Precise Kinematic Positioning with Simultaneous GPS Pseudorange and Carrier Phase Measurements

Sien-Chong Wu and Thomas P. Yunck

Jet Propulsion Laboratory, California Institute of Technology

BIOGRAPHIES

Sien-Chong Wu received his B.S.E.E. from the National Taiwan University, Taipei, Taiwan, and his Ph.D. from the University of Waterloo, Ontario, Canada. He is currently a Technical Group Leader in the Tracking Systems and Applications Section at JPL. He has been involved with the development of various tracking systems for deep-space as well as near-Earth space vehicles, and their applications to precision geodesy. His current interest is in the area of precise orbit determination using GPS.

Thomas **P.** *Yunck* received his B.S.E.E. from Princeton University, and his Ph.D. in Systems & Information Science from Yale University. He is currently Deputy Manager of the Tracking Systems and Applications Section at JPL, where he is involved in the application of GPS to precise orbit determination and geodesy.

ABSTRACT

This paper investigates three precise kinematic positioning techniques with simultaneous GPS pseudorange and carrier phase measurements. These techniques are simple, efficient and well suited for real-time positioning applications. A simulation/covariance analysis is carried out comparing the relative performance of these techniques. The analysis indicates that GPS carrier phase measurements significantly strengthen kinematic positioning. When wide-area differential GPS for reducing SA clock errors and GPS ephemeris errors is available, a low Earth orbiting satellite can be positioned to 0.5- 0.8 m in accuracy (3-D RSS). The accuracy is superior to positioning with pseudorange measurements alone by a factor of 2 to 3. Relative positioning performance and complexity between the three techniques are. compared.

INTRODUCTION

The capability of the Global Positioning System (GPS) for high-precision orbit determination of Earth satellites has long been studied [1], and lately demonstrated on TOPEX/Poseidon to few-centimeter accuracy [2,3]. Such high-precision positioning requires the collection and processing of GPS data onboard as well as from a network of ground tracking sites over a period of time. This, obviously, can only be clone after the fact. On the other hand, the unique continuous 3-D coverage of GPS suggests its potential for real-time, precise positioning of moving platforms Of particular interest is meter-level real-time positioning of aircraft and low Earth satellites in the presence of high dynamics where only kinematic positioning is deemed appropriate. The ultimate accuracy of real-time kinematic positioning is limited by uncertainties in GPS ephemerides and clock instability due to GPS Selective Availability (SA). The SA effects can be removed by differencing the GPS measurements with data from a local or regional network of ground receivers **[4,5].** Wide- area differential GPS (WADGPS) positioning [6–9] would reduce both GPS ephemerides and clock errors by uploading to the users (e. g., via geosynchronous satellites) *corrections* to GPS clocks and ephemerides determined by a larger network of ground receivers. To keep the onboard processing simple, the use of only pseudorange measurements has been proposed for these techniques.

It is well known that GPS pseudorange measurements provide absolute but crude positioning information while carrier phase measurements provide precise information of positional change. Combining the two data types yields far better positioning accuracy. However, each continuous pass of carrier phase measurements is accompanied by an unknown phase bias. These biases have to be removed or estimated in the filtering process. For real-time precision positioning, therefore, the need for an efficient onboard estimation technique becomes a great challenge. Different estimation techniques can be used to deal with these biases and to combine the information of the two data types. This paper investigates three efficient techniques for kinematic positioning using *both* GPS pseudorange and continuous carrier phase data types.

in the following. sections, the functional algorithms for the three positioning techniques are described; their superior tracking accuracy over the pseudorange-only tracking is demonstrated; their relative efficiency and weaknesses are discussed; and their performance with regional as well as wide- area differential GPS corrections is compared.

