

COORDINATION AND CONTROL OF MULTIPLE MICROSPACECRAFT MOVING IN FORMATION

P. K.C. Wang

Department of Electrical Engineering
University of California
Los Angeles, CA 90024

F.Y. Hadaegh

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, CA 91109

Abstract: The problem of coordination and control of multiple microspacecraft (MS) moving in formation is considered. Here, each MS is modeled by a rigid body with fixed center of mass. First, various schemes for generating the desired formation patterns are discussed. Then, explicit control laws for formation keeping and relative attitude alignment based on near-est neighbor-tracking are derived. The necessary data which must be communicated between the MS to achieve effective control are examined. The time-domain behavior of the feedback-controlled MS formation for typical low earth orbits is studied both analytically and via computer simulation. The paper concludes with a discussion of the implementation of the derived control laws, and the integration of the MS formation coordination and control system with a proposed inter-spacecraft communication/computing network.

1. INTRODUCTION

Recently, the use of multiple microspacecraft for future space exploration and commercial space-based communication systems has been proposed. In particular, the use of multiple spacecraft for a structural link-free interferometer in a low earth orbit (LEO) was first suggested by Stachnik and his coworkers in 1984 [1], [2]. Its feasibility was further studied by Johnson and Neck [3], and DeCou [4]. In these works, emphasis was placed on seeking low earth orbits without thrusting or with a minimum amount of control effort. Moreover, only a small number of spacecraft was considered. The use of a large number of spacecraft for a space-based global communication system in LEO such as the IridiumSM LEO Mobile Communication System has been proposed recently [5]. These proposals call for a fleet of spacecraft moving in formation in which the relative position and attitude

between the spacecraft are closely controlled. There are many possible ways for generating and maintaining a given formation. To ensure the robustness of formation with respect to external perturbations and spacecraft malfunctions, it is essential to maintain communication link and to monitor the relative position and velocity between the spacecraft. Along a different vein, the interaction dynamics of autonomous free-flying mobile robots [6], [7], and also various *simple* navigation strategies for such robots moving in formation [8], [9] have been studied recently. The simplicity of the navigation strategies was motivated from those used by pilots in precision formation flying of aircraft, and also by marching bands in an open field. They are extremely simple and surprisingly effective. In precision flying of aircraft in formation, one or more members of the fleet are designated as leaders, the remaining members track their nearest neighbors according to given rules. In a marching band, there are also designated leaders for the band who provide the basic reference paths. The remaining members of the band navigate by tracking certain members of the band in their neighborhood (eg. the nearest and the farthest members).

Here, we consider some of the basic problems associated with the coordination and control of multiple microspacecraft moving in formation. Attention is focused on the derivation of simple, robust, implementable control laws for formation keeping and relative attitude alignment. Some of the abovementioned concepts and approaches in deriving navigation strategies for multiple mobile robots will be utilized in the derivation of control laws for multiple microspacecraft moving in formation. Various problems associated with initial formation acquisition involving sequential launching of spacecraft into appropriate orbits will not be considered here.

This paper begins with the development of suitable mathematical models for both rotational and translational motions of micro-spacecraft. Various schemes for generating the desired formation patterns are discussed. This is followed by the derivation of control laws for formation keeping and relative attitude alignment. The necessary data which must be communicated between the MS to achieve effective control are examined. The time-domain behavior of the feedback-controlled MS formation for typical LFO's is studied both analytically and via computer simulation. The paper concludes with a discussion of the implementation of the derived control laws, and the integration of the MS formation coordination and control system with a proposed inter-spacecraft communication/computing network.

2. MATHEMATICAL MODEL

A microspacecraft (abbreviated by "MS" hereafter) is characterized by its small mass and volume (less than 10 kg and 0.1 m³ respectively). It is essentially rigid. Hence it can be modelled by a rigid body with fixed center of mass over a sufficiently short control time interval. In what follows, we shall consider mathematical models for the rotational and relative translational motions separately.

To describe the motion of a MS fleet in formation, we regard each MS as a point mass moving in free space under the influence of a gravitational field. We assume that the fleet consists of N microspacecraft. We shall use the following coordinate systems for deriving the equations of motion for the MS in the three-dimensional Euclidean space R³: (i) an inertial coordinate system \mathcal{F}_0 with orthonormal basis $\mathcal{B}_0 = \{e_x, e_y, e_z\}$; and (ii) a set of moving coordinate systems \mathcal{F}_i , $i = 1, \dots, N$. The origin O_i and the axes of \mathcal{F}_i are at the center of mass and along the principal axes of inertia of the i-th MS respectively.

2.1 Rotational Motion: Let $\mathcal{B}_i = \{e_{ix}, e_{iy}, e_{iz}\}$ denote an orthonormal basis associated with the moving coordinate system (abbreviated by "MCS" hereafter) \mathcal{F}_i , and $[w]_i$ denote the representation of the vector w with respect to basis \mathcal{B}_i . Then the basis vectors in \mathcal{B}_0 and \mathcal{B}_i are related by a linear transformation C , defined by

$$e_{ix} = C_i e_x, \quad e_{iy} = C_i e_y, \quad e_{iz} = C_i e_z, \quad i = 1, \dots, N, \quad (1)$$

whose representation w , with respect to basis \mathcal{B}_0 is given by the direction cosine matrix:

$$C(q_i) = q_{14}^2 - \hat{q}_i^T \hat{q}_i I + 2\hat{q}_i \hat{q}_i^T - 2q_{14} Q(\hat{q}_i);$$

$$Q(\hat{q}_i) \triangleq \begin{bmatrix} 0 & -q_{13} & q_{12} \\ q_{13} & 0 & -q_{11} \\ -q_{12} & q_{11} & 0 \end{bmatrix}; \quad \hat{q}_i \triangleq \begin{bmatrix} q_{11} \\ q_{12} \\ q_{13} \end{bmatrix}, \quad (2)$$

where $q_i = [q_1^T, q_{14}]^T$ denotes the unit quaternion with q_{1j} being the Euler symmetric parameters [10] defined by

$$q_{1j} = \varepsilon_{1j} \sin(\phi_i/2), \quad j = 1, 2, 3; \quad q_{14} = \cos(\phi_i/2), \quad (3)$$

where ϕ_i is the principal angle and the C_{1j} 's are the components of the principal vector- of rotation ℓ_i defined by

$$\ell_i = \varepsilon_{11} e_x + \varepsilon_{12} e_y + \varepsilon_{13} e_z = \varepsilon_{11} e_{ix} + \varepsilon_{12} e_{iy} + \varepsilon_{13} e_{iz} \quad (4)$$

The Euler symmetric parameter-s q_{ij} satisfy the constraint:

$$\sum_{j=1}^3 q_{ij}^2 = 1, \quad i = 1, \dots, N, \quad (5)$$

implying that q_i lies on the unit sphere in R^3 .

The time derivative of q_{ij} is related to the angular velocity $\omega_i = \omega_{1x}e_{1x} + \omega_{1y}e_{1y} + \omega_{1z}e_{1z}$ of the MCS \mathcal{F}_i relative to the inertial coordinate system \mathcal{F}_0 by

$$\left. \begin{aligned} d\hat{q}_i/dt &= (q_{i4}\omega_i - \omega_i \times \hat{q}_i)/2, \\ dq_{i4}/dt &= -(\omega_i \cdot \hat{q}_i)/2, \end{aligned} \right\} \quad (6)$$

where $w \times v$ and $w \cdot v$ denotes respectively the cross and scalar products of vectors w and v in R^3 . Equation (6) has the following matrix representation:

$$dq_i/dt = \Omega([\omega_i]_i)q_i, \quad i = 1, \dots, N, \quad (6')$$

where $[\omega_i]_i = [\omega_{ix}, \omega_{iy}, \omega_{iz}]^T$ and

$$\Omega([\omega_i]_i) = \frac{1}{2} \begin{bmatrix} Q([\omega_i]_i) & [\omega_{i1}] \\ -[\omega_i]_i^T & 0 \end{bmatrix} \quad (7)$$

where Q is defined in (2). It follows from (5) and the skew-symmetry of $\Omega([\omega_i]_i)$ that system (6') is Euclidean-norm invariant and $\|q_i(t)\|^2 = q_i(t)^T q_i(t) = 1$ for all t .

Let d/dt_0 and d/dt_i denote the time derivative operators with respect to \mathcal{F}_0 and the MCS \mathcal{F}_i respectively; and \mathcal{D}_i the time derivative operator d/dt_0 in the MCS \mathcal{F}_i defined by

$$\mathcal{D}_i w \triangleq dw/dt_i + w \times W, \quad w \in R^3. \quad (8)$$

The angular velocities of the MS or the MCS \mathcal{F}_i relative to \mathcal{F}_0 are given by the following Euler's equations relating the time derivative of the angular momentum $I_i \omega_i$ with respect to \mathcal{F}_0 to the control torque τ_{ci} :

$$\begin{aligned} \mathcal{D}_i (I_i \omega_i) &\triangleq d(I_i \omega_i)/dt_i + \omega_i \times (I_i \omega_i) \\ &= I_i d\omega_i/dt_i + \omega_i \times (I_i \omega_i) = \tau_{ci}, \quad i = 1, \dots, N, \end{aligned} \quad (9)$$

where I_i is the tensor of inertia associated with the i -th MS; The time derivative of ω_i may also be taken with respect to \mathcal{F}_0 . Equation (9) has the following representation with respect to basis \mathcal{B}_i :

$$\frac{d}{dt} \begin{bmatrix} \omega_{ix} \\ \omega_{iy} \\ \omega_{iz} \end{bmatrix} = - \begin{bmatrix} I_{i1} \omega_{iy} \omega_{iz} \\ I_{i2} \omega_{ix} \omega_{iz} \\ I_{i3} \omega_{ix} \omega_{iy} \end{bmatrix} + \begin{bmatrix} \tau_{cix}/I_{ix} \\ \tau_{ciy}/I_{iy} \\ \tau_{ciz}/I_{iz} \end{bmatrix}, \quad i = 1, \dots, N, \quad (9')$$

where

$$I_{11} = (I_{1z} - I_{1y})/I_{1x}, I_{12} = (I_{1x} - I_{1z})/I_{1y}, I_{13} = (I_{1y} - I_{1x})/I_{1z}, \quad (10)$$

where I_{1x} , I_{1y} and I_{1z} are the principal moments of inertia of the i -th MS. Thus the equations for the rotational motion of the i -th MS are given by (6), (7), (9), and (10).

2.2 Translational Motion: In formation keeping, we are interested in the relative motion between any pair of MS. The equation of motion for the center of mass for the i -th MS relative to any MCS \mathcal{F}' is given by

$$dp_i/dt' + \omega \times p_i = f_{c1} + f_{g1}, \quad (11)$$

where p_i is the linear momentum of the i -th MS defined by

$$p_i \triangleq M_i \dot{y} + \omega \times \rho_i + d\rho_i/dt', \quad (12)$$

where M_i is the mass of the MS; ω and v_0 are the angular velocity of rotation and the velocity of the origin of \mathcal{F}' relative to the inertial coordinate system \mathcal{F}_0 respectively; the vector ρ_i is the position of the MS relative to \mathcal{F}' ; f_{c1} and f_{g1} are the control and gravitational forces respectively; and d/dt' is taken with respect to \mathcal{F}' .

Now, consider a pair of MS labeled by subscripts i and j . Let the moving coordinate system \mathcal{F}' be \mathcal{F}_i as defined earlier (See Fig.1). Let the vector ρ_{j1} denote the position of the j -th MS relative to \mathcal{F}_i . Applying (11) and (12) to the i -th and j -th MS with $\rho = 0$, $\omega = \omega_i$ and $v_0 = v_1$ and identifying p with p_{j1} , we obtain

$$D_i(p_i) \triangleq dp_i/dt_i + \omega_i \times p_i = (f_{c1} + f_{g1}), \quad (13a)$$

$$p_i \triangleq M_i(v_i + \omega_i \times p_i + dp_i/dt_i) = M_i v_i; \quad (13b)$$

$$D_i(p_j) \triangleq dp_j/dt_i + \omega_i \times p_j = (f_{c_j} + f_{g_j}), \quad (14a)$$

$$p_j \triangleq M_j(v_i + \omega_i \times \rho_{j1} + d\rho_{j1}/dt_i). \quad (14b)$$

Equations (13a) and (13b) imply

$$D_i(v_i) \triangleq dv_i/dt_i + \omega_i \times v_i = (f_{c1} + f_{g1})/M_i. \quad (15)$$

Combining (14a) and (14b) gives

$$D_i^2(\rho_{j1}) + D_i(v_i) = (f_{c_j} + f_{g_j})/M_j, \quad (16)$$

which, in view of (15), reduces to

$$\mathcal{D}_1^2(\rho_{j1}) = (f_{c_j} + f_{g_j})/M_j - (f_{c_1} + f_{g_1})/M_1 \quad (16')$$

where \mathcal{D}_1^2 denotes the time derivative operator d^2/dt_0^2 in the MCS \mathcal{F}_1 given by

$$\mathcal{D}_1^2(\rho_{j1}) \triangleq d^2\rho_{j1}/dt_1^2 + (d\omega_1/dt_1) \times \rho_{j1} + 2\omega_1 \times d\rho_{j1}/dt_1 + \omega_1 \times (\omega_1 \times \rho_{j1}). \quad (17)$$

The vectors ρ_{j1} can be expressed in terms of the basis vectors $\{e_x, e_y, e_z\}$ or $\{e_{1x}, e_{1y}, e_{1z}\}$ associated with the inertial coordinate system \mathcal{F}_0 or the MCS \mathcal{F}_1 respectively, i.e.

$$\rho_{j1} = P_{j1x} e_{1x} + P_{j1y} e_{1y} + P_{j1z} e_{1z} = \rho_{j1x} e_x + \rho_{j1y} e_y + \rho_{j1z} e_z. \quad (18)$$

The vector components corresponding to coordinate systems \mathcal{F}_0 and \mathcal{F}_1 are related by

$$\begin{bmatrix} \rho_{j1x} \\ \rho_{j1y} \\ \rho_{j1z} \end{bmatrix} = C(q_1) \begin{bmatrix} \rho_{j1x} \\ \rho_{j1y} \\ \rho_{j1z} \end{bmatrix}, \quad (19)$$

where $C(q_1)$ is the direction cosine matrix given by (2).

3. FORMATION KEEPING

The movement of a fleet of MS in formation can be achieved in many ways. Here, we shall consider the simplest approach based on nearest neighbor tracking [8]. In what follows, various schemes for generating the desired formation patterns will be discussed first. Then, control laws for formation keeping for particular types of desired formation patterns will be developed.

3.1 Formation Patterns: Here, given a fleet of N microspacecraft with $N \geq 2$, we assign a proper subset of the fleet as *group leaders* or guardians whose motions will serve as *reference motions* for the remaining MS. We consider a few possible schemes for generating a formation.

(i) The MS fleet is partitioned into groups \mathcal{G}_m , $m = 1, \dots, M$. Each group has a leader whose motion serves as a reference motion for the remaining MS in that group. Let the first member in group \mathcal{G}_m be the group leader whose motion is denoted by $\mathcal{R}_1^m = \{r_1^m(t) \in R^3: t \in \bar{I}_T\}$, where $\bar{I}_T = [0, T]$ is a given time interval. Let the desired motion for the second MS in \mathcal{G}_m be specified by

$$d_2^m(t) = r_1^m(t) + h_2^m(t), \quad t \in \bar{I}_T, \quad (2.0)$$

where $h_2^m(t) \in R^3$ is a specified nonzero *deviation vector* defined for all $t \in \bar{I}_T$. The second MS tries to track the motion of its leader such that the norm

of the tracking error

$$E_2^m(t) \triangleq d_2^m(t) - r_2^m(t) \quad (21)$$

is within a specified bound for all $t \in \bar{I}_T$. Similarly, the desired motion for the i -th MS in that group is given by

$$d_1^m(t) = r_{i-1}^m(t) + h_1^m(t). \quad (22)$$

The i -th MS tries to track the $(i-1)$ -th MS such that the norm of the tracking error

$$E_i^m(t) \triangleq d_i^m(t) - r_i^m(t) \quad (23)$$

is within a specified bound. Assuming zero tracking error for all MS, then

$$d_1^m(t) \triangleq d_{i-1}^m(t) + h_1^m(t) = r_1^m(t) + \sum_{k=1}^i h_k^m(t). \quad (24)$$

Thus, for any fixed time $t \in \bar{I}_T$, the point set

$$\mathcal{P}_m(t) = \{r_1^m(t), r_1^m(t) + h_2^m(t), \dots, r_1^m(t) + \sum_{k=2}^{K_m} h_k^m(t)\} \quad (25)$$

defines a *desired formation pattern at time t* for the group \mathcal{G}_m , where K_m is the number of MS in \mathcal{G}_m . Figure 2a shows a group of MS moving along the same circular orbit led by a group leader-. Note that the desired formation pattern at any time t is completely defined by the motion of the leader and the set of all deviation vectors $\{h_i^m(t), i = 2, \dots, K_m\}$. In more complex situations, the deviation vector $h_i^m(t)$ for the i -th MS may depend on the positions and/or velocities of its neighbors. This leads to more complex MS fleet dynamics. It may be of interest to determine the asymptotic behavior of the formation pattern as $t \rightarrow \infty$.

(ii) In addition to partitioning the fleet into groups as described in (i), the fleet may also assign one or more *fleetleaders whose* motions serve as reference motions for the *groupleaders* in a similar way as for a group. A fleet leader may be a group leader or a member of the fleet who does not belong to any group. Figure 2b shows a fleet of MS consisting of three groups. The middle group leader serves also as the fleet leader. The motions of the fleet leaders and group leaders define a formation pattern themselves.

(iii) In general situations, more than one MS in the fleet may be chosen as leaders or guardians for the MS fleet. Their motions serve as a "skeleton pattern" for the fleet. The desired motion for the remaining MS may be determined by *all* the MS in their specified neighborhood. For example, for

the i -th MS, its neighbors may be either specified, or taken as the set of all the MS which are within c -distance from the i -th MS. In the latter case, the neighbors corresponding to any MS are not specified *a priori*, and they may vary with time.

