[\frac{Q^2}{8\mu^2k^2}\left(1-\left(\frac{r-\mu}{r+\mu}\right)^{2k}\right)\right]. \tag{7.6b}$$

These expressions are, in genera], singular. However, if we will choose the parameter k as $k = +1/2$, we will obtain an interesting result. For example, for $k = 1/2$ the expressions (7.6) become:

$$\phi(r) = \mp i\,\frac{Q^2}{4\mu^2}\left(1-\frac{r-\mu}{r+\mu}\right), \tag{7.7a}$$

$$w(r) = (r+\mu)^2\exp\left[\frac{Q^2}{2\mu^2}\left(1-\frac{r-\mu}{r+\mu}\right)\right], \tag{7.7b}$$

$$u(r) = \frac{1}{v(r)} = \left(\frac{r-\mu}{r+\mu}\right)\exp\left[\frac{Q^2}{2\mu^2}\left(\frac{r-\mu}{r+\mu}-1\right)\right]. \tag{7.7c}$$

This is an interesting modification of the Fock solution (see case (iii) and expression (7.5)) in the presence of the complex scalar and electromagnetic fields. Solution (7.7) is labeled by the means of two parameters μ and Q. This solution has two regular events horizons $r_\pm$, which correspond to the physical mass μ_0 and the charge Q. of the black hole as follows:

$$r_\pm = \mu_\pm = \frac{1}{2}\left(\mu_0 \pm \sqrt{\mu_0^2 - 2Q_0^2}\right), \qquad Q = Q_0. \tag{7.8}$$

The expression (7.8) limits the possible value of the physical mass to be $\mu_0 \geq \sqrt{2}Q_0$. in the next Section we will show that the scalar curvature corresponding to the solution (7.7) is also regular on surfaces (7.8).

The presence of the i in the expression for ϕ in (7.7a) might be interpreted as changing the sign in front of the scalar field term in the Lagrangian density L_M (1) to be:

$$L_M = -\frac{1}{16\pi}\sqrt{-g}\left(R + 2g^{mn}\nabla_m\varphi\nabla_n\varphi + e^{-2\varphi}g^{mn}g^{kl}F_{mk}F_{nl}\right), \tag{7.9}$$

where we denote $\varphi = -@$. The Lagrangians of that type correspond to the theory with a complex scalar field [1]-[4]. The real part of this field is dilaton and its imaginary part is axion. In the case (7.7), the obtained solution depends only on the dilaton field with the axion having been taken to be a constant. Unfortunately, the negative kinetic term - $g^{mn}\nabla_m\varphi\nabla_n\varphi$ in (7.9) generally leads to a theory without stable states. This result is allows infinitely many negative energy states when the system is quantized [2], [8]. However, this statement should be separately discussed in the case of the solution (7.7).

(v). In the case of $k = 0$ the general solution of Eqs.(6.5)-(6.7) becomes:

$$\phi(r)|_{k=0} = \pm\frac{1}{2}\frac{1}{\sqrt{1+a^2}}\ln\left(\frac{r+\mu}{r-\mu}\right) - \frac{a}{1+a^2}\ln\left[1 \pm \sqrt{1+a^2}\frac{Q}{2\mu}\ln\left(\frac{r-\mu}{r+\mu}\right)\right], \tag{7.10a}$$

$$w(r)|_{k=0} = (r^2 - \mu^2)\left[1 \pm \sqrt{1+a^2}\frac{Q}{2\mu}\ln\left(\frac{r-\mu}{r+\mu}\right)\right]^{\frac{2}{1+a^2}}\left(\frac{r+\mu}{r-\mu}\right)^{\pm\frac{a}{\sqrt{1+a^2}}}. \tag{7.10b}$$