BACKGROUND

GPS kinematic positioning techniques rely solely on GPS measurements for the determination of the user position. In other words, the positions between different epochs are not related by any orbit dynamics. Such positioning is therefore free from dynamic errors; and there is no need for an orbit integrator, simplifying the onboard computation

To simplify the onboard estimation process, GPS pseudorange measurements alone have been proposed in the past for most mal-time positioning applications. in this paper, we propose the usc of both pseudorange and carrier phase measurements for better positioning accuracy. in the following analysis, we shal J assure that the onboard GPS receiver is capable of producing pseudorange and carrier phase measurements from multiple G1'S satellites. The pseudorange measurements at a *single epoch* are used for a crude determination of the user 3-D position and clock. These are then used as the nominal values for the calculation of measurement partials, and are combined with the carrier phase measurements in the subsequent precise estimation. Pseudorange measurements arc valuable in speeding up the convergence of the phase bias estimates, and thus the user position and clock estimates. On the other hand, carrier phase measurements provide tics between successive time points, thus improving the positioning accuracy.

Without any data differencing with ground measurements, the positioning accuracy will be limited by the huge GPS clock error attributed to SA dithering. Other error sources include GPS ephemeris error, and ionospheric delay if only single-frequency measurements are available on board. SA effects can be removed by re-transmitting a ground receiver's GPS measurements (compressed and time-tagged) and taking differences with the user measurements. A network of 6 ground sites will suffice for worldwide coverage; but measurements from only one or two ground sites tire needed at any onc time. The proposal wide-area differential GPS will estimate GPS ephemerides, ionospheric and tropospheric delays and SA effects using GPS measurements from a network of ground stations and broadcast the estimates to users for differencing.

The three positioning techniques to be investigated will be called the non-differential technique, the simple time-differential technique. and the modified tim-differential technique. The time-differential operation applies to carrier phase measurements only. **No** time-diffel-cntial operation is to be applied to pseudorange measurements in any of the techniques.

THE NON-DIFFERENTIAL TECHNIQUE

The estimation process can efficiently be carried out in terms of a Square-Root Information Filter (SRIF) formulation [10]. With this formulation, the derivation and explanation of the estimation process are also made clear.

The estimated parameters include the 4-D user *current* state (position and clock) and carrier phase biases, totaling 12 with a receiver of 8 channels. All these are treated as white-noise parameters; the 4-D user state is to be reset (i.e., white-noise updated) at every epoch whereas each of the phase biases is to be reset only when its continuity ceases.

The SRIF matrix equation of the estimation is

$$A\ Y = Z \qquad (1)$$

where A is the matrix of measurement partials; Y is the vector of estimated parameters; and Z is the measurement vector. The crux of the estimation process is the triangularization of A. Let the triangularized SRIF matrix equation be

$$R\ Y = Z' \qquad (2)$$

where the triangularized matrix R can be written as

$$R = \begin{pmatrix} X & C \\ & B \end{pmatrix} \qquad (3)$$

Here, X is a triangular matrix associating the 4-D user current state; C is a rectangular matrix correlating the 4-D state and the phase biases; and B is a triangular matrix associating the phase biases.

It is well known that a white-noise update (stochastic update in general) of a parameter involves the permutation of that parameter to the top of the SRIF matrix; after re-triangularization, the top row of the matrix which contains the smoothing information is put aside. Such permutation and re-triangularization are not needed for resetting, parameters which are originally placed at the top of the parameter list. Because the 4-D user state is to be reset far more frequently than the phase biases, it is recommended tobc placed before the phase biases for better efficiency. Since only white-noisruixiatc is involved, a generalized stochastic updating filter is not needed. Also, for the real-time application of interest here., the smoothing information can be discarded altogether. Them simplify the filtering process, which is outlined in the following.

The phase biases are identified by the. receiver channel numbers and are independent of which GPS arc being observed. Changes of GPS being observed in any channel provide the flags for phase breaks.

The white-noise update for the 4-D state, which occurs at every epoch, can be done by simply discarding X and C while leaving B alone. The white-noise update fora phase bias when its continuity ceases requires the following operations on B: (1) move the column associating the bias

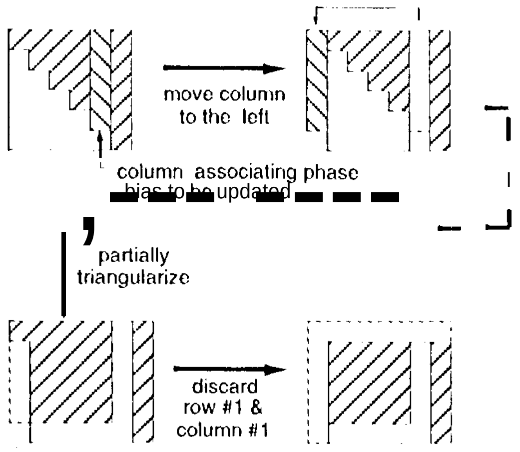

Fig. 1. The SRIF matrix operations for the white-noise update of a phase bias

to be updated to the left side of B; (2) partially triangular B so that all but one element in the first column are zeroed and (3) discard the first row and the first column. Fig. 1 depicts these matrix operations on B where the fifth bias out of the six assumed is being white-noise updated. in this figure, the shaded areas imply non-zero elements anct the clear area imply zero elements.