Let $\mathcal{J}_i(t) \subset \{1, \dots, N\} \setminus \{i\}$ denote the index set consisting of the labels of all the neighbors of the i -th MS whose position at time t is $r_j(t)$. Let $\mathcal{N}_i(t)$ denote the point set $\{r_j(t) : j \in \mathcal{J}_i(t)\}$ specifying the positions of all the neighbors of i -th MS. Let $\rho_{jk}(t) \triangleq r_j(t) - r_k(t)$. Suppose that $\{\rho_{jk}(t), j \in \mathcal{J}_i(t), k \neq j\}$ is linearly independent, where k is any element in $\mathcal{J}_i(t)$. Then the convex hull of $\mathcal{N}_i(t)$ (denoted by $\text{Co}\{\mathcal{N}_i(t)\}$) is a simplex. We may set the desired motion $d_i(t)$ for the i -th MS as the barycenter of $\text{Co}\{\mathcal{N}_i(t)\}$, i.e.

$$d_i(t) = \frac{1}{N_i} \sum_{j \in \mathcal{J}_i(t)} r_j(t), \quad (26)$$

where N_i is the number of elements in $\mathcal{J}_i(t)$.

Figure 2c shows a fleet of ten MS moving in formation. Here, the first MS is the fleet leader, and the neighboring MS corresponding to any MS are specified *a priori*. In particular, the MS labelled 3, 5, 7, and 9 are taken as neighbors for the sixth MS whose desired position at time t is taken to be the barycenter of the simplex formed by these four neighbors. If for the fifth MS, we assign the second, sixth, and eighth MS as its neighbors. Here, it is desirable to use the barycenter associated with only the second and eighth MS as the desired position for the fifth MS.

The choice of the barycenter of $\text{Co}\{h_i(t)\}$ as $d_i(t)$ is suitable for MS fleets moving in formation along straightline spatial paths. It is unsuitable for MS fleets moving in formation along curved spatial paths, since the barycenter of any pair of MS lies along the chord joining the centers of mass of the MS. In this case, we make use of (22) for specifying $d_i(t)$.

3.2 Control Laws: In what follows, we shall derive control laws for the cases where $d_i(t)$ is given by (26) or (22).

Case 1: Let the desired motion for the i -th MS correspond to the barycenter of the simplex $\text{Co}\{\mathcal{N}_i(t)\}$ defined by (26). The tracking error for the i -th MS is defined by $E_i(t) = d_i(t) - r_i(t)$ which, in view of (26), can be written as

$$E_i(t) = \frac{1}{N_i} \sum_{j \in \mathcal{J}_i(t)} r_j(t) - r_i(t)$$

$$= \frac{1}{N_1} \sum_{j \in \mathcal{J}_1(t)} (r_j(t) - r_1(t)) = \frac{1}{N_1} \sum_{j \in \mathcal{J}_1(t)} \rho_j^{(1)}(t) \quad (2-f)$$

Making use of (16'), we obtain the following differential equation for E_1 :

$$\begin{aligned} \mathcal{D}_1^2(E_1) &\triangleq \ddot{E}_1 + \dot{\omega}_1 \times E_1 + 2\omega_1 \times \dot{E}_1 + \omega_1 \times (\omega_1 \times E_1) \\ &= \frac{1}{N_1} \sum_{j \in \mathcal{J}_1(t)} (f_{gj}/M_j - f_{g1}/M_1 + u_{cj}) - u_{c1}, \end{aligned} \quad (28)$$

where $u_{cj} = f_{cj}/M_j$, $\dot{\omega}_1 \triangleq d\omega_1/dt_1 = d\omega_1/dt_0$, $\dot{E}_1 \triangleq dE_1/dt_1$, and $\ddot{E}_1 \triangleq d^2E_1/dt_1^2$.

To derive a control law for the i -th MS, we consider the following positive definite function of (E_1, \dot{E}_1) defined on \mathbb{R}^6 :

$$V_{11} = (K_{11} E_1 \cdot E_1 + \dot{E}_1 \cdot \dot{E}_1)/2, \quad K_{11} > 0. \quad (29)$$

The time rate-of-change of V_{11} along any trajectory of (28) is given by

$$\begin{aligned} dV_{11}/dt &= \dot{E}_1 \cdot \left\{ \frac{1}{N_1} \sum_{j \in \mathcal{J}_1(t)} (f_{gj}/M_j - f_{g1}/M_1 + u_{cj}) - 2\omega_1 \times \dot{E}_1 \right. \\ &\quad \left. - I_1^{-1}(-\omega_1 \times (I_1 \omega_1) + \tau_{c1}) \times E_1 - \omega_1 \times (\omega_1 \times E_1) + K_{11} E_1 - u_{c1} \right\} \\ &= \dot{E}_1 \cdot \left\{ \frac{1}{N_1} \sum_{j \in \mathcal{J}_1(t)} (f_{gj}/M_j - f_{g1}/M_1 + u_{cj}) \right. \\ &\quad \left. - I_1^{-1}(-\omega_1 \times (I_1 \omega_1) + \tau_{c1}) \times E_1 - (\omega_1 \cdot E_1)\omega_1 + (K_{11} + \|\omega_1\|^2)E_1 - u_{c1} \right\}. \end{aligned} \quad (30)$$

In the derivation of (30), we have used (9) to eliminate the term $d\omega_1/dt_1$ in (28). Now, if we set

$$\begin{aligned} u_{c1} &= \frac{1}{N_1} \sum_{j \in \mathcal{J}_1(t)} (f_{gj}/M_j - f_{g1}/M_1 + u_{cj}) - I_1^{-1}(-\omega_1 \times (I_1 \omega_1) + \tau_{c1}) \times E_1 \\ &\quad - \omega_1 \cdot E_1 \omega_1 + (K_{11} + \|\omega_1\|^2)E_1 + K_{21} \dot{E}_1, \end{aligned} \quad (31)$$

where K_{21} is a positive constant > 0 , then

$$dV_{11}/dt = -K_{21} \|\dot{E}_1\|^2 \leq 0. \quad (32)$$

In physical implementation, it is desirable to express the control law in terms of the variables referenced with respect to the body coordinate system of the MS. For control law (31), its representation with respect to basis $\mathcal{B}_1 = \{e_{1x}, e_{1y}, e_{1z}\}$ of the MCS \mathcal{F}_1 has the form:

$$[u_{c1}]_1 = \{B_1([\omega_1]_1) + (K_{11} + \|\omega_1\|^2)[I] - [\omega_1]_1 [\omega_1]_1^T\} [E_1]_1 + Q([E_1]_1) \Pi_{p1}^{-1} [\tau_{c1}]_1$$

$$\frac{1}{N} \sum_{j \in \mathcal{E}} g_j(t) ([\Delta u_{gj}^i]_1 + [u_{cj}^i]_1) + K_{21} [\dot{E}_1]_1, \quad i = 1, \dots, N, \quad (33)$$

where

$$\Pi_{p1} = \begin{bmatrix} I_{1x} & 0 & 0 \\ 0 & I_{1y} & 0 \\ 0 & 0 & I_{1z} \end{bmatrix}; \quad B_1([\omega_1]_1) = \begin{bmatrix} 0 & -I_{13} \omega_{1x} \omega_{1y} & I_{12} \omega_{1x} \omega_{1z} \\ I_{13} \omega_{1x} \omega_{1y} & 0 & -I_{11} \omega_{1y} \omega_{1z} \\ -I_{12} \omega_{1x} \omega_{1z} & I_{11} \omega_{1y} \omega_{1z} & 0 \end{bmatrix}, \quad (34)$$

and $Au_{gj}^i \triangleq f_{gj} / M_j - f_{g1} / M_1 = \Delta u_{gxj}^i e_{1x} + \Delta u_{gyj}^i e_{1y} + \Delta u_{gzj}^i e_{1z}$; [11 denotes the 3 x 3 identity matrix. For this control law, the equation for $[E_1]_1$ of the corresponding feedback-controlled system is given by

$$[\ddot{E}_1]_1 + (K_{21}[I] + 2Q([\omega_1]_1(t)))[\dot{E}_1]_1 + K_{11}[E_1]_1 = 0. \quad (35)$$

Since $Q([\omega_1]_1)$ is skew-symmetric, all solutions- $[E_1, \dot{E}_1]_1(t) \rightarrow (0, 0) \in \mathbb{R}^6$ as $t \rightarrow \infty$ for any $K_{11}, K_{21} > 0$.

In the special case where the MS are identical and move in a gravitational field which is essentially uniform over the spatial region containing the MS fleet, the term Au_{gj}^i in (33) can be neglected.

Case 2: We assume that each MS moves in a LEO, and the desired motion for the i -th MS is given by

$$d_i(t) = r_{i-1}(t) + h_i(t), \quad (36)$$

where $h_i(t)$ is a specified deviation vector. Here, the tracking error is defined by $E_i(t) \triangleq d_i(t) - r_i(t)$. Thus,

$$E_i(t) = r_{i-1}(t) + h_i(t) - r_i(t) = \rho_{(i-1)1} + h_i(t). \quad (37)$$

Using (17), we obtain the following differential equation for E_i :

$$\begin{aligned} \mathcal{D}_1^2(E_i) &\triangleq \ddot{E}_i + \dot{\omega}_1 \times E_i + 2\omega_1 \times \dot{E}_i + \omega_1 \times (\omega_1 \times E_i) \\ &= \mathcal{D}_1^2(h_i) + f_{g(i-1)} / M_{i-1} - f_{g1} / M_1 + u_{c(i-1)} - u_{c1}, \end{aligned} \quad (38)$$

where

$$\mathcal{D}_1^2(h_i) \triangleq \ddot{h}_i + \dot{\omega}_1 \times h_i + 2\omega_1 \times \dot{h}_i + \omega_1 \times (\omega_1 \times h_i). \quad (39)$$

Assuming a central Newtonian gravitational force field, the gravitational force acting on the k -th MS has the form:

$$f_{gk} = -\mu M_k r_k / \|r_k\|^3, \quad (40)$$

where μ is the geocentric gravitational constant (the product of the gravitational constant and the mass of the Earth); r_k is the vector

specifying the position of the mass center of the k-th MS relative to the inertial frame; and $\|r_k\|$ the Euclidean norm of r_k . Let $e_{r_1} = r_1/\|r_1\|$. We can write

$$r_1 = \|r_1\| e_{r_1} = \|r_1\| \{ (e_{r_1} \cdot e_{ix}) e_{ix} + (e_{r_1} \cdot e_{iy}) e_{iy} + (e_{r_1} \cdot e_{iz}) e_{iz} \} r_1 = r_1 + \rho_{j1}, \quad (41)$$

For a fleet of LEO MS moving in formation at approximately the same altitude, we have $\|r_1\|/\|r_{1-1}\| \approx 1$. Thus, the components of $(f_{g(1-1)}^{/M_{1-1}} - f_{g1}^{/M_1})$ in (38) with respect to basis \mathcal{B}_1 can be approximated by

$$\begin{aligned} & (f_{g(1-1)}^{/M_{1-1}} - f_{g1}^{/M_1})_{ik} \\ &= \omega_{10}^2 \{ \|r_1\| (e_{r_1} \cdot e_{ik}) - (\|r_1\|/\|r_{1-1}\|)^3 (\rho_{(1-1)ik} + \|r_1\| (e_{r_1} \cdot e_{ik})) \} \\ &\approx -\omega_{10}^2 \rho_{(1-1)ik}, \quad k = x, y, z, \end{aligned} \quad (42)$$

where $\omega_{10}^2(t) = \mu/\|r_1(t)\|^3$ is the orbital angular speed of the i-th MS about the origin of the inertial coordinate system \mathcal{F} at time t. Substituting (42) into (38) and making use of the identity $\omega_1 \times (\omega_1 \times E_1) = (\omega_1 \cdot E_1) \omega_1 - \|\omega_1\|^2 E_1$ lead to

$$\begin{aligned} \ddot{E}_1 + \dot{\omega}_1 \times E_1 + 2\omega_1 \times \dot{E}_1 + (\omega_1 \cdot E_1) \omega_1 + (\omega_{10}^2 - \|\omega_1\|^2) E_1 \\ = \mathcal{D}_1^2(h_1) + \omega_{10}^2 h_1 + u_{c(1-1)} - u_{c1}. \end{aligned} \quad (43)$$

When $\omega_{10} \approx \omega_0$ for all i (43) can be approximated by

$$\begin{aligned} \ddot{E}_1 + \dot{\omega}_1 \times E_1 + 2\omega_1 \times \dot{E}_1 + (\omega_1 \cdot E_1) \omega_1 + (\omega_{10}^2 - \|\omega_1\|^2) E_1 \\ = \mathcal{D}_1^2(h_1) + \omega_0^2 h_1 + u_{c(1-1)} - u_{c1}. \end{aligned} \quad (44)$$

To derive a control law for the i-th MS, we consider again the positive definite function V_{11} defined by (29). Here, dV_{11}/dt is given by

$$\begin{aligned} dV_{11}/dt &= \dot{E}_1 \cdot \{ \mathcal{D}_1^2(h_1) + \omega_{10}^2 h_1 + u_{c(1-1)} - u_{c1} - I_1^{-1} (-\omega_1 \times (I_1 \omega_1) + \tau_{c1}) \} \times E_1 \\ &\quad - 2\omega_1 \times \dot{E}_1 - (\omega_1 \cdot E_1) \omega_1 - (\omega_{10}^2 - \|\omega_1\|^2 - K_{11}) E_1 \} \\ &= \dot{E}_1 \cdot \{ 2\omega_1 \times \dot{h}_1 + \ddot{h}_1 + (\omega_1 \cdot (h_1 - E_1)) \omega_1 + (\omega_{10}^2 - \|\omega_1\|^2) (h_1 - E_1) \\ &\quad + u_{c(1-1)} - u_{c1} + I_1^{-1} (-\omega_1 \times (I_1 \omega_1) + \tau_{c1}) \times (h_1 - E_1) + K_{11} E_1 \}, \end{aligned} \quad (45)$$

where $\dot{h}_1 = dh_1/dt_1$ and $\ddot{h}_1 = d^2h_1/dt_1^2$. Now, if we set

$$\begin{aligned} u_{c1} &= I_1^{-1} (-\omega_1 \times (I_1 \omega_1) + \tau_{c1}) \times (h_1 - E_1) + (\omega_1 \cdot (h_1 - E_1)) \omega_1 \\ &\quad + (\omega_{10}^2 - \|\omega_1\|^2) (h_1 - E_1) + K_{11} E_1 + K_{21} \dot{E}_1 + 2\omega_1 \times \dot{h}_1 + \ddot{h}_1 + u_{c(1-1)}, \end{aligned} \quad (46)$$

where K_{21} is a positive constant > 0 , then

$$dV_{11}/dt = -K_{21} \|\dot{E}_1\|^2 \leq 0. \quad (47)$$

The above control law has the following representation with respect to basis \mathcal{B}_1 of the MCS \mathcal{F}_1 :

$$\begin{aligned} [u_{c1}]_1 = & \{B_1([\omega_1]_1) + (\omega_{10}^2 - \|\omega_1\|^2)[I] - [\omega_1]_1[\omega_1]_1^T\} [h_1 - E_1]_1 + 2Q([\omega_1]_1)[\dot{h}_1]_1 \\ & - Q([h_1 - E_1]_1)\Pi_{p1}^{-1}[\tau_{c1}]_1 + K_{11}[E_1]_1 + K_{21}[\dot{E}_1]_1 + [\ddot{h}_1]_1 + [u_{c(1-1)}]_1 \end{aligned} \quad (48)$$

As in Case 1, the equation for $[E_1]_1$ corresponding to the feedback-controlled system is given by (35).

Remark 1: Control laws (31) and (46) are model dependent in the sense that they depend on the parameters of the model. They correspond to state-feedback linearization controls which involve partial cancellation of the terms in (28) and (44). Assuming perfect cancellation, there is no coupling between the equations for the tracking errors of MS given by (35). In physical situations, perfect cancellation is not achievable due to inaccurate knowledge of the model parameter-values, sensor errors, and actuator saturation. Therefore it is of importance to determine the effect of imperfect cancellation on the behavior of the feedback-controlled system. Here, we model this imperfection by introducing a persistent disturbance N in (35) as follows:

$$\begin{aligned} \ddot{E}_1]_1 + (K_{21}[I] + 2Q([\omega_1]_1(t)))[\dot{E}_1]_1 + K_{11}[E_1]_1 \\ = N(t, [\omega_1(t)]_1, [\rho_{j1}(t)]_1, [u_{cj}(t)]_1, [E_1]_1, [\dot{E}_1]_1). \end{aligned} \quad (49)$$

We require that the zero state of (49) to be *totally stable*, i.e. given any $\epsilon > 0$, there exist two positive numbers $\delta_1(\epsilon)$ and $\delta_2(\epsilon)$ such that if

$$\|([E_1]_1, [\dot{E}_1]_1)(0)\| < \delta_1(\epsilon), \quad (50)$$

and

$$\|N(t, [\omega_1(t)]_1, [\rho_{j1}(t)]_1, [u_{cj}(t)]_1, [E_1]_1, [\dot{E}_1]_1)\| < \delta_2(\epsilon) \quad (51)$$

for $\|([E_1]_1, [\dot{E}_1]_1)(t)\| < \epsilon$ and all $t \geq 0$, then the corresponding solutions to (49) satisfy

$$\|([E_1]_1, [\dot{E}_1]_1)(t)\| < \epsilon \text{ for all } t \geq 0. \quad (52)$$

Since B_1 is a skew-symmetric matrix, the zero state of (35) is uniformly asymptotically stable for any $K_{11}, K_{21} > 0$. Then, it follows from a well-known theorem of Malkin [11] that the zero state of (49) is totally stable.

Remark 2: Note that control laws (33) (resp. (46)) for the i -th MS require the knowledge of its own attitude control law τ_{c1} and the *control* laws u_{c1} of all its neighbors (resp. $(i-1)$ -th MS). The latter information must be transmitted to the i -th MS. Note also that control law (46) can be rewritten as

$$\begin{aligned} u_{c1} = & I_1^{-1}(-\omega_1 \times (I_1 \omega_1) + \tau_{c1}) \times E_1 - (\omega_1 \cdot E_1) \omega_1 - (\omega_{10}^2 - \|\omega_1\|^2) E_1 \\ & + K_{11} E_1 + K_{21} \dot{E}_1 + u_{c(i-1)} + \mathcal{D}_1^2(h_1) + \omega_{10}^2 h_1. \end{aligned} \quad (46')$$

The term $(\mathcal{D}_1^2(h_1) + \omega_{10}^2 h_1)$ in (46') represents a feed-forward control. When the norm of this term is large, the norm of u_{c1} is also large, which is undesirable. This situation may be alleviated by replacing the term by a suitable scalar multiple of $(\mathcal{D}_1^2(h_1) + \omega_{10}^2 h_1)$. In the important special case where the MS move in a circular LEO and the deviation vector h_1 rotates about the Earth's center with angular velocity $\omega_1 \cong \omega_{10}$, then $\mathcal{D}_1^2(h_1) + \omega_{10}^2 h_1 \cong 0$ or h_1 is close to a solution of the simple harmonic oscillator equation $d^2 h_1 / dt^2 + \omega_{10}^2 h_1 = 0$.