With Q = O and an arbitrary a, this result is the usual solution for the scalar field given by (7.5). For an arbitrary value of both parameters a and Q, the expressions (7.10) are representing the solution with the naked singularity at $r = \mu$. The physical mass and electric charge for this case are related as:

$$\mu_0\sqrt{1+a^2} = Q_0 \pm a\mu. \tag{7.10C}$$

If we set the parameter $\mu = 0$, the result (7.10) will take the form:

$$\phi(r)|_{\substack{\mu=0\\k=0}} = -\frac{a}{1+a^2}\ln\left[1 \mp \sqrt{1+a^2}\,Q\right], \tag{7.11a}$$

$$w(r)|_{\substack{\mu=0\\k=0}} = r^2\left[1 \mp \frac{\sqrt{1+a^2}\,Q}{r}\right]^{\frac{2}{1+a^2}}. \tag{7.11b}$$

It easy to see from (7. 10c), that the physical mass μ_0 is, in this case, generated just by the electric charge. This result is also represents the naked singularities. Depending on the sign in front of the square root in the expressions (7.11), they occur at $r_+ = 0$ for the positive sign and at $r_- = [0; \sqrt{1+a^2}Q]$ for the negative.

(vi). And finally, in the case $k = \pm 1/2$, the general solution becomes a Garfinke-Horowitz-Strominger [5] type solution in harmonic coordinates.

A). Thus with $k = -1/2$ and a positive sign in front of the $\sqrt{1+A^2}$ in the experssion (6.6) one might get:

$$\phi^-(r) = \frac{a}{1+a^2}\ln\left(1 - \mu\frac{1+\sqrt{1+A_*^2}}{r+\mu\sqrt{1+A_*^2}}\right), \tag{7.12a}$$

$$w^-(r) = \left(r + \mu\sqrt{1+A_\star^2}\right)^2 \left(1 - \mu\frac{1+\sqrt{1+A_\star^2}}{r+\mu\sqrt{1+A_\star^2}}\right)^2, \tag{7.12b}$$

$$u^-(r) = \frac{1}{v^-(r)} = \left(1 + \mu\frac{1-\sqrt{1+A_\star^2}}{r+\mu\sqrt{1+A_\star^2}}\right)\left(1 - \mu\frac{1+\sqrt{1+A_\star^2}}{r+\mu\sqrt{1+A_\star^2}}\right)^{\frac{1-a^2}{1+a^2}}, \tag{7.12c}$$

where function $A_\star^2$ is defined from (6.6) as:

$$A_\star^2 = (1+a^2)\frac{Q^2}{\mu^2}. \tag{7.13}$$

The horizons R_1 and R_2 are related to the mass μ_0 and charge Q. of the hole according to:

$$2\mu_0 = R_1 + \left(\frac{1-a^2}{1+a^2}\right)R_2, \qquad Q_0^2 = \frac{R_1R_2}{1+a^2} = Q^2, \tag{7.14a}$$

$$R_1 = \mu(\sqrt{1+A_\star^2} - 1), \qquad R_2 = \mu(\sqrt{1+A_\star^2} + 1). \tag{7.14b}$$

Note that horizons $R_{1,2}$ correspond to $(r_\pm)$ which are located at $r_\pm = \mu_\pm$

$$\mu_\pm = \frac{1}{1-a^2}\left(\mu_0 a^2 \pm \sqrt{\mu_0^2 - (1-a^2)Q_0^2}\right). \tag{7.14c}$$

B). In case of $k = 1/2$ and a negative sign in front of the $\sqrt{1+A^2}$ in (6.6), one will obtain the following result:

$$\phi^+(r) = \frac{a}{1+a^2}\ln\left(1 + \mu\frac{1-\sqrt{1+A_\star^2}}{r+\mu\sqrt{1+A_\star^2}}\right), \tag{7.15.}$$

$$w^+(r) = \left(r + \mu\sqrt{1+A_\star^2}\right)^2\left(1 + \mu\frac{1-\sqrt{1+A_\star^2}}{r+\mu\sqrt{1+A_\star^2}}\right)^{\frac{2a^2}{1+a^2}}, \tag{7.15b}$$

$$u^+(r) = \frac{1}{v^+(r)} = \left(1 - \mu\frac{1+\sqrt{1+A_\star^2}}{r+\mu\sqrt{1+A_\star^2}}\right)\left(1 + \mu\frac{1-\sqrt{1+A_\star^2}}{r+\mu\sqrt{1+A_\star^2}}\right)^{\frac{1-a^2}{1+a^2}}. \tag{7.15c}$$