The measurement update can efficiently be done in two steps: (1) partially triangularize the SRIF matrix for the new measurements so that only X is fully triangularized, which also yields a new C and (2) combine the. un-triangularized part of the new measurements with previous B and triangularize to yield the updated B.

Fig. 2 provides a functional sketch of the above SRIF matrix operations for two consecutive epochs. It was assumed that no phase bias update occurs between t_{n-1} and t_n whereas a bias update occurs between t_n and t_{n+1}. In this figure, changes in fill pattern indicate changes in the matrix elements as a result of the matrix operation,

Although it may appear to be complicated, the actual

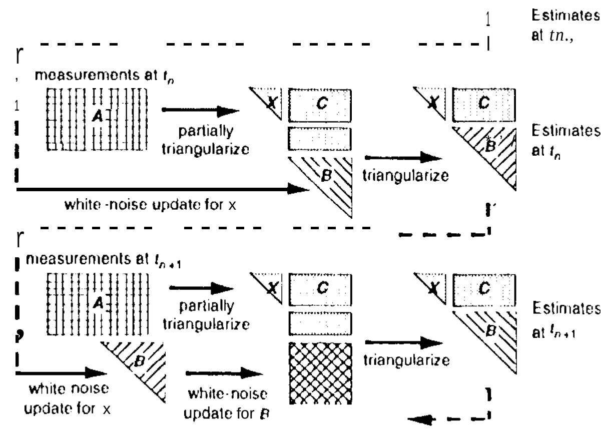

Fig. 2. A functional sketch of the SRIF matrix operations for the estimate of X

algorithm is quite simple. Both the white-noise updates and the measurement updates involve only simple matrix operations of small sizes and can be carried out fast. Hence, the technique is well suited for real-time applications.

THE SIMPLE TIME-DIFFERENTIAL TECHNIQUE

The use of GPS carrier phase measurements requires the estimation of phase biases in the above ncm-differential technique. These phase biases can be removed from the estimation by time differencing the. carrier phase measurements, leaving only the 4-D state to be estimated. The differencing can be taken either relative to a fix epoch or between consecutive time points. The latter is suitable for the time-differential techniques to be investigated,

The vectors of *linearized* non-differential carrier phase and pseudorange measurements at a time t_n have the following functional forms:

$$\phi_n = A_n X_n + B\,; \qquad n = 0, 1, \ldots, N \qquad (4)$$

and

$$P_n = A_n X_n \qquad (5)$$

where X is the 4-D user current state vector, B the vector of carrier phase biases and *A the* matrix of measurement partials.

The simple time-differential carrier phase measurements are formed by taking differences of (4) between consecutive time points to eliminate the phase biases B:

$$\Delta\phi_n = \phi_n - \phi_{n-1}\,; \qquad n = 1,2, \ldots, N$$
$$= A_n X_n - A_{n-1} X_{n-1} \qquad (6)$$

With $\Delta\phi_n$ observing 4 or more GPS satellites, one can make a precise estimate of the change in the. 4-D state

$$\Delta X_n = X_n - X_{n-1} \qquad (7)$$

Let this estimate be $\Delta\hat{X}_n$. Also, let the crude estimate of X_n using the (non-differential) pseudorange measurements be $\tilde{X}_n$. Then the combined estimate of the state can be written as

$$\hat{X}_n = \frac{n}{n+1}\left(\hat{X}_{n-1} + \Delta\hat{X}_n\right) + \frac{1}{n+1}\tilde{X}_n$$
$$\hat{X}_0 = \tilde{X}_0 \qquad (8)$$