3.3 Formation Pattern Stability: Given a fleet of N microspacecraft each having its own desired motion $d_i = d_i(t)$ defined for all $t \geq 0$, the formation pattern at time t for the fleet is specified by the point set $\mathcal{P}(t) = \{d_i(t), i = 1, \dots, N\}$. We introduce an error measure $A(t)$ for the MS fleet with respect to $\mathcal{P}(t)$ as follows:

$$\Delta(t) = \left(\sum_{i=1}^N \{ \sigma_{11} \|E_i(t)\|^2 + \sigma_{21} \|\dot{E}_i(t)\|^2 \} \right)^{1/2} \quad (53)$$

where σ_{11} and σ_{21} are specified positive weighting coefficients. We define stability of a desired formation pattern as follows:

Definition: A given desired formation $\mathcal{P} = \mathcal{P}(t), t \geq 0$, for the MS fleet is said to be *stable*, if given any real number $\epsilon > 0$, there exists a $\delta > 0$ such that $\Delta(t) < \delta \Rightarrow A(t) < \epsilon$ for all $t \geq 0$. If, in addition, $\Delta(t) \rightarrow 0$ as $t \rightarrow \infty$, then the desired formation pattern is said to be *asymptotically stable*.

It is evident from (35) that if each MS applies control law (33) or (46), then the desired formation pattern is asymptotically stable. Here, asymptotic stability is only local in the sense that the convergence of $A(t)$ to 0 as $t \rightarrow \infty$ is attained if the deviation of the initial formation pattern at $t = 0$ from the desired one is sufficiently small. In physical situations, the possibility of collision between MS must also be considered.

4. ATTITUDE CONTROL

Let the desired attitude and angular velocity of the i -th MS at time t

relative to the inertial coordinate system \mathcal{F}_0 be specified respectively by the MCS $\mathcal{F}_1^d(t)$ and $\omega_1^d(t)$, which may depend on the attitude and angular velocity of its neighbors. For example, the desired attitude and angular velocity of the i -th MS correspond exactly to $\mathcal{F}_{i-1}(t)$ and $\omega_{i-1}(t)$ respectively. A more complex situation involves MS moving in LEO where the desired attitude and angular velocity of the i -th MS are related to the attitude and angular velocity of the $(i-1)$ -th MS by a specified rotation and angular velocity increment $\Delta\omega_1$ respectively. In the foregoing situations, it is of interest to control the *relative* attitudes and angular velocities between the MS. In physical situations, the measurement or estimation of spacecraft attitude and angular velocities with respect to an inertial coordinate system can be performed using star trackers and/or sun sensors. The measurement and estimation of the relative attitude and angular velocities of spacecraft require more complex sensor and estimation systems. Therefore it is of interest to derive control laws which are expressed in terms of the instantaneous attitude and angular velocity of each MS with respect to the inertial coordinate system \mathcal{F}_0 . In what follows, we shall derive control laws for the i -th MS which are expressed in terms of the instantaneous attitudes and angular velocities of the MS relative to the inertial coordinate system \mathcal{F}_0 or relative to the MCS \mathcal{F}_1 .

Let the unit quaternion corresponding to $\mathcal{F}_1^d(t)$ relative to the inertial coordinate system \mathcal{F}_0 be denoted by $q_1^d(t) = [\hat{q}_1^d(t), q_{14}^d(t)]^T$. We assume that q_1^d and ω_1^d are consistent in the sense that they satisfy

$$\left. \begin{aligned} d\hat{q}_1^d/dt_0 &= (q_{14}^d \omega_1^d - \omega_1^d \times \hat{q}_1^d)/2, \\ dq_{14}^d/dt_0 &= -(\omega_1^d \cdot \hat{q}_1^d)/2. \end{aligned} \right\} \quad (54)$$

and

$$d(I_1 \omega_1^d)/dt_0 = I_1 d\omega_1^d/dt_0 + \omega_1^d \times (I_1 \omega_1^d) \stackrel{\Delta}{=} \tau_{c1}^d, \quad (55)$$

We introduce the deviations

$$\delta q_1 \stackrel{\Delta}{=} q_1^d - q_1 = [\hat{q}_1^d - \hat{q}_1, q_{14}^d - q_{14}]^T, \quad \delta \omega_1 \stackrel{\Delta}{=} \omega_1^d - \omega_1. \quad (56)$$

Evidently, δq_1 satisfies

$$\left. \begin{aligned} d\delta\hat{q}_1/dt_0 &= (q_{14}^d \omega_1^d - q_{14} \omega_1 - \omega_1^d \times \hat{q}_1^d + \omega_1 \times \hat{q}_1)/2, \\ d\delta q_{14}/dt_0 &= -(\omega_1^d \cdot \hat{q}_1^d - \omega_1 \cdot \hat{q}_1)/2. \end{aligned} \right\} \quad (57)$$

To derive an attitude control law, we consider the following positive definite function V_{11} defined on R^7 :

$$V_{11} = K_{q1} V'_{11} + V''_{11}, \quad (58)$$

where K_{q1} is a given positive constant and

$$V'_{11} = \delta q_{14}^2 + \delta \hat{q}_1 \cdot \delta \hat{q}_1, \quad V''_{11} = (\delta \omega_1 \cdot I_1 \delta \omega_1)/2. \quad (59)$$

The time derivatives of V'_{11} and V''_{11} along the solutions of the equations for $(\delta q_1, \delta \omega_1)$ are given by

$$\begin{aligned} dV'_{11}/dt &= \delta q_{14} (-\omega_1^d \cdot \hat{q}_1^d + \omega_1 \cdot \hat{q}_1) + \delta \hat{q}_1 \cdot (q_{14}^d \omega_1^d - q_{14} \omega_1 - \omega_1^d \times \hat{q}_1^d + \omega_1 \times \hat{q}_1) \\ &= (q_{14}^d \delta \hat{q}_1 - \delta q_{14} \hat{q}_1^d - \hat{q}_1^d \times \delta \hat{q}_1) \cdot \delta \omega_1; \end{aligned} \quad (60)$$

$$\begin{aligned} dV''_{11}/dt &= \{(I_1 d\delta \omega_1/dt_0) + d(I_1 \delta \omega_1)/dt_0\} \cdot \delta \omega_1/2 \\ &= \{d(I_1 \delta \omega_1)/dt_0 + (\omega_1 \times (I_1 \omega_1) - \omega_1^d \times (I_1 \omega_1^d))/2\} \cdot \delta \omega_1 \\ &= \{\tau_{c1}^d - \tau_{c1} - (\omega_1^d \times (I_1 \delta \omega_1))/2\} \cdot \delta \omega_1. \end{aligned} \quad (61)$$

Thus,

$$\begin{aligned} dV_{11}/dt &= \{K_{q1} (q_{14}^d \delta \hat{q}_1 - \delta q_{14} \hat{q}_1^d - \hat{q}_1^d \times \delta \hat{q}_1) \\ &\quad + \tau_{c1}^d - \tau_{c1} - \omega_1^d \times (I_1 \delta \omega_1)/2\} \cdot \delta \omega_1. \end{aligned} \quad (62)$$

Now, if we set

$$\tau_{c1} = K_{q1} (q_{14}^d \delta \hat{q}_1 - \delta q_{14} \hat{q}_1^d - \hat{q}_1^d \times \delta \hat{q}_1) + \tau_{c1}^d - \omega_1^d \times (I_1 \delta \omega_1)/2 + K_{\omega 1} I_1 \delta \omega_1, \quad (63)$$

where $K_{\omega 1}$ is a positive constant, then

$$dV_{11}/dt = -K_{\omega 1} \delta \omega_1 \cdot I_1 \delta \omega_1 \leq 0. \quad (64)$$

Thus $V_{11}(t) \leq V_{11}(0)$ for all $t \geq 0$ implying uniform boundedness of $\|\delta \omega_1(t)\|$ for all $t \geq 0$. From (64), we have

$$\begin{aligned} d^2 V_{11}/dt^2 &= -2K_{\omega 1} dV''_{11}/dt \\ &= -2K_{\omega 1} \{K_{q1} (\delta q_{14} \hat{q}_1^d - q_{14}^d \delta \hat{q}_1 + \hat{q}_1^d \times \delta \hat{q}_1) - K_{\omega 1} I_1 \delta \omega_1\}. \end{aligned} \quad (65)$$

Thus,

$$\begin{aligned} |d^2 V_{11}/dt^2| &\leq 2K_{\omega 1} (K_{q1} \|\delta q_{14} \hat{q}_1^d - q_{14}^d \delta \hat{q}_1 + \hat{q}_1^d \times \delta \hat{q}_1\| \|\delta \omega_1\| + K_{\omega 1} \delta \omega_1 \cdot I_1 \delta \omega_1) \\ &\leq 2K_{\omega 1} (K_{q1} \|\delta \omega_1\| + K_{\omega 1} \delta \omega_1 \cdot I_1 \delta \omega_1). \end{aligned} \quad (66)$$

Since $\|\delta \omega_1(t)\|$ is uniformly bounded for all $t \geq 0$, $d^2 V_{11}/dt^2$ is also uniformly bounded. Consequently dV_{11}/dt is uniformly continuous for $t \geq 0$. From Barbalat's Lemma [12], we conclude that $(dV_{11}/dt)(t) \rightarrow 0$ as $t \rightarrow \infty$, or $\omega_1(t) \rightarrow \omega_1^d(t)$ as $t \rightarrow \infty$. But it does not follow that $\delta q_{14}(t) \rightarrow 0$ and $\delta \hat{q}_1(t) \rightarrow 0$ as $t \rightarrow \infty$.

To proceed further, we make use of the fact that the quaternion $(\hat{\Delta}q_1, \Delta q_{14})$ of the desired attitude or \mathcal{F}_1^d relative to \mathcal{F}_1 is related to the quaternions (\hat{q}_1^d, q_{14}^d) for \mathcal{F}_1^d , and (\hat{q}_1, q_{14}) for \mathcal{F}_1 relative to the inertial coordinate system \mathcal{F}_0 by [131

$$\Delta\hat{q}_1 = q_{14} \hat{q}_1^d - q_{14}^d \hat{q}_1 - \hat{q}_1 \times \hat{q}_1^d, \quad \Delta q_{14} = q_{14} q_{14}^d + \hat{q}_1 \cdot \hat{q}_1^d. \quad (67)$$

When \mathcal{F}_1 coincides with \mathcal{F}_1^d (i.e. $\delta\hat{q} = 0$ and $\delta q_{14} = 0$), we have $\Delta\hat{q} = 0$ and $\Delta q_{14} = 1$. Now, making use of the identity

$$\delta q_{14} \hat{q}_1^d - q_{14}^d \delta\hat{q}_1 + \hat{q}_1^d \times \delta\hat{q}_1 = q_{14}^d \hat{q}_1 - q_{14} \hat{q}_1^d - \hat{q}_1 \times \hat{q}_1^d, \quad (68)$$

control law (63), in view of (67), can be rewritten as:

$$\tau_{c1} = -K_{q1} \Delta\hat{q}_1 + \tau_{c1}^d - \omega_1^d \times (I_1 \delta\omega_1)/2 + K_{\omega1} I_1 \delta\omega_1, \quad (69)$$

which, except for the addition of $-\omega_1^d \times (I_1 \delta\omega_1)/2$ term and I_1 in the last term, has the same form as that proposed by Wen and Kreutz-Delgado [141]. It can be verified that, their result (Theorem 2 with corrections [15] and minor modifications) is applicable to control law (69). Thus, we conclude that $\Delta\hat{q}_1(t)$ and $\delta\omega_1(t) \rightarrow 0$ as $t \rightarrow \infty$ for any positive K_{q1} and $K_{\omega1}$. Moreover, if

$$\delta\omega_1(0) \cdot I_1 \delta\omega_1(0)/2 < 2K_{q1} (1 + \Delta q_{14}(0)), \quad (70)$$

the rate of convergence is exponential.

Remark 3: Global exponential convergence can be achieved by adding a nonlinear term to control law (69), i.e.

$$\begin{aligned} \tilde{\tau}_{c1} = & -K_{q1} \Delta\hat{q}_1 + \tau_{c1}^d - \omega_1^d \times (I_1 \delta\omega_1)/2 + K_{\omega1} \delta\omega_1/2 \\ & + \tilde{K}_1 K_{q1} (\delta q_{14}^2 + \|\delta\hat{q}_1\|^2) I_1 \delta\omega_1 / (\delta\omega_1 \cdot I_1 \delta\omega_1), \end{aligned} \quad (71)$$

where \tilde{K}_1 is a given positive constant. For this control law,

$$dV_{11}/dt = -K_{\omega1} \delta\omega_1 \cdot I_1 \delta\omega_1/2 - \tilde{K}_1 K_{q1} (\delta q_{14}^2 + \|\delta\hat{q}_1\|^2) \leq -\min\{\tilde{K}_1, K_{\omega1}\} V_{11}. \quad (72)$$

Thus, $V_{11}(t) \leq V_{11}(0) \exp(-\min\{\tilde{K}_1, K_{\omega1}\}t)$ for all $t \geq 0$. Here, although the convergence rate is determined only by \tilde{K}_1 and $K_{\omega1}$, the effective nonlinear gain $(\delta q_{14}^2 + \|\delta\hat{q}_1\|^2)/(\delta\omega_1 \cdot I_1 \delta\omega_1)$ may be large, which may result in undesirable large magnitude control torques.

Now, consider an alternate approach to the derivation of an attitude control law similar to that used in [161 and [171. Let $z = z(q)$ be a nonlinear transformation from the unit sphere in R^4 into R^3 defined by

$$z(q) = [q_1/q_4, q_2/q_4, q_3/q_4]^T, \quad (73)$$

where $q = [q_1, q_2, q_3, q_4]^T$. The vectors $z_1 \triangleq z(q_1)$ correspond to the Gibbs vector or Cayley-Rodrigues parameter-s [II]. Again we consider the quaternion $(\hat{q}_1, \Delta q_{14})$ of the desired attitude or \mathcal{F}_1^d relative to \mathcal{F}_1 . We define $\Delta z_1 = \hat{\Delta q}_1 / \Delta q_{14}$. It can be readily verified that

$$d\Delta z_1/dt = (\delta\omega_1 - \delta\omega_1 \times \Delta z_1 + (\delta\omega_1 \cdot \Delta z_1)\Delta z_1)/2. \quad (74)$$

Consider the functional

$$V_{21} = K_{z1} \Delta z_1 \cdot \Delta z_1 + V_{11}^n, \quad (75)$$

where V_{11}^n is defined in (59), and K_{z1} is a given positive constant. By direct computation using (74), we obtain

$$d(K_{z1} \Delta z_1 \cdot \Delta z_1)/dt = K_{z1} (1 + \|\Delta z_1\|^2) \Delta z_1 \cdot \delta\omega_1, \quad (76)$$

which, in view of (61), leads to

$$dV_{21}/dt = \delta\omega_1 \cdot \{K_{z1} (1 + \|\Delta z_1\|^2) \Delta z_1 + -C_{ci}^d - \tau_{ci} - (\omega_1^d \times (I_1 \delta\omega_1))/2\}. \quad (77)$$

Now, if we set

$$\begin{aligned} \tau_{ci} = & T_{ci}^d + K_{z1} (1 + \|\Delta z_1\|^2) \Delta z_1 - (\omega_1^d \times (I_1 \delta\omega_1))/2 \\ & - K_{\omega1} I_1 \delta\omega_1 / 2 - K_{z1} \|\Delta z_1\|^2 I_1 \delta\omega_1 / (\delta\omega_1 \cdot I_1 \delta\omega_1), \end{aligned} \quad (78)$$

then

$$dV_{21}/dt = -V_{11} - K_{\omega1} V_{21} \leq -\min\{K_{\omega1}, 1\} V_{21}. \quad (79)$$

Thus, $V_{21}(t) \leq V_{21}(0) \exp(-\min\{K_{\omega1}, 1\}t)$ for all $t \geq 0$, implying exponential stability of the equilibrium state $(\Delta z_1, \delta\omega_1) = (0, 0)$.

Remark 4: The Gibbs vector becomes singular when the rotation angle is an odd multiple of π . Thus, control law (78) is useful for small angle maneuvers. This limitation does not occur in control laws (69) and (71).

Now, we apply the foregoing controls to the special case where $q_1^d = q_{1-1} + \Delta q_1$ and $\omega_1^d = \omega_{1-1} + \Delta\omega_1$, where Δq_1 and $\Delta\omega_1$ are specified consistent increments. For this case,

$$\begin{aligned} \delta q_1 &= q_{1-1} + \Delta q_1 - q_1 = [\hat{q}_{1-1} + \Delta \hat{q}_1 - q_1, q_{(1-1)4} + \Delta q_{14} - q_{14}]^T \\ \delta \omega_1 &= \omega_{1-1} + \Delta \omega_1 - \omega_1. \end{aligned} \quad (80)$$

Thus, control law (63) takes on the form

$$\tau_{ci} = K_{q1} \{-(q_{(1-1)4} + \Delta q_{14}) \hat{q}_1 + q_{1-1} (\hat{q}_1 + \Delta \hat{q}_1)\} + \tau_{c(1-1)} + I_1 d(\Delta \omega_1)/dt_0$$

$$\begin{aligned}
& + (\omega_{i-1} + \Delta\omega_1) \times I_1 \Delta\omega_1 / 2 - \omega_{i-1} \times I_1 (\omega_{i-1} - \omega_1) / 2 + \Delta\omega_1 \times I_1 \omega_1 / 2 \\
& + \Delta\omega_1 \times I_1 \omega_{i-1} + K_{\omega_1} I_1 (\omega_{i-1} + \Delta\omega_1 - \omega_1). \tag{81}
\end{aligned}$$

5. IMPLEMENTATION OF CONTROL LAWS

We observe that the implementation of control law (33) (resp. (48)) for formation keeping requires a knowledge of $[E_1]_1, [\dot{E}_1]_1, [\omega_1]_1, [\tau_{c1}]_1$ and $[u_{c_j}]_1$ for $j \in \mathcal{J}_1(t)$ (resp. $[u_{c(i-1)}]_1$) at any time t . The quantities $[E_1]_1$ and $[\dot{E}_1]_1$ can be determined from $[\rho_{j1}]_1$ and $[\dot{\rho}_{j1}]_1$ which require measurement of the position and velocity of the j -th neighboring MS relative to the i -th MS. These quantities can also be obtained by transmitting the position and velocity of the j -th neighboring MS to the i -th MS. Also, the control of the j -th MS at any time must also be transmitted to the i -th MS. When one or more MS failure occurs, one may adopt the following backup schemes for control law implementation depending on the nature of failure:

(i) Inter-spacecraft Communicate on System Failure: One may obtain estimates of $[\rho_{j1}]_1$ and $[\dot{\rho}_{j1}]_1$ by using on-board optical range sensors, or by setting the relative position and velocity between the failed and active MS at their nominal values temporarily until the failure is recovered.

