

# RADIATION FROM AXISYMMETRIC WAVEGUIDE FED HORNS\*

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## I Introduction

In this paper the finite element method (FEM) is used in conjunction with the method of moments (MoM) and the mode matching technique (MM) to calculate return losses and radiation patterns for axisymmetric waveguide fed horns. The coupling of the FEM to the MoM, on one hand, and the coupling of the FEM to the MM, on the other, are performed by using boundary integrals. One advantage of this approach is that it allows for the presence of inhomogeneous materials to be included in the modelling domain as this poses no special problems for the FEM. In this respect, this method differs from the work of Berthon and Bills [1] who use only the MoM with a single waveguide mode serving as the excitation.

After describing the basic theory, the method is applied to the horn shown in Figure 1. The two dimensional modelling domain for this horn, depicted in Figure 2, clearly shows where the FEM/MoM and the FEM/MM boundary surfaces are located. Comparisons of measured and calculated far field radiation patterns and return loss are then shown. It is noted that this method generates a sparse, diagonally dominant, complex-symmetric system matrix which may be solved with standard library routines. Moreover, this FEM formulation has been shown to be free of spurious solutions [2].

## II Theory

The system matrix for the problem is found by coupling waveguide modes (MM) to the hybrid symmetric finite element method (HSFEM) as developed by Hoppe, Epp and Lee [3]. This coupling is effected by the use of two equations: the wave equation for the electric field and the matching of the tangential electric field at the waveguide/FEM interface. These equations, when enforced in a weak sense, lead to

$$\int_V \frac{1}{\mu_r} (\mathbf{V} \times \mathbf{w}^*) \cdot (\mathbf{V} \times \mathbf{E}^{FEM}) dV - \int_V k_0^2 \epsilon_r \mathbf{w}^* \cdot \mathbf{E}^{FEM} dV - j\omega\mu_0 \int_{S_{MoM}} \mathbf{w}^* \cdot (\hat{\mathbf{n}} \times \mathbf{H}^{MoM}) dS_{MoM} - j\omega\mu_0 \int_{S_{WG}} \mathbf{w}^* \cdot (\hat{\mathbf{n}} \times \mathbf{H}^{WG}) dS_{WG} = 0 \quad (1)$$

and

$$\int_{S_{WG}} \mathbf{u}^* \cdot \mathbf{E}^{FEM} dS_{WG} = \int_{S_{WG}} \mathbf{u}^* \cdot \mathbf{E}^{WG} dS_{WG} \quad (2)$$

respectively. If the test vectors in (1),  $\mathbf{w}$ , are chosen to be the set of finite element basis functions and the test vectors in (2),  $\mathbf{u}$ , are chosen to be the set of waveguide mode vectors,

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it is seen that the surface integrals in the the 1,11S of (2) and the last term of the LHS of (1) couple the FEM to the MM at their common inter face. The following matrix equation results:

$$\begin{bmatrix} Z^{EE} & B^{EJ} & 0 & B^{EB} \\ B^{JE} & G^{JJ} & G^{JM} & 0 \\ 0 & G^{MJ} & G^{MM} & 0 \\ B^{BE} & 0 & 0 & K^{BB} \end{bmatrix} \begin{pmatrix} C^E \\ C^J \\ C^M \\ b \end{pmatrix} = \begin{pmatrix} a^E \\ V^J \\ V^M \\ a^E \end{pmatrix}. \quad (3)$$

In this equation, the unknowns to be determined,  $C^E$ ,  $C^J$ ,  $C^M$ , and  $b$  represent the amplitudes of the electric field within the FEM region, the electric and magnetic surface currents on the FEM/MoM surface and the reflected waveguide mode coefficients at the FEM/WG interface, respectively. As noted in the introduction, the system matrix is complex-symmetric, sparse and diagonally dominant.

### III Results

in this section, the method outlined above is applied to the open-pipe horn antenna shown in Figure 1. This horn was designed by Tom Otoshi of the Jet Propulsion Laboratory to be used aboard the NASA Cassini Spacecraft as a low gain antenna. The two frequencies of operation of this horn are 7.175 GHz and 8.425 GHz. The radiation is RCP. It features two chokes to ensure operation in a balanced hybrid mode thus reducing cross polar radiation.