The recursive formula (8) is attractive in that it is extremely simple to calculate. However, it suffers from two weaknesses as will be made clear in the following,

Eq. (8) can be re-written as

$$\hat{X}_n = \frac{1}{n+1}\left[\sum_{i=0}^{n}\tilde{X}_i + \sum_{i=1}^{n} i\,\Delta\tilde{X}_i\right] \qquad (9)$$

This equation clearly indicates that the absolute. fix of the positioning relics on the first term in the brackets, which amounts to the mean value of $\tilde{X}_n$; the second t ern n relates the relative position bet ween different epochs t_n. In other words, the accuracy of the state estimate depends heavily cm the quality of the pseudorange estimate $\tilde{X}_n$ over the continuous pass.

Another weakness of (8) is that the correlation between $\Delta\tilde{X}_n$ cannot be taken care of. Ignoring this correlation wilt degrade the estimation accuracy. However, in the presence of other mismodeling errors such as in GPS ephemerides and SA clocks, the effects of ignoring, the data correlation is insignificant.

THE MODIFIED TIME-DIFFERENTIAL TECHNIQUE

The simple time-cliffe.rential technique above can be modified to recover the absolute positioning information embedded in the. earl-ier phase. The modification applies only to the estimation process. The pseudorange and time-differential carrier phase measurements remain the. same as in the simple time-differential technique so that carrier phase biases arc removed.

We observe from (6) that the differential phase measurements at t_n are a function of both X_n and X_{n-1}. Thus, instead of solving for ΔX_n one can solve for *both* X_n and X_{n-1}. This will require at each time point the estimation of eight filter slates (two sets of 4-D states). The estimate of X_n at t_n using $\Delta\phi_n$ and R_n is further updated at t_{n+1} when $\Delta\phi_{n+1}$ and R_{n+1} are processed.

The modified time-differential technique recovers the absolute positioning information in the carrier phase measurements. 'I'his technique remains simple and is more efficient in processing than the ncm-differential technique above when the number of carrier phase biases at any one time is higher than 4. The correlation between the measurements still cannot be taken care of. However, as with the simple time-differential technique, these effects are insignificant in the presence of other mismodeling errors such as in GPS ephemeri des and clocks.

COMPARISON OF PERFORMANCE

The algorithms for the above three kinematic positioning techniques were coded into J]'] .'s orbit estimation software system GIPSY/OASIS-I 1 [11]. A covariance analysis was carried out comparing the techniques. The user was an Earth satellite with a Sun-synchronous circular orbit at an altitude of 700 km. The. GPS Primary Satellites Constellation [12] was assumed. Ionospheric-free (dual -frequency combined) pseudorange and carrier phase measurements wer c simulated for one hour at one minute intervals. At any epoch, between five and eight GPS satellites were observed.

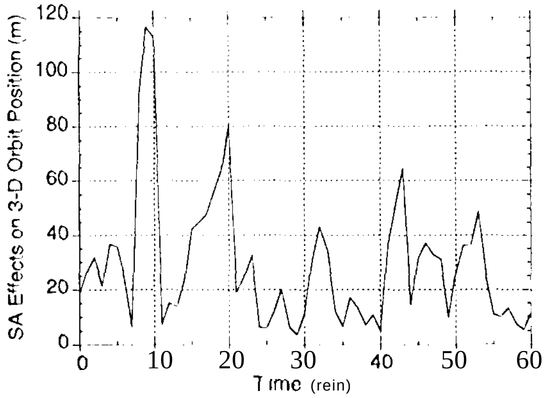

Fig. 3. User 3-D position error due to GPS SA clock dithering

Without ground GPS observations, the onboard kinematic positi oning is limited by the SA clock dithering. Pseudorange anti carrier phase measurements are equally corrupted by these effect S, which are typically tens of meters in magnitude. Consequently, carrier phase measurements would have to be heavily deweighted and thus would lose their strength and the positioning would rely on pseudorange measurements only. Fig. 3 shows the 3-D user positioning error due to the simulated 5A dithering.

The SA effects can be removed if, at any epoch, each GPS satellite observed by the user is also observed by a ground station and the information re-transmitted to the user for differencing. 1 'or this study, four globally distributed stations at Goldstone, CA; Canberra, Australia; Madrid, Spain and Usuda, Japan were assumed. At any epoch, each GPS satellite observed by the user was also sin ultaneously observed by one (and only one) of these stations. The common GPS view periods are shown in Fig. 4.