(ii) Overall Spacecraft Failure: Here the failure is sufficiently severe such that the MS is no longer useful. In this case, the MS should be removed from the formation by deorbiting or by manual retrieval. If the failed MS is not replaced, then it is necessary to reconfigure the formation. The control laws for steering the remaining active MS from the old to the new formation requires separate consideration. This aspect will be discussed elsewhere.

We note also that in the derivation of foregoing control laws, no constraints have been imposed on the magnitude of the control variables. In the presence of bounded controls, one expect that the rate of decay of $\|([E_1]_1, [\dot{E}_1]_1)(t)\|$ and $\|(\delta\omega_1, \delta q_1)(t)\|$ to zero would be reduced when one or more of the control variables takes on its extreme values.

Finally, in physical situations, it is necessary to consider discrete-time versions of the proposed control laws. In view of the limited fuel on-board, it is generally undesirable to have continuously acting controls. Therefore the system response corresponding to the control laws derived here serves as a basis for comparison between the idealized and the actual responses.

6. FLEET COORDINATION

For a fleet of MS, one may require complete autonomy in each MS in the sense that all the decisions for determining its future behavior are made on-board without the assistance of external agents. Although this approach provides enhanced operational reliability, it may not be cost effective since each MS must contain all the essential hardware and software for coordination and control. An alternative approach is to require each MS to have only the basic hardware and software for attitude control and orbital maneuvering. The more complex tasks in fleet coordination and control are shared by all the MS in the fleet. Moreover, some of the MS may be equipped with special hardware and software to perform particular tasks for the entire fleet. In what follows, we shall describe a particular scheme for fleet coordination with the aid of a simple example.

Consider a fleet of ten MS moving in a formation pattern shown in Fig. 2c. Here, only the fleet leader, the eighth and tenth MS are equipped with inertial guidance hardware and software for determining their position with respect to a specified inertial frame. The remaining MS are equipped only with sensors for determining the displacements and attitudes relative to the fleet leader or their neighbors.

The fleet coordination is achieved with the aid of an inter-spacecraft, communication network (eg. radio or optical links). This network has the following basic functions:

(i) Communicating the necessary data for fleet formation-keeping and relative attitude control;

(ii) Linking the computers in the MS to form a distributed computing network thereby increasing the computational capability of the MS fleet for more computationally intensive tasks such as on-board image processing.

In the realization of the first function, each fleet leader broadcasts its position and attitude with respect to a specified inertial frame, and the remaining MS broadcast their positions and velocities relative to their leaders to achieve formation alignment. Moreover, when certain MS malfunction, the fleet leaders may transmit the instructions for reconfiguring the formation to the remaining MS. In the case where a fleet of MS is used for planetary exploration, certain tasks such as concerted mapping using multiple cameras in the MS require the communication of attitude alignment information between the MS. The second function permits the performance of complex tasks which cannot be performed by a single MS.

7, SIMULATION STUDY

The main objective of this simulation study is to determine the performance of the proposed control laws for formation keeping and attitude regulation in the presence of actuator saturation, variations in spacecraft parameters, and loss of communication between MS.

First, we consider a simple case involving a fleet of four microspacecraft whose desired motions are along an inclined circular orbit O_1 about the Earth as sketched in Fig.3. For convenience, we introduce a geocentric fixed cartesian coordinate frame \mathcal{F}_o with origin O at the Earth's center along with a spherical coordinate system (r, θ, ϕ) with orthonormal basis $\{e_r, e_\theta, e_\phi\}$. Let the first MS be the fleet leader moving in orbit O_1 with inclination angle $(\pi/2 - \varphi_{inc})$ and ascending node along the Y-axis. Its motion in spherical coordinates is given by

$$\begin{aligned} r_1(t) &= r_o, \quad \theta_1(t) = \cos^{-1}\{\cos(\varphi_{inc})\cos(\theta_o - \omega_o t)\}, \\ \phi_1(t) &= \tan^{-1}\{-\tan(\theta_o - \omega_o t)/\sin(\varphi_{inc})\}, \end{aligned} \quad (82)$$

where r_o is a given orbital radius, and $\omega_o = \sqrt{\mu/r_o^3}$. The i -th follower MS tries to track the $(i-1)$ -th MS along the same orbit with a fixed angle $\Delta\theta$ in the orbital plane, i.e.

$$\begin{aligned} r_i^d(t) &= r_o, \quad \theta_i^d(t) = \cos^{-1}\{\cos(\varphi_{inc})\cos(\theta_{i-1}(t) - \Delta\theta)\}, \\ \phi_i^d(t) &= \tan^{-1}\{-\tan(\theta_{i-1}(t) - \Delta\theta)/\sin(\varphi_{inc})\}, \quad i = 2, 3, 4. \end{aligned} \quad (83)$$

Here, for simplicity, we have set the desired orbital radius for all MS to r_o . Since the desired orbit O_1 is circular, we adopt control law (48) for formation keeping, where the deviation vectors $h_i(t)$ are given by

$$\begin{aligned} h_2(t) &= (-2r_o \sin(\varphi_{inc}) \sin(\Delta\theta/2) \sin(\theta_o - \omega_o t - \Delta\theta/2) e_x \\ &\quad - 2r_o \sin(\Delta\theta/2) \cos(\theta_o - \omega_o t - \Delta\theta/2) e_y \\ &\quad + 2r_o \cos(\varphi_{inc}) \sin(\Delta\theta/2) \sin(\theta_o - \omega_o t - \Delta\theta/2) e_z \end{aligned} \quad (84)$$

$$\begin{aligned} h_i(t) &= (-2r_o \sin(\varphi_{inc}) \sin(\Delta\theta/2) \sin(\theta_o - \omega_o t - i\Delta\theta/2) e_x \\ &\quad - 2r_o \sin(\Delta\theta/2) \cos(\theta_o - \omega_o t - i\Delta\theta/2) e_y \\ &\quad + 2r_o \cos(\varphi_{inc}) \sin(\Delta\theta/2) \sin(\theta_o - \omega_o t - i\Delta\theta/2) e_z, \quad i = 3, 4. \end{aligned} \quad (85)$$

Evidently, $h_i(t)$ satisfies

$$d^2 h_i(t)/dt^2 + \omega_o^2 h_i(t) = 0 \quad (86)$$

for all t and $i = 2, 3, 4, 5$.

To specify the desired attitude of the i-th MS, we introduce the (1-2-3) Euler angles $(\Theta_1, \Psi_1, \Phi_1)$ corresponding to a rotation of Θ_1 about the X-axis followed by a rotation of Ψ_1 about the rotated Y-axis, and a rotation of Φ_1 about the rotated Z-axis. Evidently, the desired Euler angles for the i-th MS are:

$$\begin{aligned} \Theta_2^d(t) &= \theta_o - \omega_o t - \Delta\theta; & \Psi_2^d(t) &= \varphi_{inc}; & \Phi_2^d(t) &= 0. \\ \Theta_1^d(t) &= \Theta_{1-1}(t) - \Delta\theta; & \Psi_1^d(t) &= \varphi_{inc}; & \Phi_1^d(t) &= 0, \quad i = 3,4. \end{aligned} \quad (87)$$

The corresponding direction cosine matrix has the form:

$$C(\Theta_1^d(t), \Psi_1^d(t), \Phi_1^d(t)) = \begin{bmatrix} \cos(\varphi_{inc}) \sin(\varphi_{inc}) \sin(\Theta_1^d(t)) - \sin(\varphi_{inc}) \cos(\Theta_1^d(t)) & & \\ 0 & \cos(\Theta_1^d(t)) & \sin(\Theta_1^d(t)) \\ \sin(\varphi_{inc}) - \cos(\varphi_{inc}) \sin(\Theta_1^d(t)) & \cos(\varphi_{inc}) \cos(\Theta_1^d(t)) & \end{bmatrix}. \quad (88)$$

Thus, the desired attitude of the i-th MS can be expressed in terms of the following quaternions:

$$\begin{aligned} q_{11}^d(t) &= \sin(\Theta_1^d(t)/2) \cos(\varphi_{inc}/2), & q_{12}^d(t) &= \cos(\Theta_1^d(t)/2) \sin(\varphi_{inc}/2), \\ q_{13}^d(t) &= \sin(\Theta_1^d(t)/2) \sin(\varphi_{inc}/2), & q_{14}^d(t) &= \cos(\Theta_1^d(t)/2) \cos(\varphi_{inc}/2), \end{aligned} \quad i = 2,3,4 \quad (89)$$

Assuming that we require every MS to spin about its z-axis with constant angular speed ω_s , the desired angular velocity for the i-th MS is given by

$$\omega_1^d = \omega_o \cos(\varphi_{inc}) e_x + \omega_o \sin(\varphi_{inc}) e_z + \omega_s e_{1z} = \omega_o e_{1x} + \omega_s e_{1z}, \quad (90)$$

where $\{e_{1x}^d, e_{1y}^d, e_{1z}^d\}$ corresponds to the basis of the body coordinate system \mathcal{F}_1^d associated with the i-th MS with the desired attitude.

Figure 4 shows a typical time-domain response of the MS fleet with formation-keeping control law (48) and attitude control law (69). The MS parameter values used in the simulation study are given in Table 1. Figure 5 shows the corresponding time-domain response when actuator saturation is introduced. It can be seen that the error decay is prolonged in the presence of actuator saturation as expected. The corresponding time-domain response of the MS fleet with the $[u_{c(1-1)}]_1$ term in (48) set to zero (to simulate the loss of communication between the MS) was also determined. The results do not differ significantly from those shown in Fig.4. Next, the effect of inertia perturbations on the time-domain response of the MS fleet was studied. It was found that the qualitative behavior of the response is

essentially identical to that of the unperturbed case. Figure 6 shows typical results for the case where the actual values of I_{x1} , I_{y1} , and I_{z1} are 2/3 of those used in attitude control law computation. The observed robustness of quaternion-feedback attitude control laws with respect to inertia perturbations was discussed earlier in [14], [171]-[191]. Finally, the time-domain response of the MS fleet was computed for the case with attitude control law (69) but with proportional-plus-rate formation-keeping control law given by

$$\mathbf{u}_{c1} = K_{11} \mathbf{E}_1 + K_{21} \dot{\mathbf{E}}_1 + \mathcal{D}_1^2(\mathbf{h}_1) + \omega_{10}^2 \mathbf{h}_1 + \mathbf{u}_{c(1-1)}, \quad (91)$$

where the values of K_{11} and K_{21} are identical to those used in Fig. 4. It was found that the response does not differ significantly from those shown in Fig. 4, although there are noticeable perturbations in the control forces (see Fig. 7).

Next, we consider a more complex formation consisting of five MS moving in formation. As in the previous case, the first MS is the fleet leader whose motion is specified by orbit O_1 given by (82.). The second and third MS try to move along the same orbit O_1 as in the previous case. The desired motions for the fourth and fifth MS correspond to two circular orbits with the same inclination angle $(\pi/2 - \varphi_{1nc})$, but with ascending nodes at $(r, \theta, \phi) = (r_0, \pi/2, \Delta\phi)$ and $(r_0, \pi/2, -\Delta\phi)$ respectively, where $\Delta\phi$ is a given positive angle $< \pi/2$. The desired motions of the centers of mass of the second and third MS in the spherical coordinates are given by (83). But the desired motions for the fourth and fifth MS in spherical coordinates are given by

$$\begin{aligned} r_4^d(t) &= r_5^d(t) = r_0, & \theta_4^d(t) &= \theta_5^d(t) = \cos^{-1}(\cos(\varphi_{1nc})\cos(\theta_2(t))), \\ \phi_4^d(t) &= \tan^{-1} \left\{ \frac{\sin(\theta_2(t))\cos(\Delta\phi) + \cos(\theta_2(t))\sin(\Delta\phi)\sin(\varphi_{1nc})}{\sin(\theta_2(t))\sin(\Delta\phi) - \cos(\theta_2(t))\cos(\Delta\phi)\sin(\varphi_{1nc})} \right\}, \\ \phi_5^d(t) &= \tan^{-1} \left\{ \frac{-\sin(\theta_2(t))\cos(\Delta\phi) + \cos(\theta_2(t))\sin(\Delta\phi)\sin(\varphi_{1nc})}{\sin(\theta_2(t))\sin(\Delta\phi) + \cos(\theta_2(t))\cos(\Delta\phi)\sin(\varphi_{1nc})} \right\}. \end{aligned} \quad (92)$$

The desired (1-2-3) Euler angles for the MS are given by

$$\begin{aligned} \Theta_4^d(t) &= \Theta_5^d(t) = 0, & \Psi_4^d(t) &= \Psi_5^d(t) = \varphi_{1nc}, \\ \Phi_4^d(t) &= \Delta\phi, & \Phi_5^d(t) &= -\Delta\phi. \end{aligned} \quad (93)$$

Figure 7 shows an exaggerated sketch of the desired orbits of the MS fleet under the assumption that the tracking errors of all follower MS are zero. Note that in (93) only the desired azimuth angles depend on the those of the

neighboring MS, while the desired elevations angles are absolute. The desired attitude of the fourth MS can be expressed in terms of the following quaternions:

$$q_{44}^d(t) = \{1 + \cos(\Delta\phi)\cos(\varphi_{1nc}) - \sin(\Delta\phi)\sin(\varphi_{1nc})\sin(\theta_2(t)) + \cos(\theta_2(t))(\cos(\Delta\phi) + \cos(\varphi_{1nc}))\}^{1/2}/2; \quad (94a)$$

$$q_{41}^d(t) = \{\sin(\Delta\phi)\sin(\varphi_{1nc})\cos(\theta_2(t)) + \sin(\theta_2(t))(\cos(\Delta\phi) + \cos(\varphi_{1nc}))\}/(4q_{44}^d(t)); \quad (94b)$$

$$q_{42}^d(t) = \{\sin(\varphi_{1nc})(1 + \cos(\Delta\phi)\cos(\theta_2(t))) - \sin(\Delta\phi)\sin(\theta_2(t))\}/(4q_{44}^d(t)); \quad (94c)$$

$$q_{43}^d(t) = \{\cos(\Delta\phi)\sin(\varphi_{1nc})\sin(\theta_2(t)) + \sin(\Delta\phi)(\cos(\theta_2(t)) + \cos(\varphi_{1nc}))\}/(4q_{44}^d(t)). \quad (94d)$$

The quaternions corresponding to the desired attitude of the fifth MS have the same form as (92) except with $\Delta\phi$ replaced by $-\Delta\phi$.

As in the previous case, we require each MS spins about its z-axis with constant angular speed ω_g . Thus, $\omega_i^d(t)$, $i = 2, 3, 4, 5$, are also given by (90). The deviation vectors $h_i(t)$ for the second and third MS are given by (84) and (85) as in the previous case. Here, both the fourth and fifth MS try to align with the position of the second MS. I'bus, the deviation vector $h_4(t)$ is given by

$$h_4(t) = r_o \{((1 - \cos(\Delta\phi))\sin(\varphi_{1nc})\cos(\theta_o - \omega_o t - 2\Delta\theta) + \sin(\Delta\phi)\sin(\theta_o - \omega_o t - 2\Delta\theta))e_x + (\sin(\Delta\phi)\sin(\varphi_{1nc})\cos(\theta_o - \omega_o t - 2\Delta\theta) + (\cos(\Delta\theta) - 1)\sin(\theta_o - \omega_o t - 2\Delta\theta))e_y\}. \quad (95)$$

The deviation vector $h_5(t)$ has the same form as (94) except with $\Delta\phi$ replaced by $-\Delta\phi$. Evidently, $h_i(t)$ satisfies (86) for $i = 2, 3, 4, 5$.

Figure 8 shows a typical time--domain response of the MS fleet with formation keeping control law [48] and attitude control law (69) in the presence of actuator saturation. The qualitative behavior of the response is similar to that in the simple case involving four MS.

8. CONCLUDING REMARKS

In this paper, control laws for a fleet of MS moving in formation have been derived based on nearest-neighbor tracking using a simplified model for a rigid MS. These control laws require the knowledge of the relative displacements and attitudes of the MS and its neighbors. Simulation results

based on a generic MS model showed that the derived control laws are effective in formation and relative attitude alignment provided that the magnitude of the initial deviation from the desired state is sufficiently small so that collisions between the MS do not occur. In the case where the MS move in formation along LEO, simulation results showed that a simple proportional-plus-rate formation-keeping control law with properly chosen values for the feedback gains provides good time-domain behavior.

Finally, in this work, important factors such as data processing time-delay and time discretization arising in physical implementation have not been taken into consideration. Nevertheless the results reveal the basic structure of the control laws and the required inter-spacecraft data required for their implementation. Finally, the problems associated with the physical implementation of the control laws in terms of the state-of-the-art hardware and fuel consumption for control are not considered here, and they require further study.

Acknowledgement

This work was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. The work of P.K.C. Wang was also supported by the California Space Institute and the National Science Foundation, ARPA/CSTO under Grant CDA-93-03148.