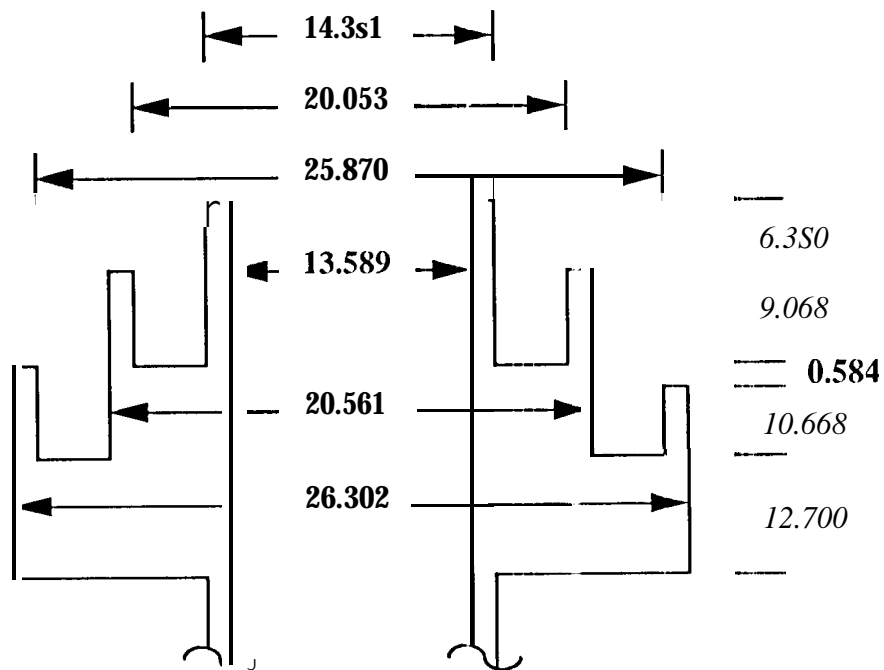
The finite element region shown in Figure 2 was meshed using approximately 35 (linear) nodes per wavelength at the highest frequency of simulation, 8.6 GHz. Also, 8 waveguide modes of azimuthal index,  $n = 1$ , were allowed to exist at the FEM/MM interface. This unnecessarily high mesh density and large number of waveguide modes was chosen in order to guarantee that the solution obtained would already have converged. The resultant system matrix is of order  $n^2 = 5073$ .

Figure 3 shows the calculated and measured return loss over a band of frequencies extending from 7.0 GHz to 8.6 GHz. The measured return loss was not time gated and thus includes the combined effects of a reflection due to the horn with a reflection due to a polarizer used to transition from rectangular to circular waveguide. Fortunately, this polarizer is placed far enough away from the horn such that the higher order modes generated do not couple into the horn. Note that there is a glitch in the measured curve at 8.47 GHz. This is due to a  $TM_{01}$  mode which begins to propagate at that frequency. Finally, Figure 4 shows the far field pattern at 8.425 GHz. Good agreement may be stated.

### References

- [1] A. Berthon and R.P. Bills. Integral equation analysis of radiating structures of revolution. *IEEE Transactions on Antennas and Propagation*, 37(2):159-170, February 1989.
- [2] G.C. Chinn, L. W. Epp and G. M. Wilkins. Determination of the eigenfrequencies of a ferrite filled cylindrical cavity resonator using the finite element method. to appear in *IEEE Transactions on Microwave Theory and Techniques*.
- [3] D.J. Hoppe, L. W. Epp and J. F. Lee. A hybrid symmetric FEM/MoM formulation applied to scattering by inhomogeneous bodies of revolution. *IEEE Transactions on Antennas and Propagation*, 42(6):798-805, June 1994.

Figure 1. Cassini Low Gain Antenna (LGA2) horn.



All dimensions in mm.

Figure 2. Computational domain.