The horizons R_1' and R_2' arc connected to physical mass μ_0 and charge Q_0 by the relations (7.14a). Expressions for these horizons may be obtained from (7.14b) by the swiching the signs $(+ \leftrightarrow -)$, namely:

$$R_1' = \mu(\sqrt{1+A_\star^2} + 1), \qquad R_2' = \mu(\sqrt{1+A_\star^2} - 1) \tag{7.16a}$$

which correspond to :

$$\mu_\pm = \frac{1}{1-a^2}\left(-\mu_0 a^2 \pm \sqrt{\mu_0^2 - (1-a^2)Q_0^2}\right). \tag{7.16b}$$

One might see that by taking the limit $(a \to \pm i)$ in the expressions (7.16b), one will immediately arrive to the result (7.7) presented above.

The solutions in both partial cases (7.12) and (7.15) will have coinciding horizon μ^* when the constant $a = 1$ (i.e. string case). This horizon might be presented as follows:

$$\mu^* = \mu_0\left(1 - \frac{Q^2}{2\mu_0^2}\right). \tag{7.17}$$

Assuming that parameters μ^* and μ_0 are both positive, for the physical mass μ_0 and the electric charge Q one will get the following relation in the string case: $\mu_0 \geq Q/\sqrt{2}$.

It is easy to see that the partial cases demonstrates a different behavior when the electric charge is taken to be zero. ‘J’bus, in the limit ($Q \to 0$), the expressions (7.12) corresponds to pure scalar gravity with the linear element ds^2 given by (7.5a) and parameter $q = (1-a^2)/(1+a^2)$. However, in the same limit, result (7.15) corresponds to the Fock solution for an arbitrary value for the constant a. Note that taking ($Q = 0$) is equivalent to extracting of the electromagnetic term from the action (1). Moreover, by choosing; the parameter k in addition to that as $k = \pm 1/2$, one will elliminate the scalar field term also. ‘1’hen, the solution in this limit should describe a static spherically symmetric distribution of matter. Because of this reason, on] y the result in the partial case (7.15) is likely corresponds to a charged black hole solution in harmonic coordinates.

VIII. THE SINGULARITIES IN THE SCALAR CURVATURE.

In order to study the structure of the singularities of the general solution, let us clarify the meaning of the parameter μ. As one might have noticed in a previous section, the parameter μ describes the locations of the horizons. To specify its meaning, we shall connect it to the physical mass μ_0 and electrical charge Q.. One may identify the physical mass μ_0 by examining $1/r$ behavior of g_{00} far away from the source, namely by using the expression for g_{00} in a weak-field approximation: $g_{00} = 1 - 2\mu_0/r + O(1/r^2)$, while ($r \to \infty$). ‘1 'hen, making of use the relations for the general solution given by Eq. (6.5) – (6.7) with parameter $C_0 = 0$, one might obtain the required connection between the parameters from (3.8):

$$\mu = \frac{1}{4k^2 - a^2}\left(\mp \mu_0 a\sqrt{1 + a^2 - 4k^2} \pm \sqrt{4k^2 - Q_0^2(4k^2 - a^2)}\right), \tag{8.1a}$$

where the signs should be chosen in order to satisfy the condition $\mu > 0$. In partial cases this result corresponds to solutions we have obtained in the previous Section. The relation (8.1) sets the condition on the values of the parameters k and a, connecting them to relations between physical mass and electrical charge:

$$\frac{Q_0^2}{\mu_0^2} \leq \frac{4k^2}{4k^2 - a^2}. \tag{8.1b}$$

The result (8.1 b) puts also the limitation on the parameters k and a as: $4k^2 - a^2 \geq 0$. Note, in the case $4k^2 - a^2 = 0$, the obtained relations (8.1 a) correspond to extreme black hole solution with $\mu_0 = Q_0$.