References

- [1] Stachnik, R., Ashlin, K.H., *Bulletin of American Astronomical Soc.* Vol.16, No.3, 1984, pp.818-827.
- [2] Stachnik, R., Melroy, R., and Arnold, D., "Multiple Spacecraft Michelson Stellar Interferometry," *Proc. SPIE, instrumentation in Astronomy V.* Vol.445, pp. 358-369, 1984.
- [3] Johnson, M.D. and Nock, K.T., "Free Flyer Optical Interferometry Trajectory Analysis, JPL 4/30-5/2, 1990,
- [4] DeCue A.B., "Multiple Spacecraft Optical Interferometry, Preliminary Feasibility Assessment," JPL Technical Internal Report D-8811, Aug. 1991.
- [5] McKenna, J.T. and Mecham, M., "Motorola Wins Iridium Pledges", *Aviation Week & Space Technology*, Aug. 1993, pp. 56--57.
- [6] Wang, P.K. C., "Interaction Dynamics of Multiple Mobile Robots with Simple Navigation Strategies," *J. Robotic Systems*, Vol 6, pp.77-101, 1989.
- [7] Wang, P.K.C., "Dynamics of Interacting Multiple Autonomous Mobile Robots in Bounded Spatial Domains)" *Int.J.Control*, Vol.50, pp.2109-2124, 1989,
- [8] Wang, P.K.C., "Navigation Strategies for Multiple Autonomous Mobile Robots Moving in Formation," *J. Robotic Systems*, Vol.8, pp.177-195, 1991.
- [9] Wang, P.K.C., "Nonlinear Interaction Dynamics of Multiple Mobile Automata," *Proc. of First World Congress of Nonlinear Analysts*,

Tempa, FL. 1992.

- [10] Wertz, J.R. (Ed.) *Spacecraft Attitude Determination and Control, I*).
Reidel Pub. Co, Boston, 1980.
- [11] Hahn, W. *Stability of Motion*, Springer-Verlag, New York, 1967.
- [12] POPOV, V.M., *Hyperstability of Automatic Control Systems*, Springer
Verlag, New York, 1973.
- [13] Ickes, B.P., "A New Method for Performing Digital Control System
Attitude Computation Using Quaternions," *AIAA J.* Vol.8, No.1, 1970,
pp. 13-17.
- [14] Wen, J.T. and Kreutz-Delgado, K, "The Attitude Control Problem,"
IEEE Trans. Auto. Control, Vol.36, No.10, Oct. 1991, pp.1148-1162.
- [15] Fjellstad, O. and Fossen, T., "Comments on The Attitude Control
Problem," *IEEE Trans. Auto. Control*, Vol.39, No.3, Mar. 1994, pp.699-700.
- [16] Mortensen, R., "On Systems of Automatic Control of the Rotation of a
Rigid Body," Electronics Res. Lab., Univ. of Calif. Berkeley, Rpt.
63-23, Nov. 1963.
- [17] Wang, P.K.C.; "Automatic Assembly of Space Stations, " *Proc.NASA/ACC
Workshop on Identification and Control of Flexible Space Structures*,
San Diego, CA. pp. 67-101, June, 1984.
- [18] Wie, B., Weiss, H., and Arapostathis, A., "Quaternion Feedback
Regulator for Spacecraft Eigenaxis Rotations, " *J. Guidance Contr.*
Vol.12, No.3, pp.375-380, 1989.
- [19] Wang, P.K.C., "A Robust. Nonlinear Attitude Control Law for- Space
Stations with Flexible Structural Components, " *Proc. 5-th VPI &SU/AIAA
Symp. on Dynamics and Control of Large Structures*, Blacksburg, VA. L.
Meirovitch, Ed, June 12-14, 1985, pp. 381-398.

M_1 (mass of MS) -- 10 kg.
 I_{ix} (moment of inertia about x-axis) -- 0.3646 kg m².
 I_{iy} (moment of inertia about y-axis) -- 0,2734 kg m².
 I_{iz} (moment of inertia about z-axis) -- 0.3125 kg m².
 r_o (desired orbital radius of MS) -- $\sqrt[3]{.13814 \times 10^6}$ m.
 $\omega_o = \sqrt{\mu/r_o^3}$ (orbital angular speed of fleet leader) -- 0.001 rad/see,
 ω_s (desired spin speed about. z-axis) --- 0.01 rad./sec
 $\varphi_{inc} = \pi/2$ - inclination angle of reference orbits -- $8.2\pi/180$ rad.
 $\Delta\phi$ (azimuthal angle associated with the ascending node of reference orbits) -- 0.2 rad.
 $\Delta\theta$ (MS separation angle) -- $\pi/120$ rad.

Table 1 Values of microspacecraft and orbital parameters for simulation study.

Figure Captions

Fig. 1 Sketch of inertial and moving coordinate systems.

Fig. 2 Examples of formation patterns.

Fig. 3 Four microspacecraft moving along an inclined orbit about the Earth.

Fig. 4 Time-domain response of the MS fleet with formation keeping control law (48) and attitude control law (69) with $K_{1i} = 0.5$, $K_{2i} = 2.0$, $K_{\omega i} = 1.5$, $K_{q i} = 0.4$, $i = 2,3,4$; MS 1 (solid curves); MS 2 (dash-dot curves); MS 3 (dashed curves). Initial states for MS:

$$[E_1(0), \dot{E}_1(0)]_1 = [1 \ 0.5 \ -0.5, \ 0001]; \quad [\omega_1(0)]_1 = [0.01 \ 0.01 \ 0.011];$$

$$q_1(0) = [0.2 \ 0.2 \ 0.2 \ 0.9380831];$$

$$[E_2(0), \dot{E}_2(0)]_2 = [-1 \ 1 \ -1, \ 0001]; \quad [\omega_2(0)]_2 = [-0.01 \ 0.01 \ -0.011];$$

$$q_2(0) = [-0.2 \ 0.15 \ -0.1 \ 0.963068];$$

$$[E_3(0), \dot{E}_3(0)]_3 = [0.5 \ -1 \ 0.5, \ 0 \ 0 \ 01]; \quad [\omega_3(0)]_3 = [0.01 \ 0 \ -0.011];$$

$$q_3(0) = [-0.1 \ 0.1 \ 0.1 \ 0.984885-];$$

Fig. 4a: Positional tracking errors (m) of MS vs. time.

Fig. 4b: Angular velocities (m/sec) of MS vs. time.

Fig. 4c: Quaternions of MS vs. time.

Fig. 4d: Control forces (N) of MS vs. time.

Fig. 4e: Control torques (N.m) of MS vs. time.

Fig. 5 Time-domain response of the MS fleet with formation keeping control law (48) and attitude control law (69) in the presence of actuator saturation, and with gains and initial states given in Fig. 4; MS 1 (solid curves); MS 2 (dash-dot curves); MS 3 (dashed curves).

Saturation levels: $|f_{c1j}| \leq 1 \text{ N}$; $|\tau_{c1j}| \leq 0.05 \text{ N.m}$, $i = 2,3,4$; $J = x, y, z$.

Fig. 6 Time-domain response of the MS fleet with formation keeping control law (48) and attitude control law (69) in the presence of inertia perturbations, and with gains and initial states given in Fig. 4. Actual values for I_{x1} , I_{y1} , and I_{z1} are 2/3 of those used in control law computation; MS 1 (solid curves); MS 2 (dash-dot curves); MS 3 (dashed curves).

Fig. 7 Plot of control forces for the case with proportional-plus-rate formation-keeping control law (91) and attitude control law (69) in the presence of actuator saturation, and with gains, initial states, and saturation levels given in Figs. 4 and 5. MS 1 (solid curves); MS 2 (dash-dot curves); MS 3 (dashed curves).

Fig. 8 Exaggerated sketch of the reference orbits of five microspacecraft moving in formation about the Earth.

Fig. 9 Time-domain response of the MS fleet with formation keeping control law (48) and attitude control law (69) with $K_{1i} = 0.5$, $K_{2i} = 2.0$, $K_{\omega i} = 1.5$, $K_{q i} = 0.4$, $i = 2,3,4$; MS 1 (solid curves); MS 2 (dash-dot curves); MS 3 (dashed curves); MS 4 (long-dashed curves).

Fig. 9 (Continued)

$$\begin{aligned} [E_1(0), \dot{E}_1(0)]_1 &= [52-5, 0001; \quad \{\omega_1(0)\}_1 = [0.02 \quad 0.020, 02); \\ q_1(0) &= [0.3 \quad 0.1 \quad 0.2 \quad 0,9273618]; \\ [E_2(0), \dot{E}_2(0)]_2 &= [-5 \quad -33, 0001; \quad \{\omega_2(0)\}_2 = [-0.01 \quad 0.015 \quad -0.01]; \\ q_2(0) &= [-0.2 \quad 0.2 \quad 0.3 \quad 0.91104331]; \\ [E_3(0), \dot{E}_3(0)]_3 &= [0.5 \quad -1 \quad 0.5, 0 \quad 0 \quad 01; \quad \{q_3(0)\}_3 = [0.01 \quad 0 \quad -0.015]; \\ q_3(0) &= [0.1 \quad 0.2 \quad 0.2 \quad 0(953939?1); \\ [E_4(0), \dot{E}_4(0)]_4 &= [1.55-3, 0001; \quad \{\omega_4(0)\}_4 = [0.015 \quad -0.01 \quad -0.01]; \\ q_4(0) &= [-0.1 \quad -0.2 \quad 0.15 \quad 0.9630681]; \end{aligned}$$

- Fig. 9a: Positional tracking errors (m) of MS vs. time.
- Fig. 9b: Angular velocities (m/see) of MS vs. time.
- Fig. 9c: Quaternions of MS vs. time.
- Fig. 9d: Control forces (N) of MS vs. time.
- Fig. 9e: Control torques (N.m) of MS vs. time.

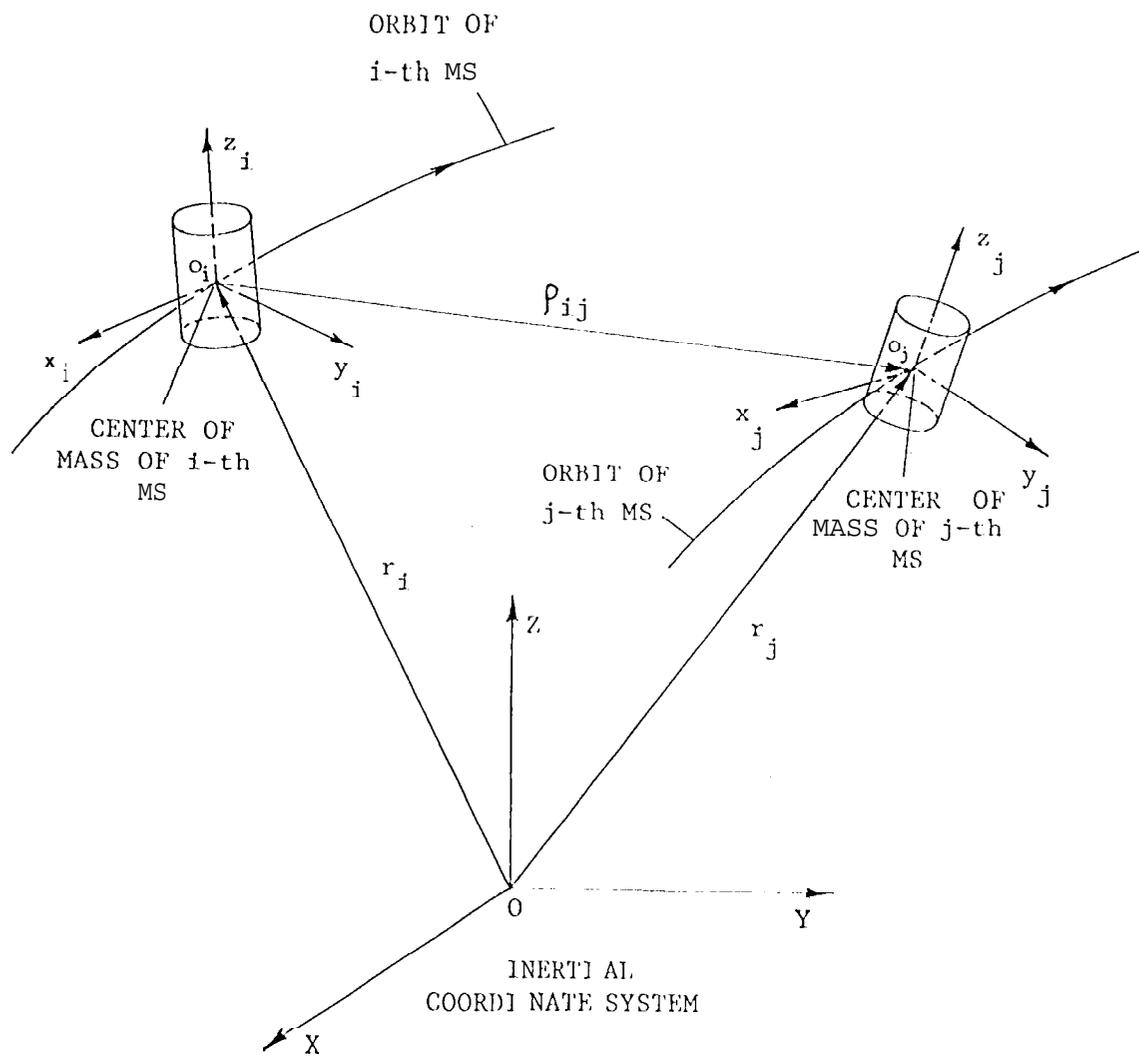
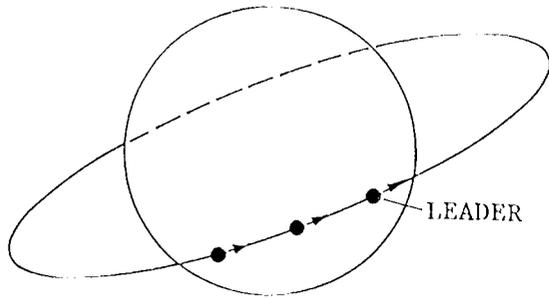
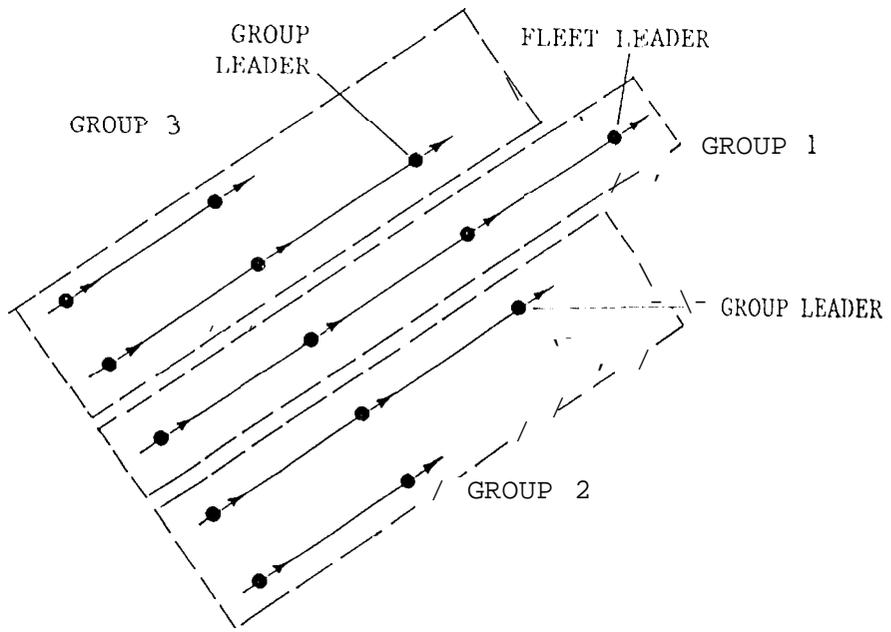


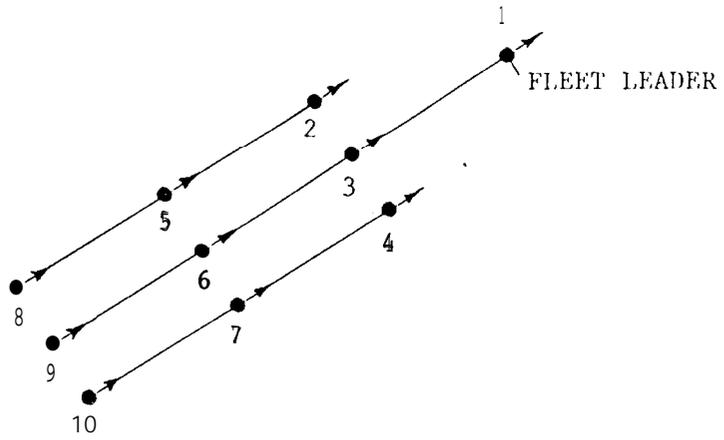
Fig.1



(a)



(b)



(c)