Fig. 5 compares the relat ive performance of the three estimation techniques under the influence of data noise only. To investigate how earl-ic.r phase measurements strcn~'then the positioning, the results using pseudorange measurements alone were also included in the figure. The data noise assumed was 1 m for pseudoranges and 1 cm for carrier phases on each recei ver. The improved positioning accuracy by adding tbc carrier phase. measurements is significant over the results using pseudorange data alone.

Fig. 5 includes only the effects of data noise. Other error sources considered include 10 cm zenith troposphere at each ground station, 5 cm in each component of station locat i on, 1 m station clocks and 2 m in each component of GPS orbit error. Although tbc troposphere and the station location can be better modeled, they arc

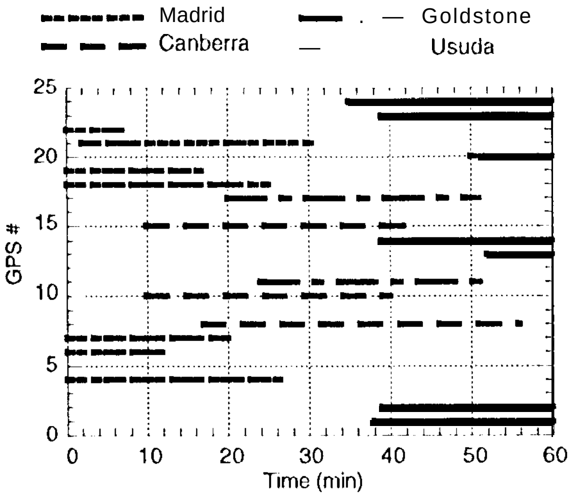

Fig. 4. Common view periods of GPS satellites between the user and four ground stations

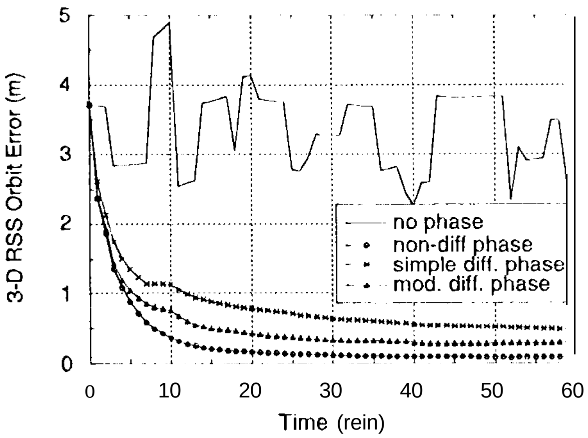

Fig. 5. User positioning error duc to data noise (lm pseudorange, lcm carrier phase)

insignificant in the presence of the dominating, GPS orbit error. The total user orbit errors are shown in Fig. 6. The higher error associated with the non-differential technique is due to the. high sensitivity of the tightly weighted non-differential phase measurements to mismodeled GPS orbit error which is far greater than the 1-cm data weight. Time-differential phase measurements, by nature, are far less sensitive to such slowly varying mismodeled errors.

The high sensitivity to mismodeled errors with the non-differential technique can be alleviated by deweighting the carrier phase measurements relative to the pseudorange measurements. However, its intrinsic optimal data weighting will also be violated; the effects due to data noise (Fig. 5) will consequently increase, nullifying its advantage over the tim-differential techniques. Fig. 7 shows the total user positioning error where the. can ier phase measurements were deweighted at 10 cm for the non-differential technique and at 2 cm for the modified time-differential technique. The simple time-differential technique is not affected by data weighting since the. two