It is well-known that the simplest way to study the behavior of the scalar curvature R is to use the gravitational field equations. Indeed, as far as the electromagnetic part of the energy-momentum tensor Eq. (2.2) is traceless, the only contribution to the curvature R comes from the scalar field ϕ. Thus, by taking the trace of the Hilbert-Einstein equations (2. 1), one might present the scalar curvature R as follows:

$$R = -8\pi T = 2g^{mn}\nabla_m\phi\nabla_n\phi = -\frac{2\phi'^2(r)}{w(r)}(r^2 - \mu^2). \tag{8.2}$$

Substituting the results for $\phi(r)$ and $w(r)$ from the Eqs. (5.1), (5.5) and (6.5) in the expression above, we will obtain the expression for the scalar curvature R corresponding to the general solution Eq. (6.5)-(6.7). In terms of the coordinate p given by (5.3), this expression can be presented as follows:

$$R(\rho) = -\frac{k^2}{2\mu^2}\frac{a^2}{(1+a^2)^2}\left(\frac{\delta}{k} + \frac{B^2\rho^2+1}{B^2\rho^2-1}\right)^2\left(\frac{B^2-1}{B^2\rho^2-1}\right)^{\frac{2}{1+a^2}}(1-\rho^{1/k})^4\rho^{(q-2)/k}, \tag{8.3}$$

where q is given by (7.5 b). Notice that the metric tensor g_{mn} and solutions for both matter fields $\phi(r)$ and $E(r)$ are all divergent when:

$$\rho^{2/k} = 0, \qquad B^2\rho^2 - 1 = 0. \tag{8.4}$$

where coordinates r and ρ are connected by Eq. (5.3). Because of this, there are physical singularities at these points. The singularity in the scalar curvature given by Eqs. (8.3) on the surface $\rho = 0$ might be eliminated in some special cases. Thus, choosing parameter $q = 2$, one might make R regular on this horizon for any real positive value of the parameter k. In case of $k < 0$ that condition should be replaced by $q = -2$. However, expression (8.3) shows that generically scalar curvature is divergent at both horizons and both singularities cannot be eliminated at the same time by any choice of parameters except the situation ($a \to \pm i$). Thus, in this partial case the solution given by (7.7) has two regular horizons $\mu_\pm$ given by the expression (7.8) and in the case of the Largangian function (7.9) the scalar curvature R might be presented as:

$$R(r) = \frac{Q^4}{2\mu^2}\frac{r^2-\mu^2}{(r+\mu)^6}\exp\left(\frac{Q^2}{2\mu^2}\left[\frac{r-\mu}{r+\mu} - 1\right]\right). \tag{8.5}$$

As one might expect, the scalar curvature is regular on the horizon $r = \mu$. An interesting case arises for the extreme black hole, (i.e. $\mu_0^2 = 2Q_0^2$), which leads from (7.8) to $\mu = \mu_+ = \mu_0/2$. As a result, the scalar curvature might be presented by the following expression

$$R(r) = 2\mu^2\frac{r^2-\mu^2}{(r+\mu)^6}\exp\left(\frac{r-\mu}{r+\mu} - 1\right). \tag{8.6}$$

Although this partial case might be interesting primarily from the pure mathematical point of view [8], the general solution obtained in this paper could be useful for the description of the quantum-mechanical phenomena in a strong gravitational fields and cosmology [2] ,[7].

IX. THE FINAL RESULTS.

Finally, by reconstructing the constant Z with the help of the relation (3.7c), namely:

$$r(p) = p + Z\left(p\ln\frac{p-\mu}{p+\mu} + 2\mu\right)$$

one might obtain the general static spherically-symmetric harmonic solution for the massless scalar and electromagnetic fields in general relativity in the following parametric form:

$$g_{mn} = \mathrm{diag}\left(\frac{p^2-\mu^2}{w(p)}, -\frac{w(p)}{p^2-\mu^2}(r'(p))^2, -w(p), -w(p)\sin^2\theta\right), \tag{9.1}$$

$$E(p) = r'(p)\frac{Q}{w(p)}e^{2a\phi(p)}, \tag{9.2}$$

where

$$r'(p) = \frac{dr}{dp} = 1 + Z\left(\ln\frac{p-\mu}{p+\mu} + \frac{2\mu p}{p^2-\mu^2}\right).$$