Fig. 2

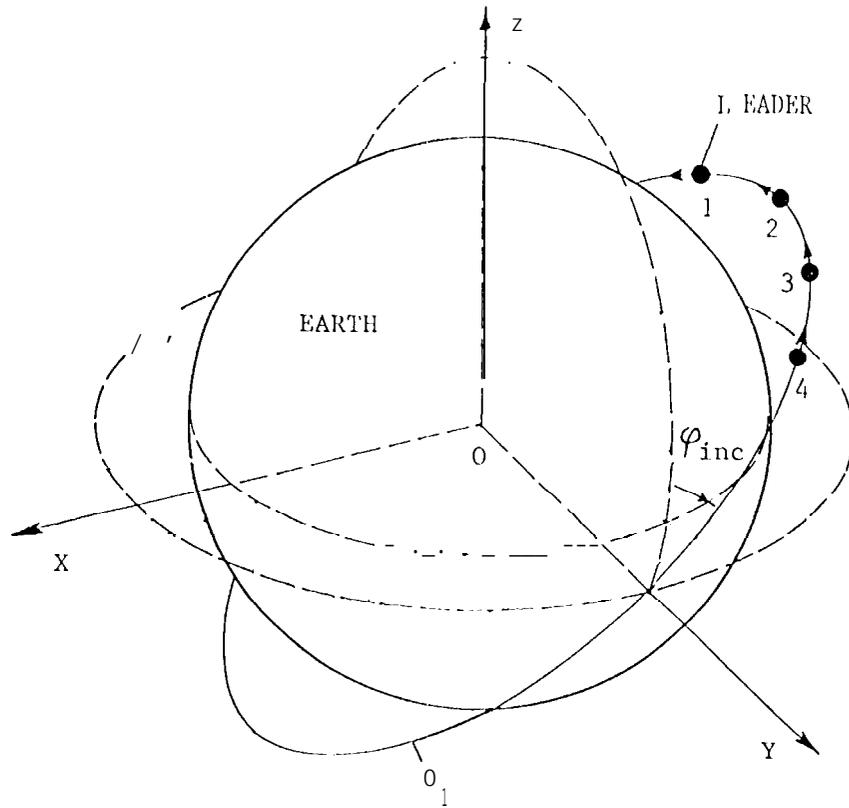


Fig.3

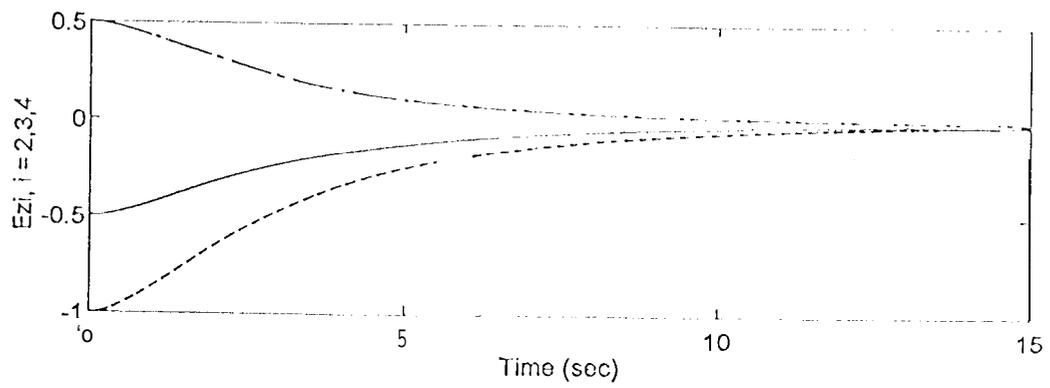
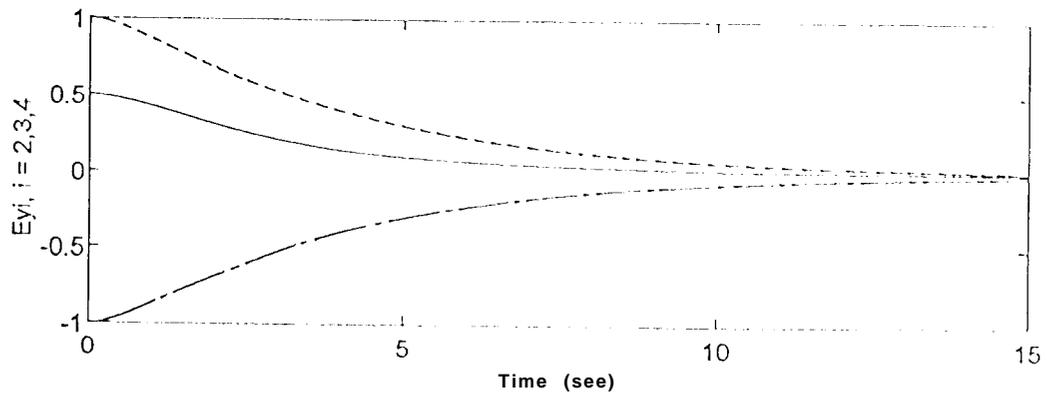
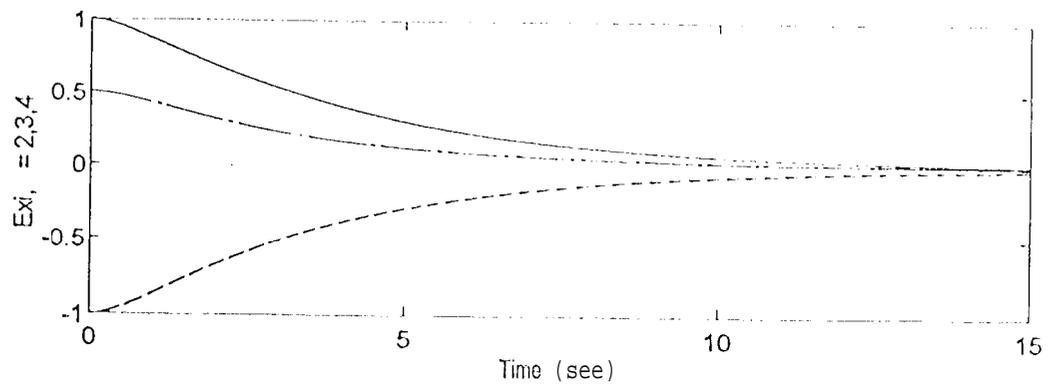


Fig.4a

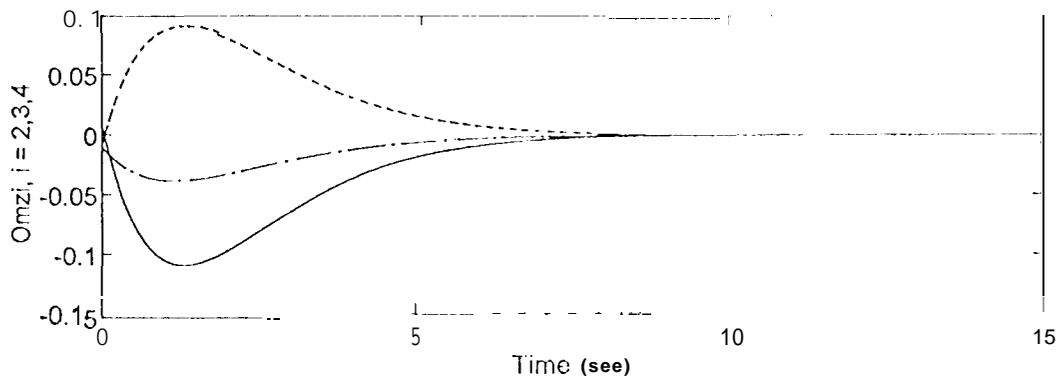
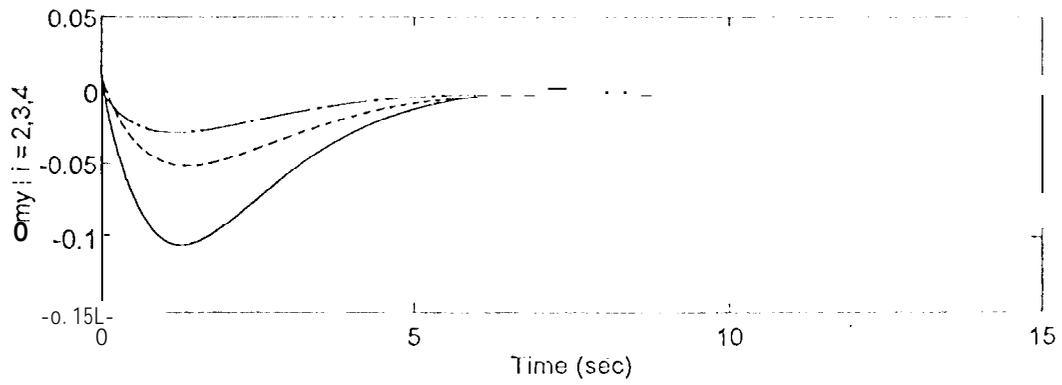
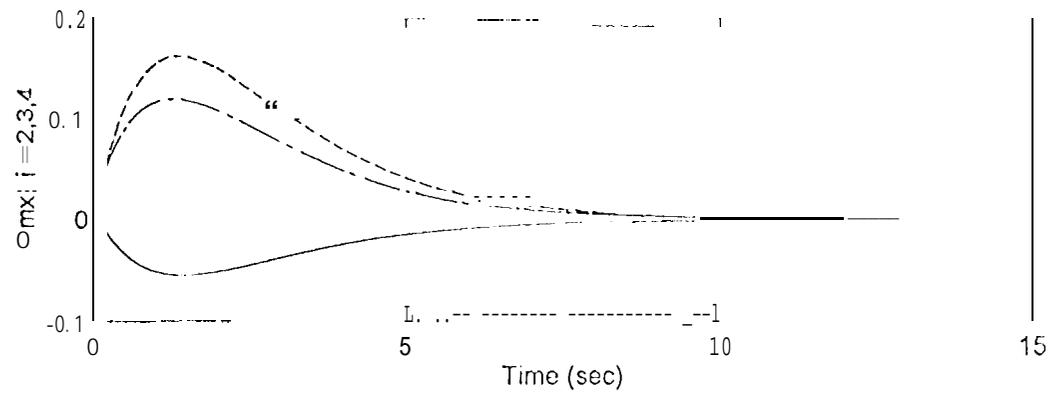


Fig. 4b

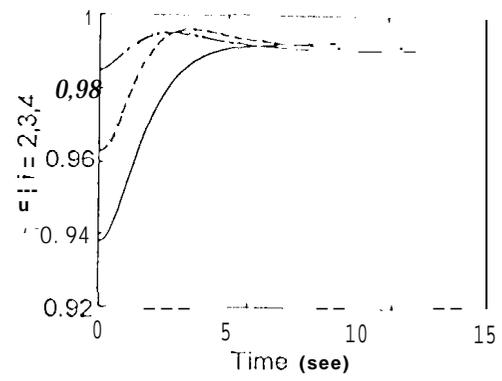
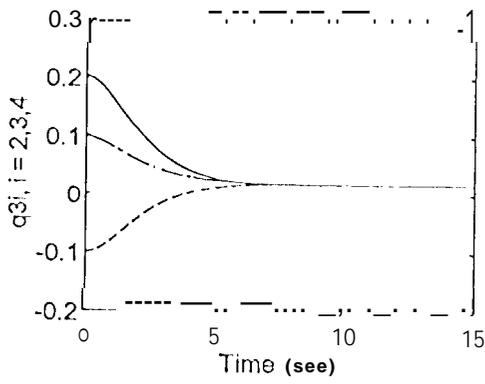
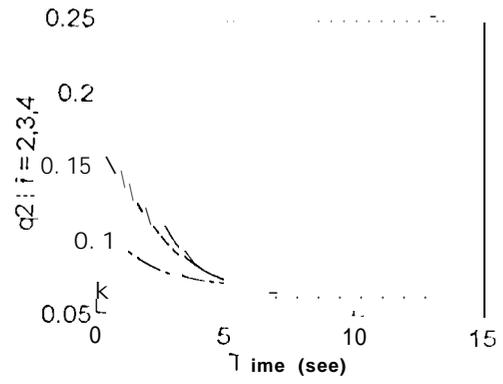
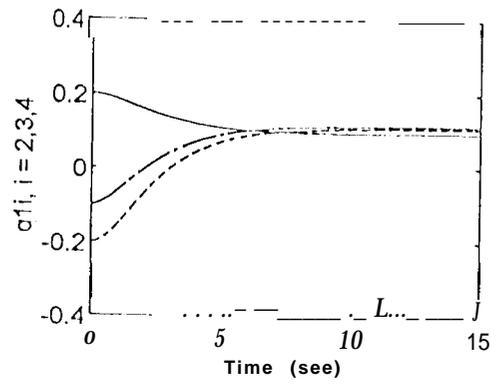


Fig.4c

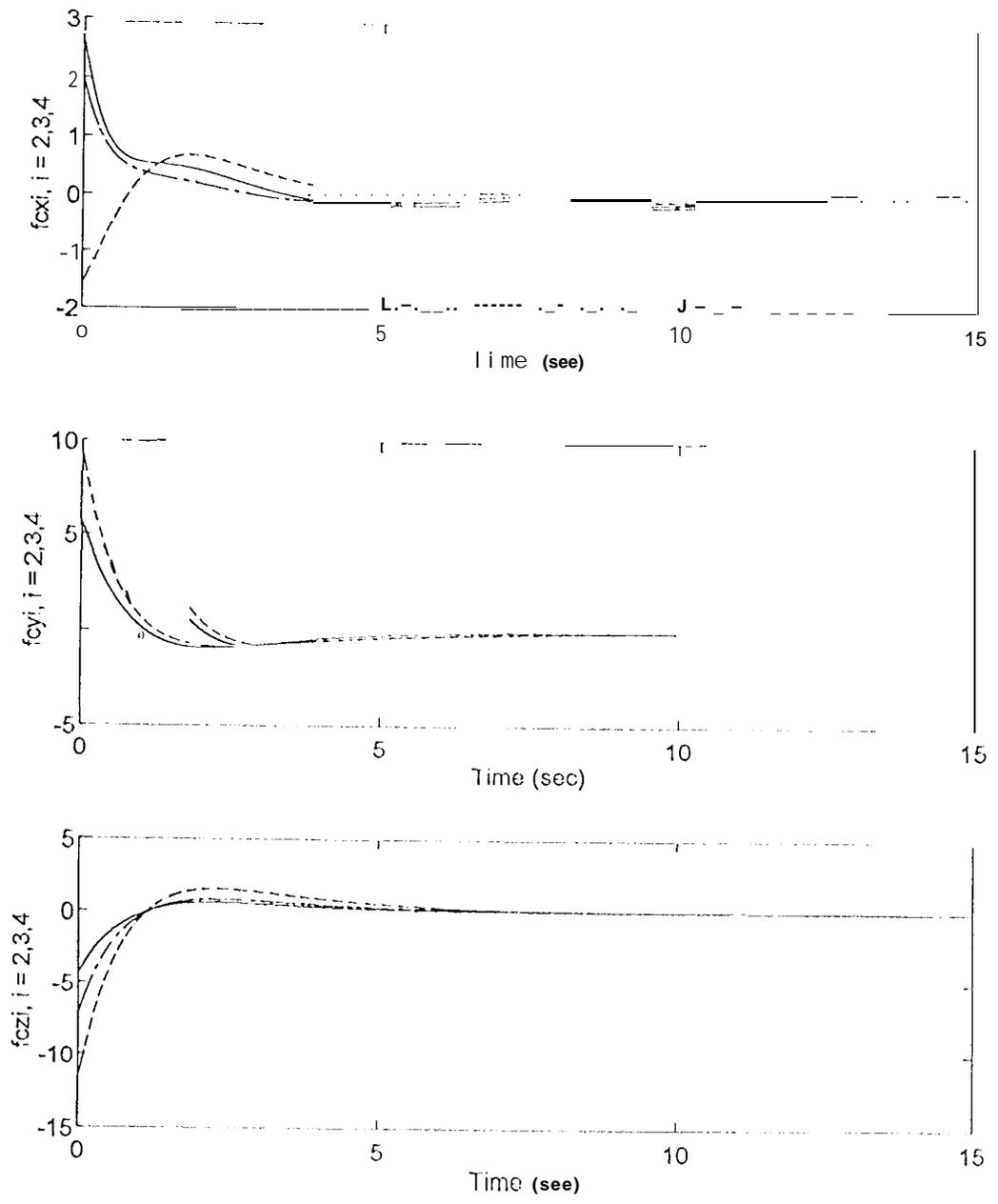


Fig.4d

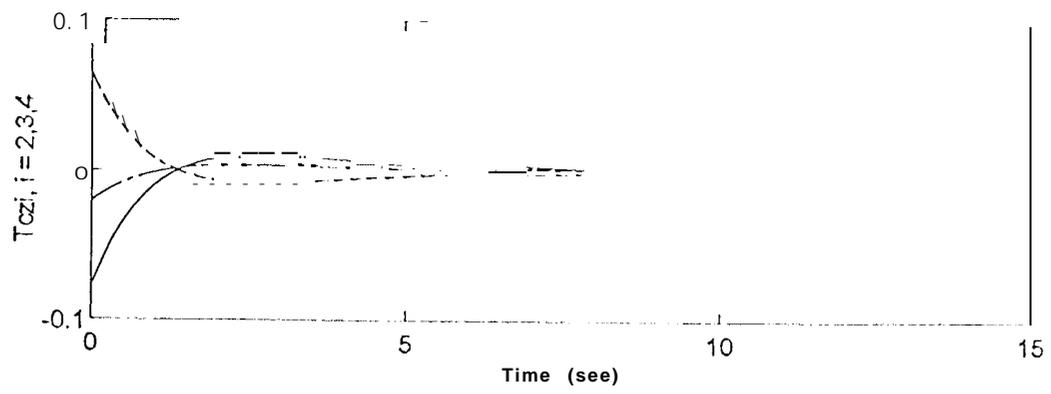
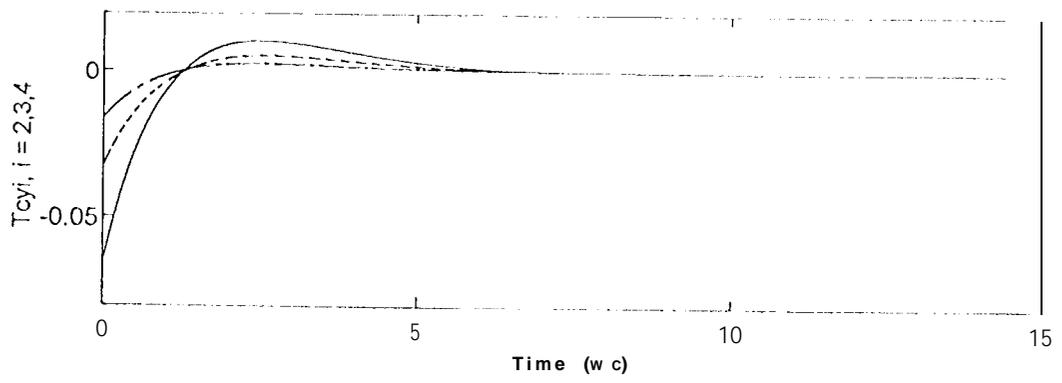
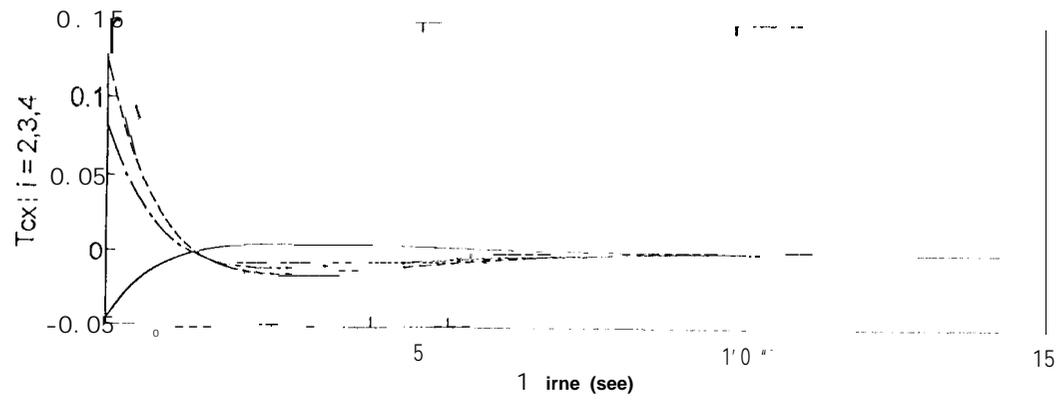