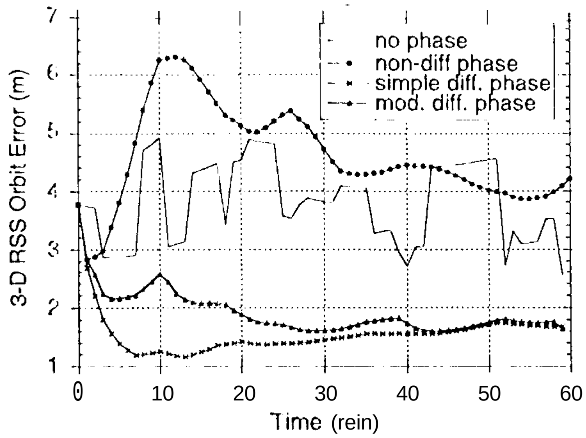

Fig. 6. Total user positioning error with regional differential GPS

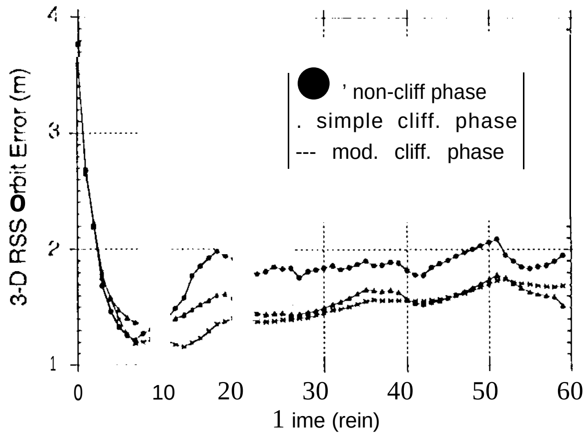

Fig. 7. Total user positioning error with regional differential GPS. Phase data were deweighted at 10 cm for non-differential technique and 2 cm for modified time-differential technique

data types are used separately and their results combined using (8).

In addition to removing SA effects, regional differential GPS also reduces the effects of GPS orbit error through common-mode cancellation. However, the degree of such cancellation diminishes as the separation between the user and the reference ground receiver increases. The reduction of GPS orbit error can be. greatly enhanced by wide-area differential GPS (WADGPS). The GPS orbit errors are now reduced in two ways. First, the orbits are better determined with the measurements from all the network receivers. Second, the effective receiver separation for common-mode cancellation, now between the user and the centroid of the network, is shortened.

For a North-American network the wide-area differential correction, after extrapolating ahead by 6 sec for real-time use, is expected to have a residual SA error of 1 % and an additional noise of 5 cm. The carrier phase data are accordingly deweighted to 5 cm. The effects of data noise

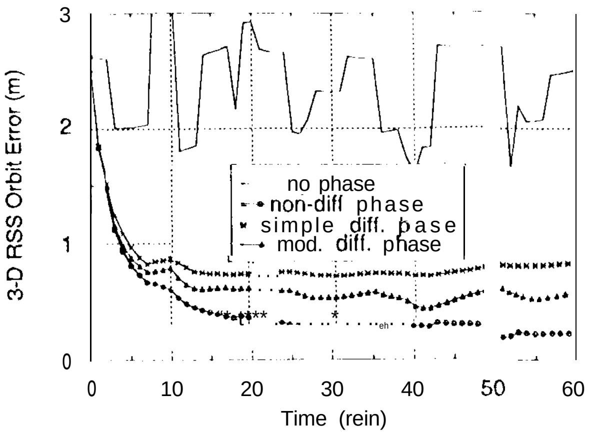

Fig. 8. User positioning error due to data noise with WADGPS

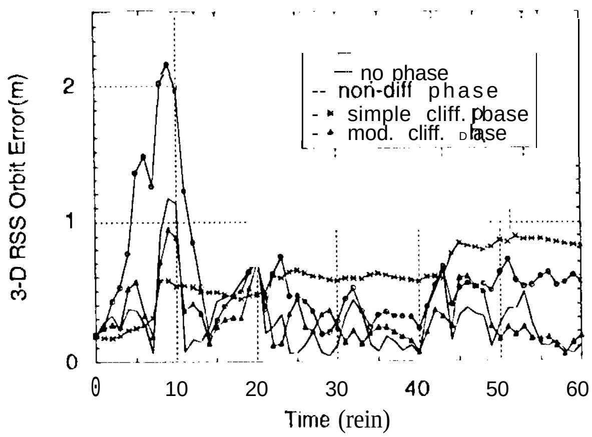

Fig. 9. User positioning error due to residual SA errors for WADGPS

and the residual SA errors are shown in Figs. 8 and 9, respectively.