The scalar field ϕ and function w are given by the expressions (6.5) and (6.7) and in parametric form they may be presented as follows:

$$\phi(p) = \frac{a(\delta - k)}{1+a^2}\ln\left(\frac{p+\mu}{p-\mu}\right) - \frac{a}{1+a^2}\ln\left[\frac{B^2}{B^2-1}\left(\frac{p-\mu}{p+\mu}\right)^{2k} - \frac{1}{B^2-1}\right], \qquad (9.3a)$$

$$w(p) = (p^2-\mu^2)\left(\frac{p+\mu}{p-\mu}\right)^q\left[\frac{B^2}{B^2-1}\left(\frac{p-\mu}{p+\mu}\right)^{2k} - \frac{1}{B^2-1}\right]^{\frac{2}{1+a^2}}, \qquad (9.3b)$$

where the constant B is given by the relations (6.6) as:

$$\frac{B^2}{B^2-1} = \frac{1}{2}\left(1 \pm \sqrt{1+A^2}\right), \qquad \frac{1}{B^2-1} = \frac{1}{2}\left(-1 \pm \sqrt{1+A^2}\right), \qquad A^2 = (1+a^2)\frac{Q^2}{4\mu^2k^2}$$

and parameters δ and q are presented by (5.6) and (7.5b) correspondingly

$$\delta = \pm\frac{1}{2a}\sqrt{1+a^2-4k^2}, \qquad q = 2\,\frac{k+a^2\delta}{1+a^2} = \frac{1}{1+a^2}\left(2k \pm a\sqrt{1+a^2-4k^2}\right).$$

Note that only the results which correspond to $q = \pm 1$ present the black hole-like solutions. Indeed, one might make sure that this condition gives as a consequence the equation: $(1+a^2)(2k\pm 1)2 = 0$. We have seen, that in both cases (either $a = \pm i$ from Eq. (7.7) or $k = \pm 1/2$ in the expressions (7.2), (7.5), (7.7), (7.12), (7.15)), the results correspond to different types of black hole solutions. This fact suggests that in order to describe physically meaningful situations the parameter k must be choosen as $k = +1/2$.

X. DISCUSSION.

We have obtained a general static spherically symmetric harmonic solution of the Einstein-Maxwell gravity coupled to the massless scalar field. The form of the singularity in the general solution is quite well defined. By the specific choice of the constant Z, this singularity in a partial cases might be transferred [8] from the components of effective metric g_{00} and g_{11} to that of g_{22} and g_{33}. However, the character of this singularity, in general, remains unchanged.

We have shown that, in the partial cases, the solution presented here corresponds to well-known solutions, It was noticed that the absence of the interaction between the matter fields, makes the existence of black holes in the presence of the scalar field problematic. The reason for this is the appearance of the singularities in the Riemann tensor invariants because of the scalar field. The presence of the arbitrary coupling constant a gives an opportunity to explore the behavior of the obtained results in both weak and strong interaction regimes. In case when the interaction between the matter fields is included, we noticed that not just the scalar field affects the solutions for both gravitational and electromagnetic fields, but also the interaction between the scalar and electromagnetic fields puts the constraints on the scalar field itself. We have seen that interaction between the matter fields drastically affects the space-time geometry and in general destroys both horizons of the solution. The only exception to this is the solution of the Garfinke-Horowitz-Strominger type for the charged dilatonic black hole, which has one regular horizon and second which is singular. Although in one of the partial cases the obtained solution describes the dilatonic black holes with two regular horizons, the question of the possibility of its physical existence is open for the moment. We believe that the general solution presented here might provide an interesting framework for studying quantum-mechanical effects in relativistic gravity.

A final question remains: whether or not this solution is stable. To study this problem might be interesting in a view of the cosmic censorship conjecture [4]. Note that the stability against an axial perturbations for solution given by expressions (7.1) has been shown in [16]. In [7] the stability of the solution presented in [5] was inferred for outside the outer horizon. However, for another related case of Einstein-Klein-Gordon equations with a quadratic self-interaction term, it was shown that static spherically symmetric solutions arc unstable [11]. ‘l’his problem in the general case of the solution (9.1)-(9.3) will be investigated in a subsequent paper.

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