Fig.4c

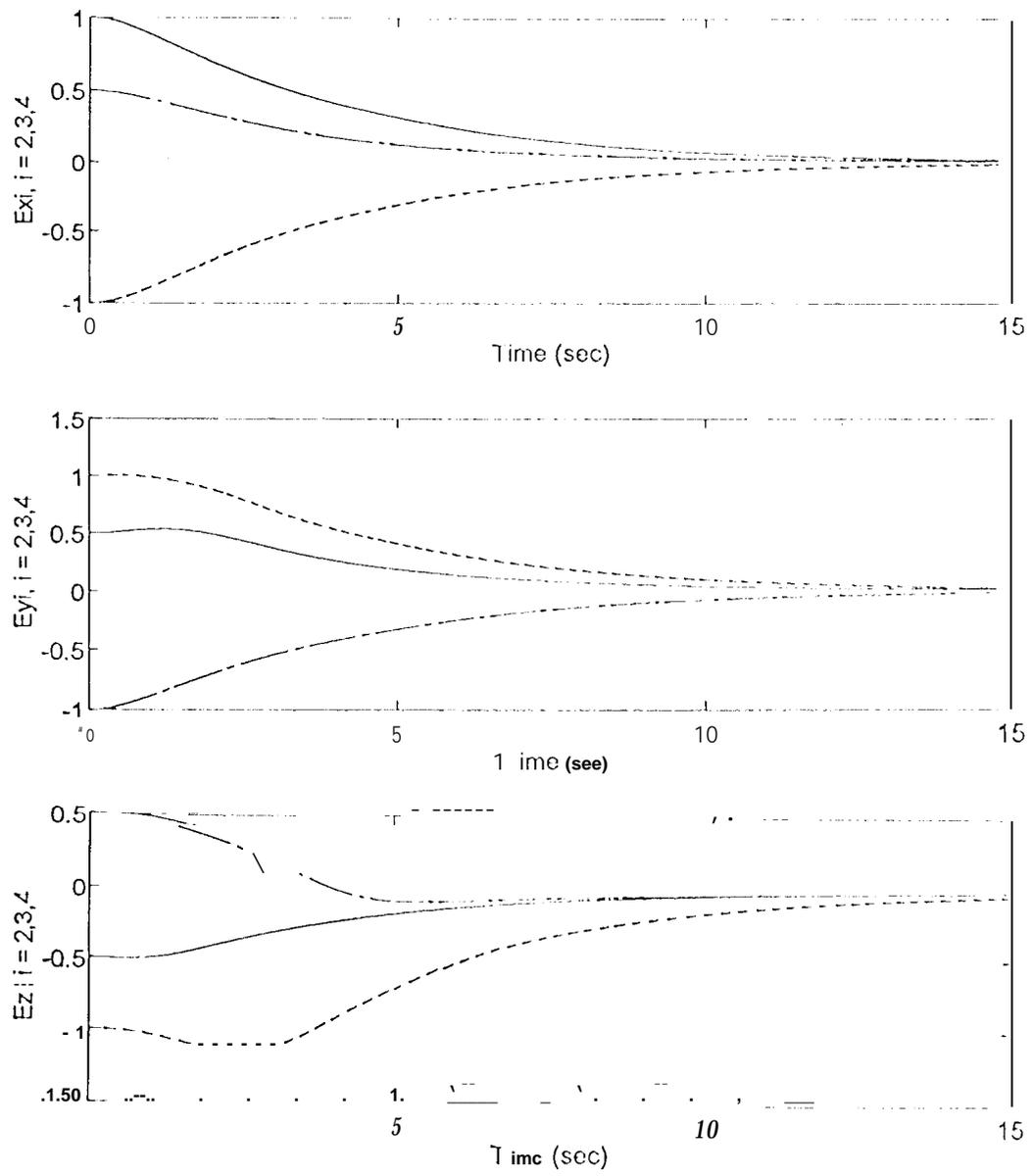


Fig. 5a

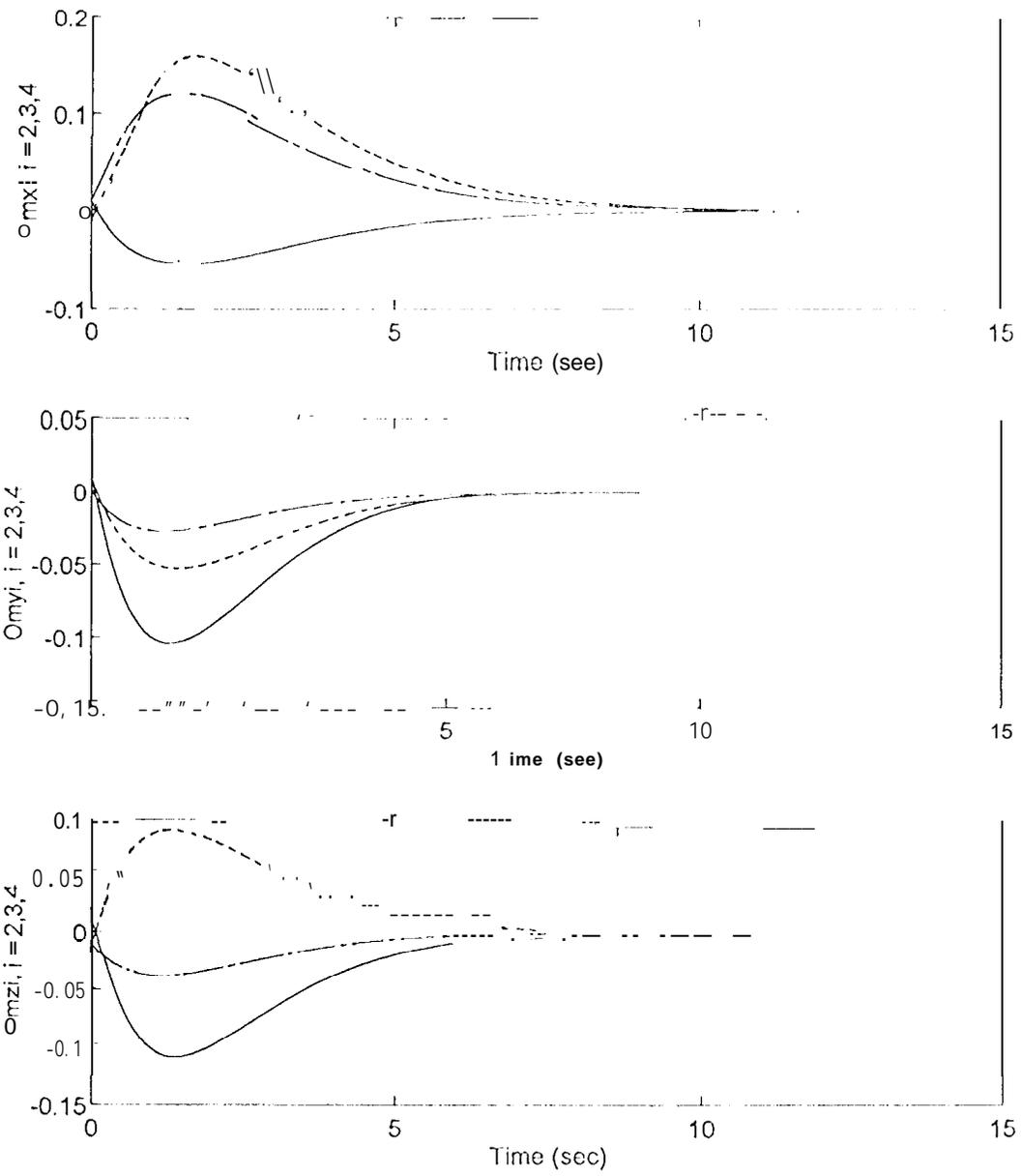


Fig. 5b

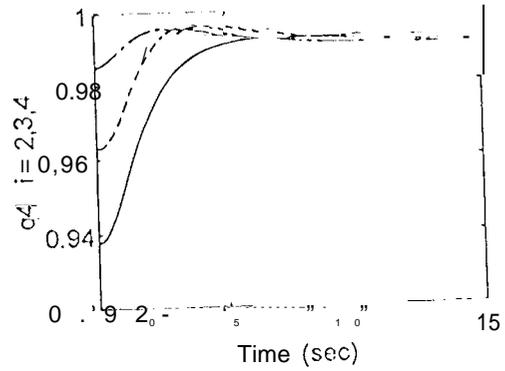
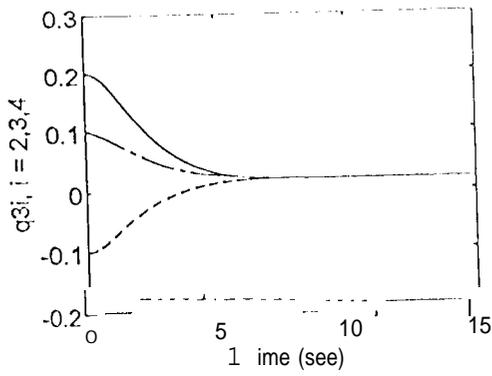
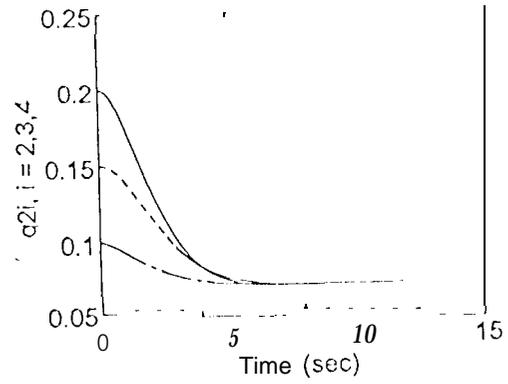
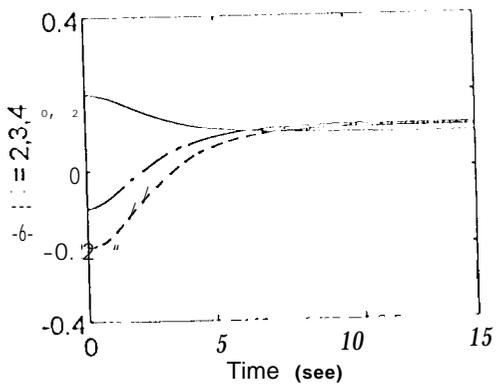


Fig.5c

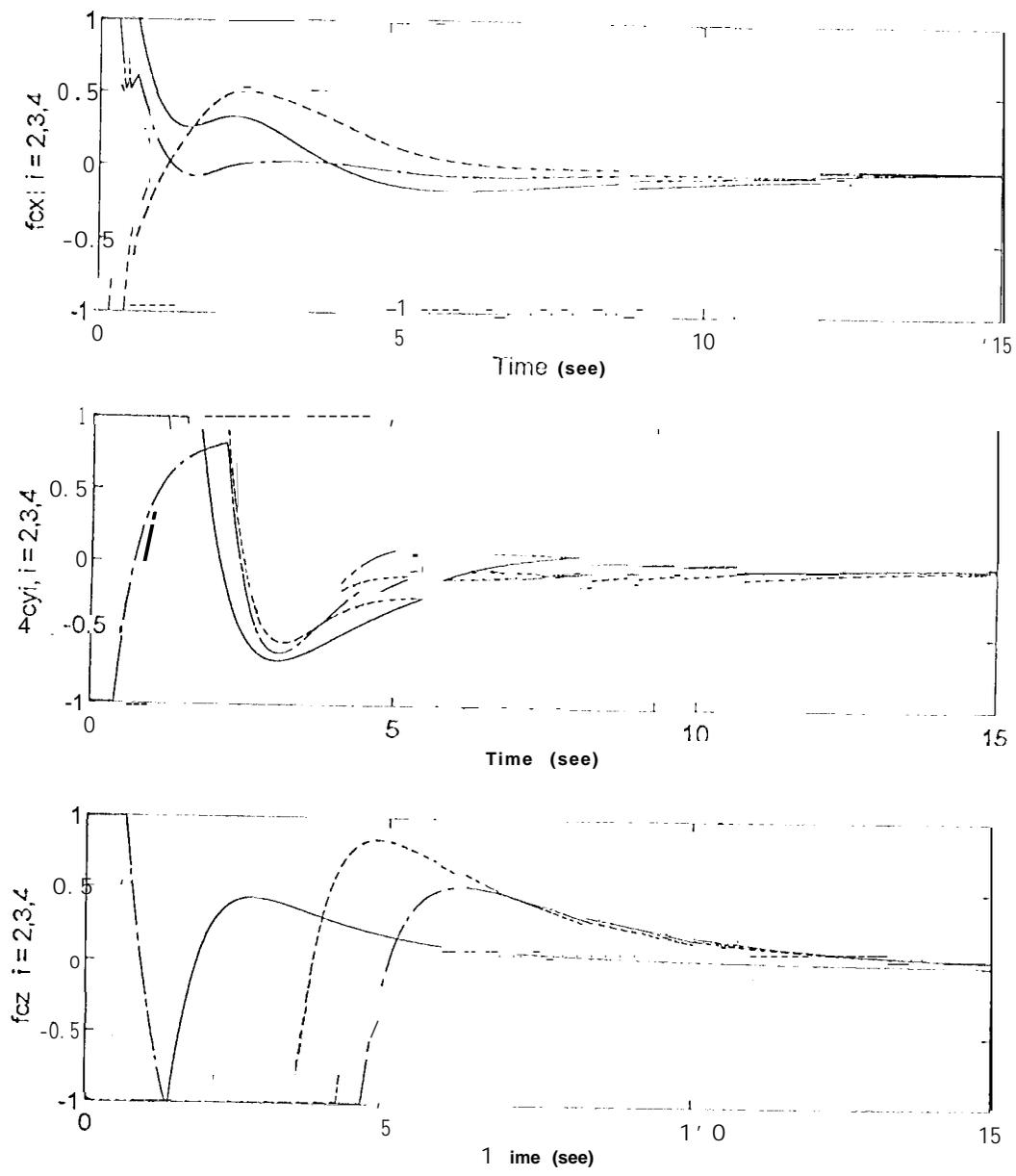


Fig. 5d

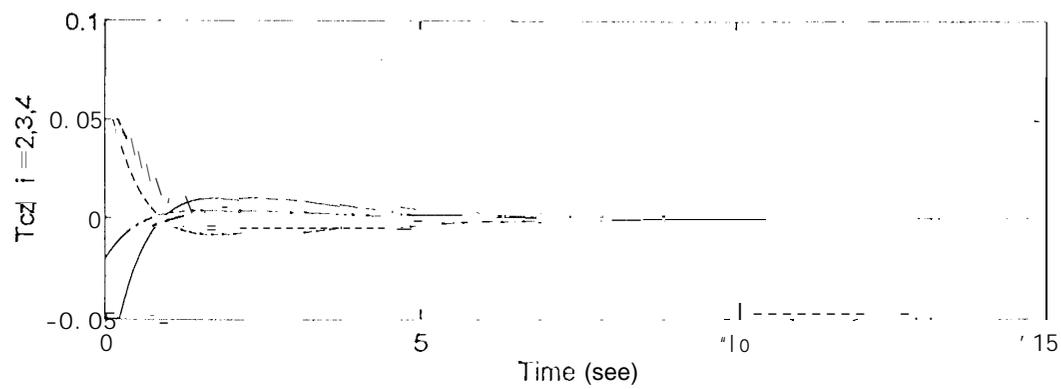
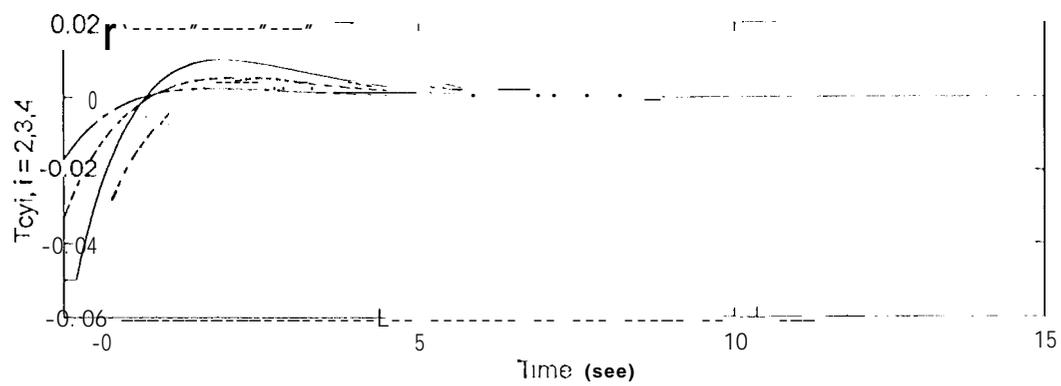
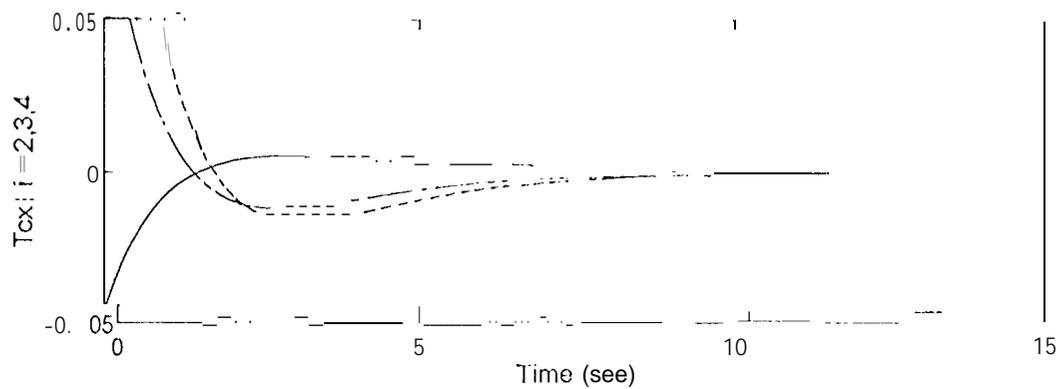