The effects of data noise for all three techniques using both carrier phase and pseudorange measurements reduce very rapidly to below 1 m. While the. non-differential technique has the lowest error for its optimal data weighting, the modified time-differential technique is least sensitive to the residual SA error. The simple time-differential technique appears to have the highest sensitivity to both data noise and the residual SA errors. Note that the non-differential technique is sensitive to a peak SA error between 5 and 9 minutes past epoch, again due to the tight weight (5 cm) on the. carrier phase measurements. This SA error could be. lowered by further deweighting the carrier phase measurements, with an increase in data noise effects. The time-differential techniques effectively reduced the peak error without requiring further data deweighting.

The wide-area correction for GPS ephemerides was assumed to have a residual 3-D RSS error of 0.8 m. This error is further reduced by a factor of -4 when the. measurements are "corrected", which is equivalent to a

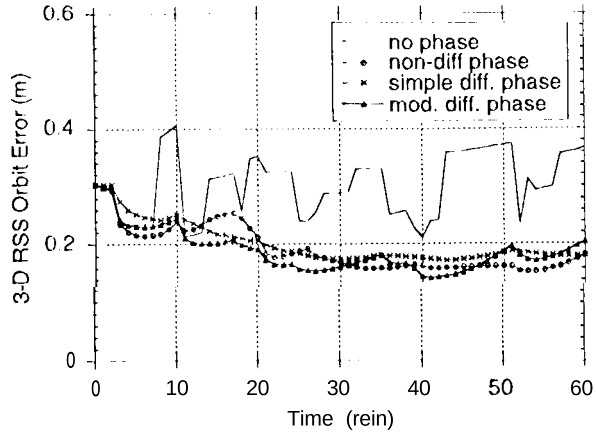

Fig. 10. Effects of residual GPS ephemeris error on user positioning with WADGPS

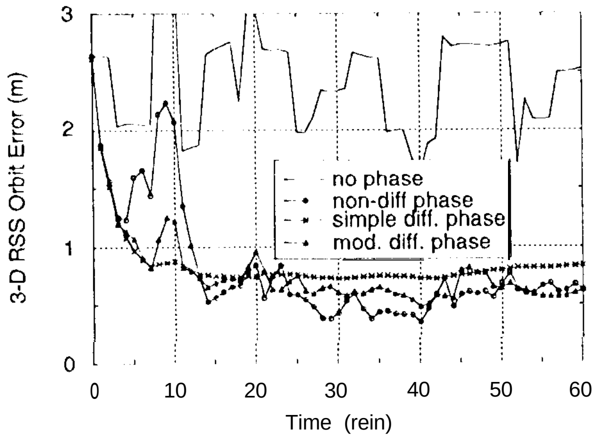

Fig. 11. Total user positioning error with WADGPS

differential operation with a fictitious station at the centroid of the North-American net work. Thus, the *differential* GPS orbit position error is about 0.2 m. The corresponding user orbit error due to this GPS orbit error is shown in Fig. 10. The effects on the three techniques using both carrier phase. and pseudorange measurements are nearly equal, and are lower as compared to the results using pseudorange measurements alone. Fig. 11 shows the total user positioning errors including the effects of data noise, the residual SA error and the GPS orbit error. The performance with the three techniques using both carrier phase and pseudorange measurements are superior to that with pseudorange alone.

OTHER APPROACHES

One way of using both pseudorange and carrier phase data types in kinematic positioning is to combine the two data types into a single data type before. the estimation process. The carrier phase data are used to cumulatively smooth the pseudorange data. The resultant data type, which is commonly called *carrier smoothed pseudorange*, will be treated as pseudorange measurements but with a lower data noise by a factor of square-root of N where N is the number of cumulative time points for the smoothing of continuous phase measurements. The effects of mismodeled systematic errors will be similar to the kinematic positioning with regular pseudorange measurements, which are higher than with the three techniques using both data types (cf. Fig.10). Although the positioning involves the estimation of a 4-D user state only, the data smoothing and the bookkeeping of the cumulative time count N for each continuous carrier phase stream requires additional computations.