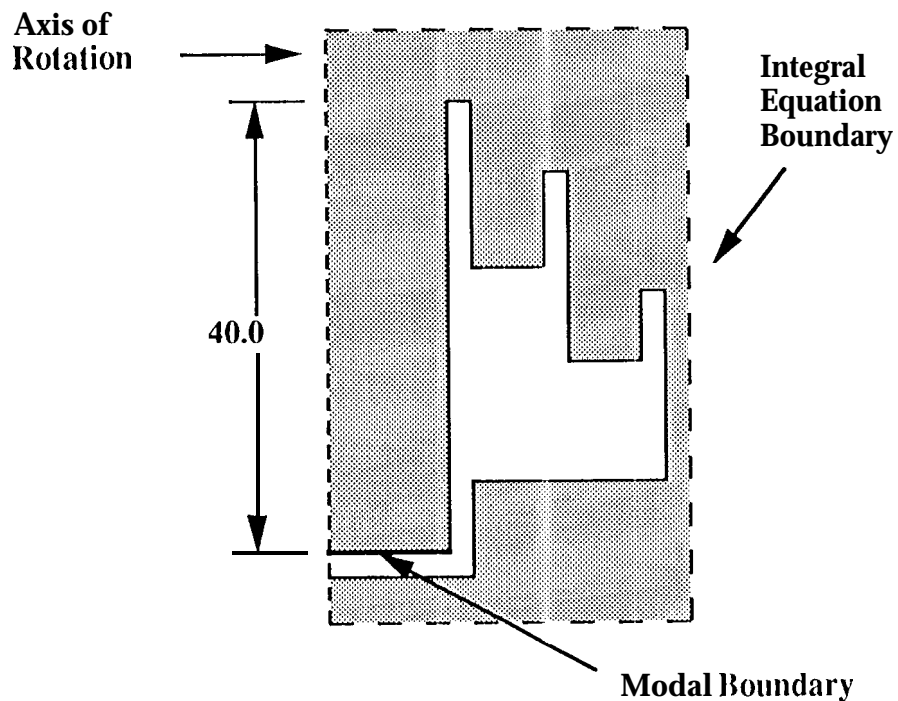


Figure 3. Measured and Calculated return loss for Cassini LGA2.

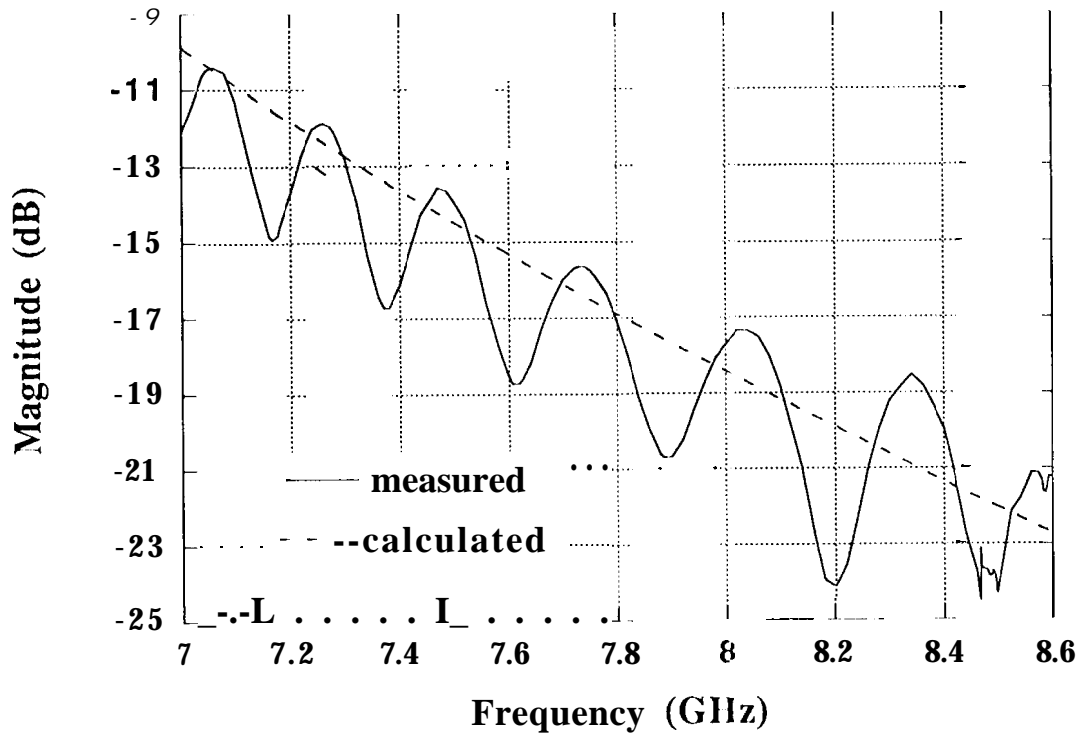


Figure 4. Far field pattern for Cassini LGA2 horn. RCP excitation at 8.425 GHz.