Another possible approach to reducing the onboard estimation parameters for kinematic positioning is that of single differencing between different GPS satellites observed. This approach will reduce the number of estimated parameters by 2 (user clock and reference phase bias) and the number of measurements by 2 (one for each data type) at any epoch. However, the saving is not without an expense. When the reference GPS satellite from which other's are differenced becomes invisible, a new reference needs to be defined and the correlations between the new and previous differenced phases need to be calculated. Whether the gain from the reduction in the numbers of estimated parameters and measurements will more than offset the added complication remain to be investigated.

CONCLUSIONS

Three precise kinematic positioning techniques using simultaneous GPS pseudorange and carrier phase measurements have been investigated. These techniques are simple to implement and efficient in computation. A 3-D positioning accuracy of 0.5 to 0.8 m can be expected when wide-area differential corrections of GPS orbits and SA clocks are available. In the case of regional differential corrections, the positioning accuracy is expected to be better than 2 m. The accuracy is superior to positioning with pseudorange measurements alone by a factor of 2 to 3 under both circumstances.

The non-differential technique requires more computation in the estimation process but has an optimal data weighting and yields higher accuracy in the absence of mismodeled systematic errors. In the presence of these errors, the earlier phase data have to be properly deweighted to keep the sensitivity to these errors low, thus deviating from its optimal data weighting. Due to the slow time varying nature of most mismodeled errors, the two time-differential techniques do not require such data deweighting. The simple time-differential technique results in higher positioning error but is extremely simple in execution. The modified time-differential technique appears to be ideal for real-time kinematic positioning for its simplicity and overall best performance.

Although a low Earth satellite has been adopted as the user in the analysis carried out in this paper, the techniques are readily applicable to other moving platforms in the air, on land or water. In particular, improved accuracy can be expected for the Department of Transport at ion's proposed Wide Area Augmentation System (WAAS) with the inclusion of carrier phase measurements using the techniques investigated in this paper. The improved accuracy would elevate WAAS one step closer to a stand-alone aircraft landing system.

ACKNOWLEDGMENT

The work described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

REFERENCES

1. T. P. Yunck, S. C. Wu, J. T. Wu and C. L. Thornton, Precise Tracking of Remote Sensing Satellites with the Global Positioning System," *IEEE Trans. Geosci. and Remote Sensing*, Vol 28, No. 1, Jan. 1990, pp. 108-116.

2. T. P. Yunck, *et al.*, "First assessment of GPS-based reduced dynamic orbit determination on TOPEX/POSEIDON," Geophys. Res. Lett., 21, 541-544, 1994.

3. W. I. Bertiger, *et al.*, "GPS Precise Tracking of Topex/Poseidon: Results and Implications," to appear in *J. Geophys. Res.*

4. W. Tang, N. Johnson and J. Graft", "Differential GPS Operation with Multiple Ground Reference Stations," *ION Proc. 2nd Int. Tech. Meeting of the Satellite Division*, Sept. 1989, pp. 319-323.

5. P. Loomis, L. Sheynblat and T. Mueller, "Differential GPS Network Design," *ION Proc. 4th Int. Tech. Meeting of the Satellite Division*, Sept. 1991, pp. 511 – 520.

6. A. Brown, "Extended Differential GPS" *Navigation*, Vol. 36, No. 3, Fall 1989, pp. 265-286,

7. C. Kee, B. W. Parkinson and P. Axelrad, "Wide Area Differential GPS," *Navigation*, Vol. 38, No. 2, Summer 1991, pp. 123145.

8. V. Ashkenazi and C. J. Hill, "Wide Area Differential GPS: A Performance Study," *Proc. ION 5th Int. Tech. Meeting of the Satellite Division*, Albuquerque, NM, Sept. 1992, pp. 589-598.

9. T. Mueller, "Wide Area Differential GPS," *GPS World*, June 1994, pp. 36 44,

10. G. J. Bierman, *Factorization Methods for Discrete Sequential Estimation*, Academic Press, NY, NY, 1977.

11. F. B. Webb and J. F. Zumberge, (ed.), *An Introduction to GIPSY/OASIS II*, JPL Document, July 1993.

12. G. B. Green, P. D. Massatt and N. W. Rhodust, "The GPS 21 Primary Satellite Constellation," *ION Proc. First Int. Tech. Meeting of the Satellite Division*, Sept. 1988, pp. 15-26.