Fig. 5e

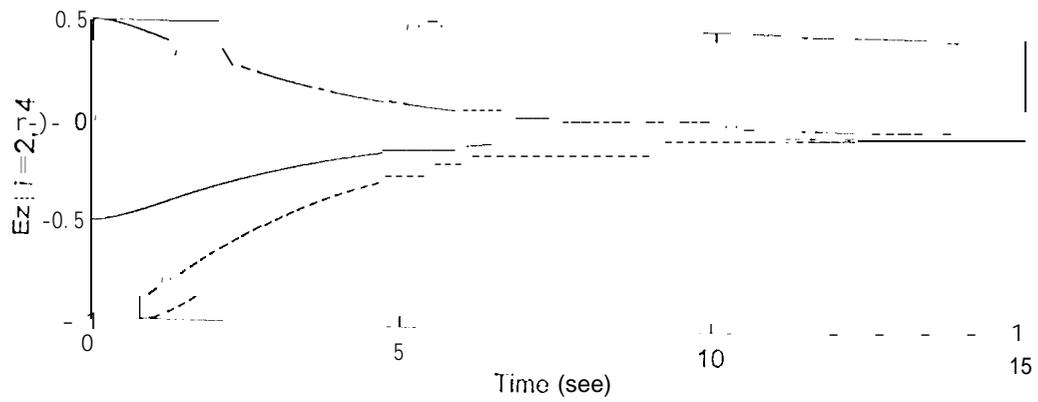
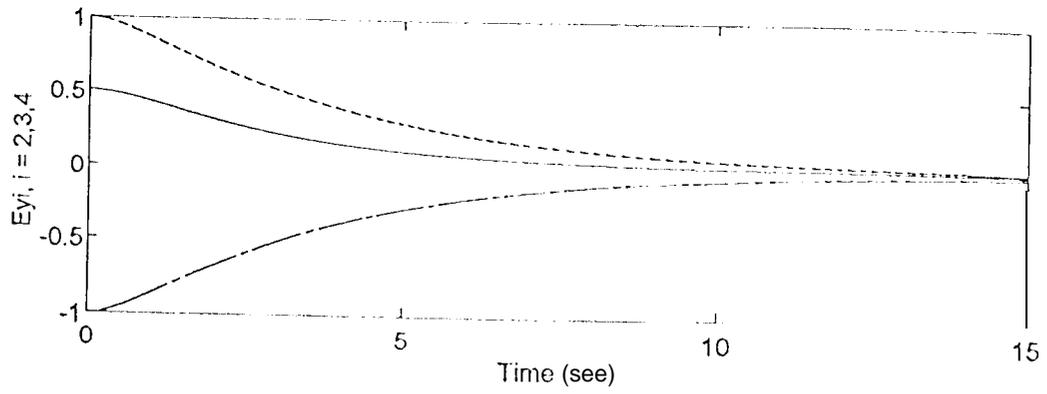
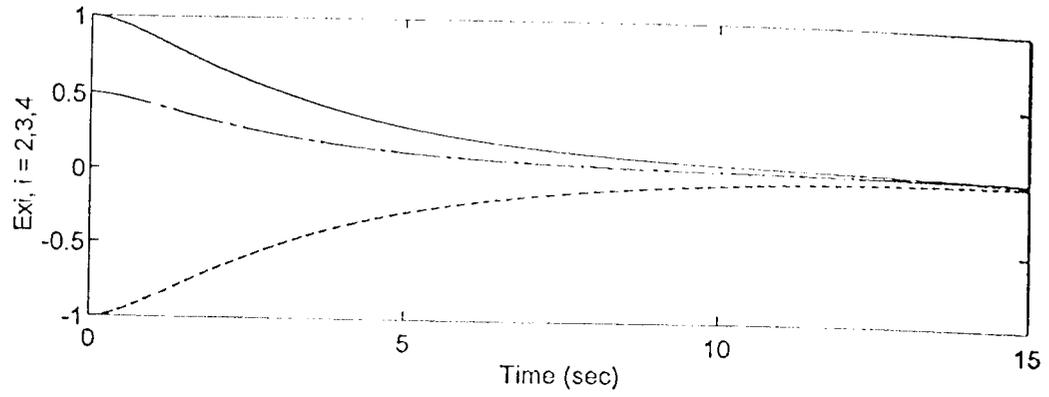


Fig.6a

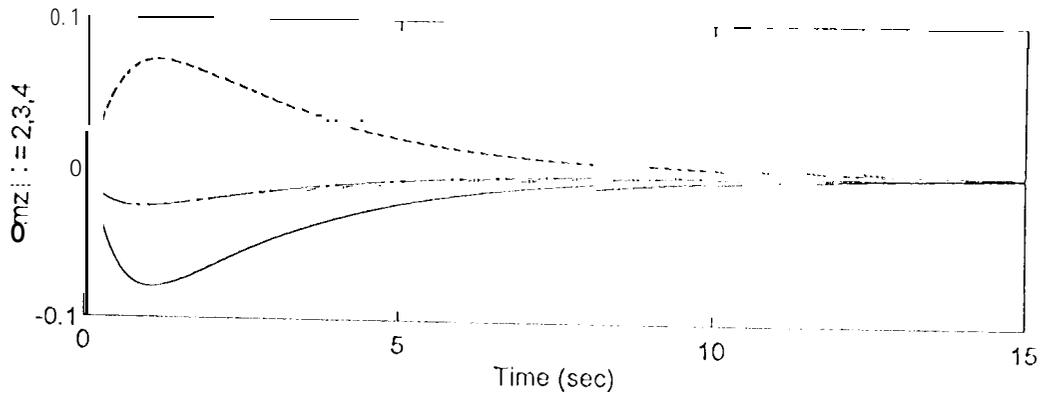
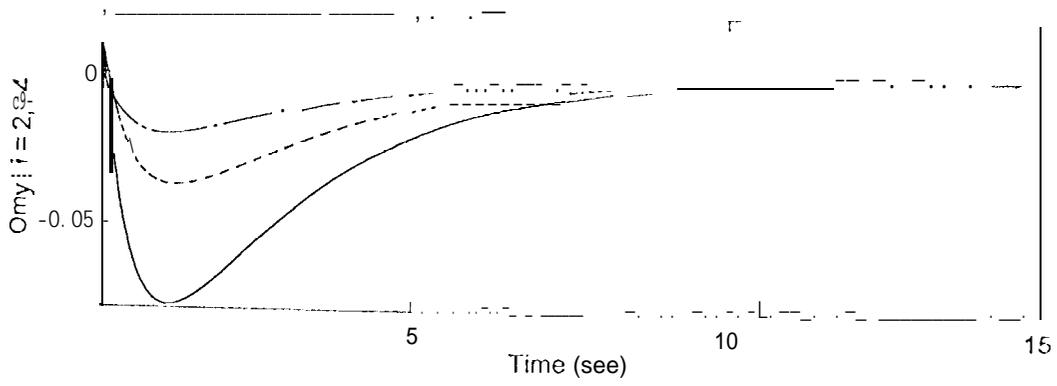
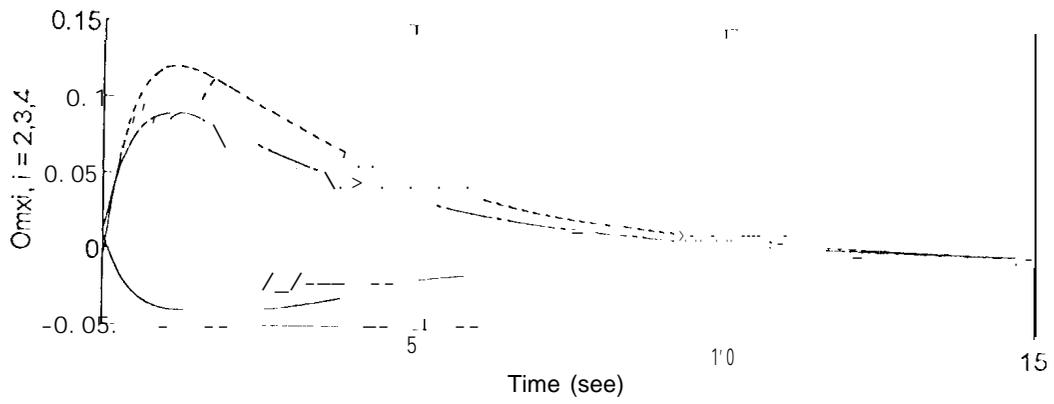


Fig. 6b

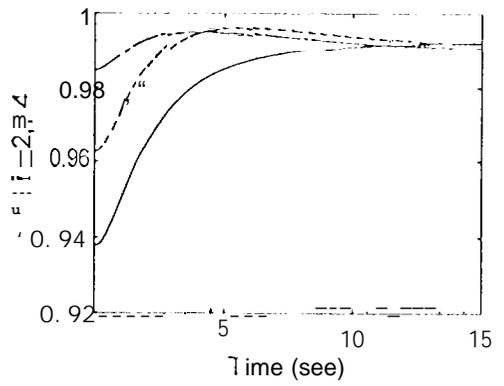
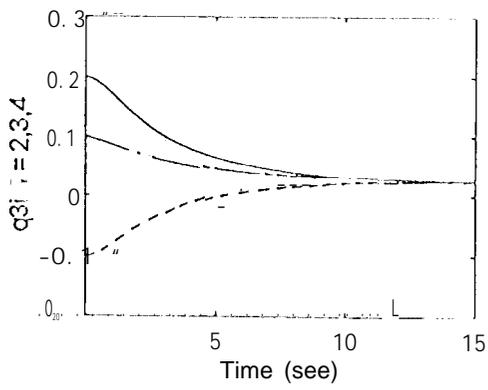
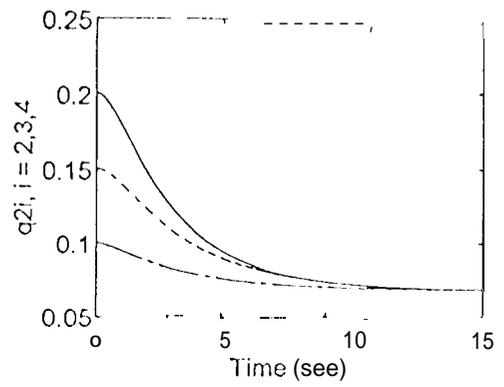
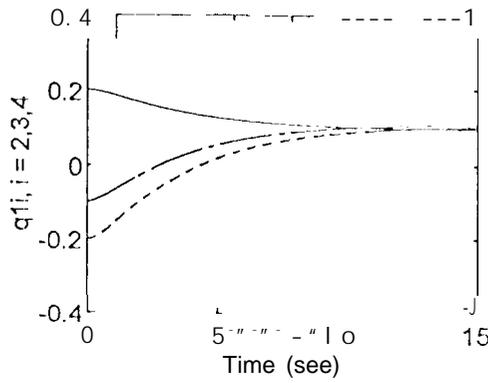


Fig.6c

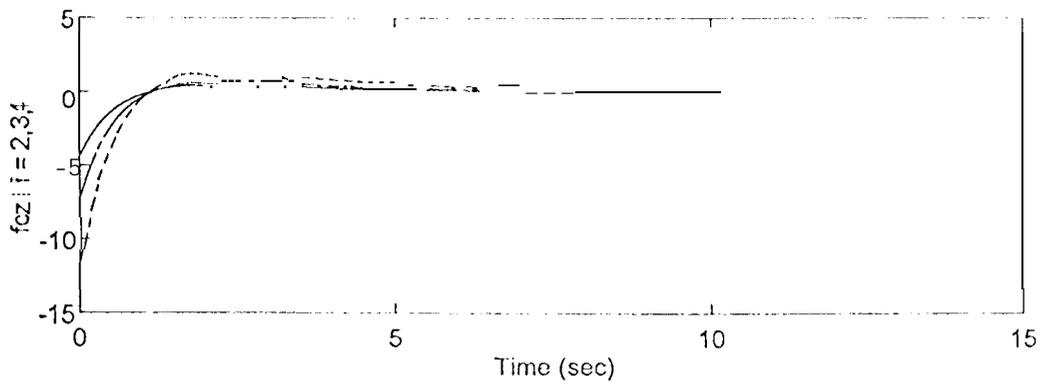
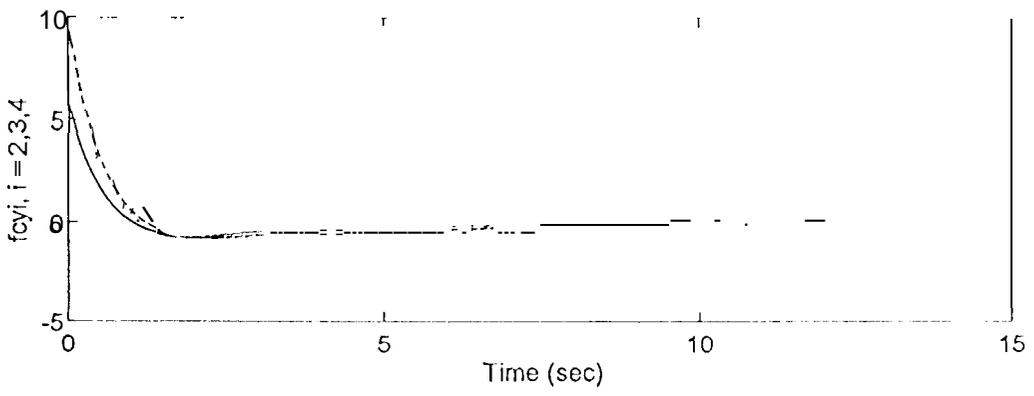
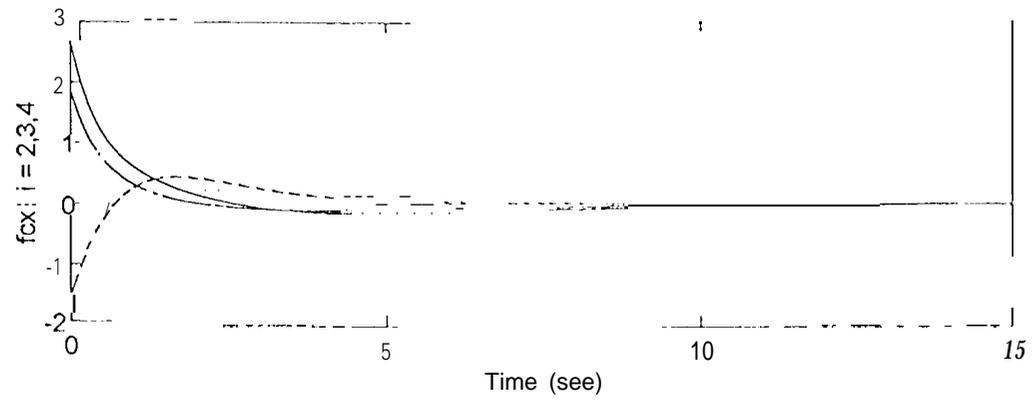


Fig.6d

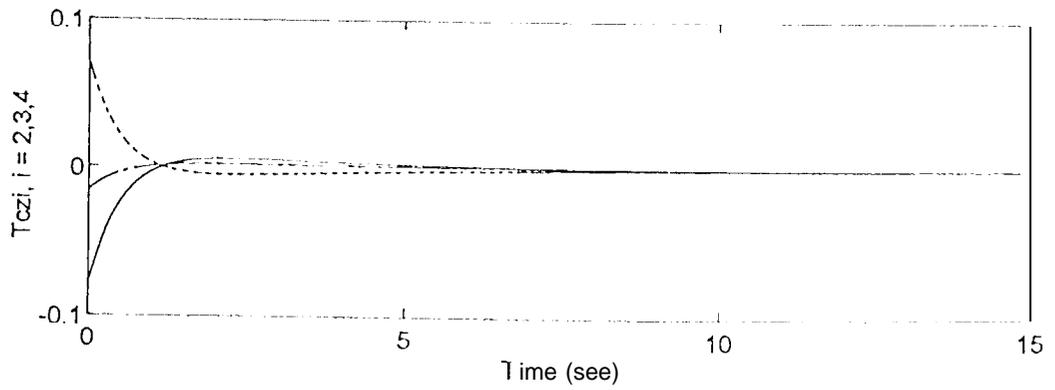
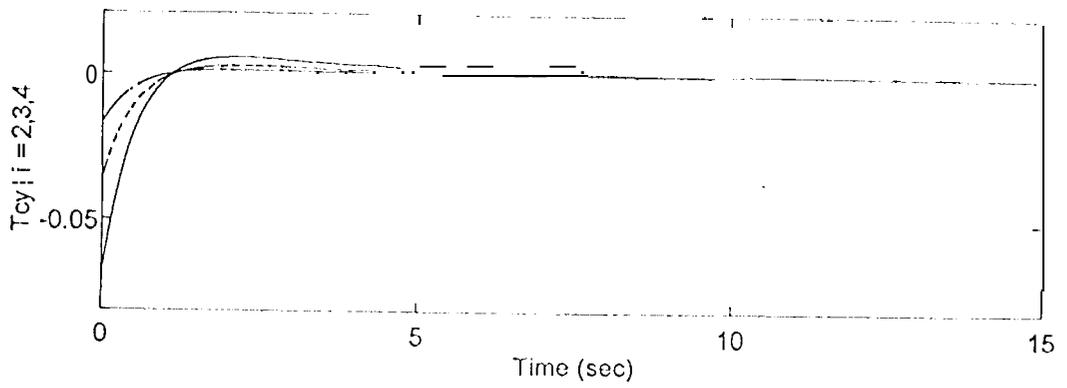
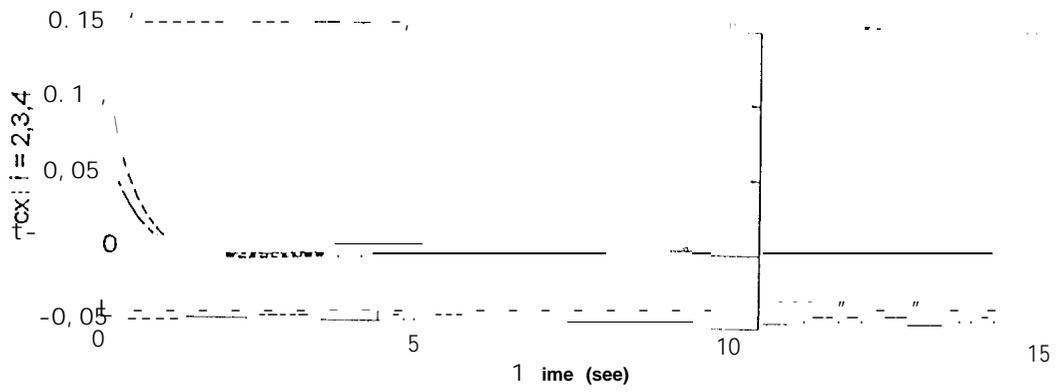


Fig. 6e

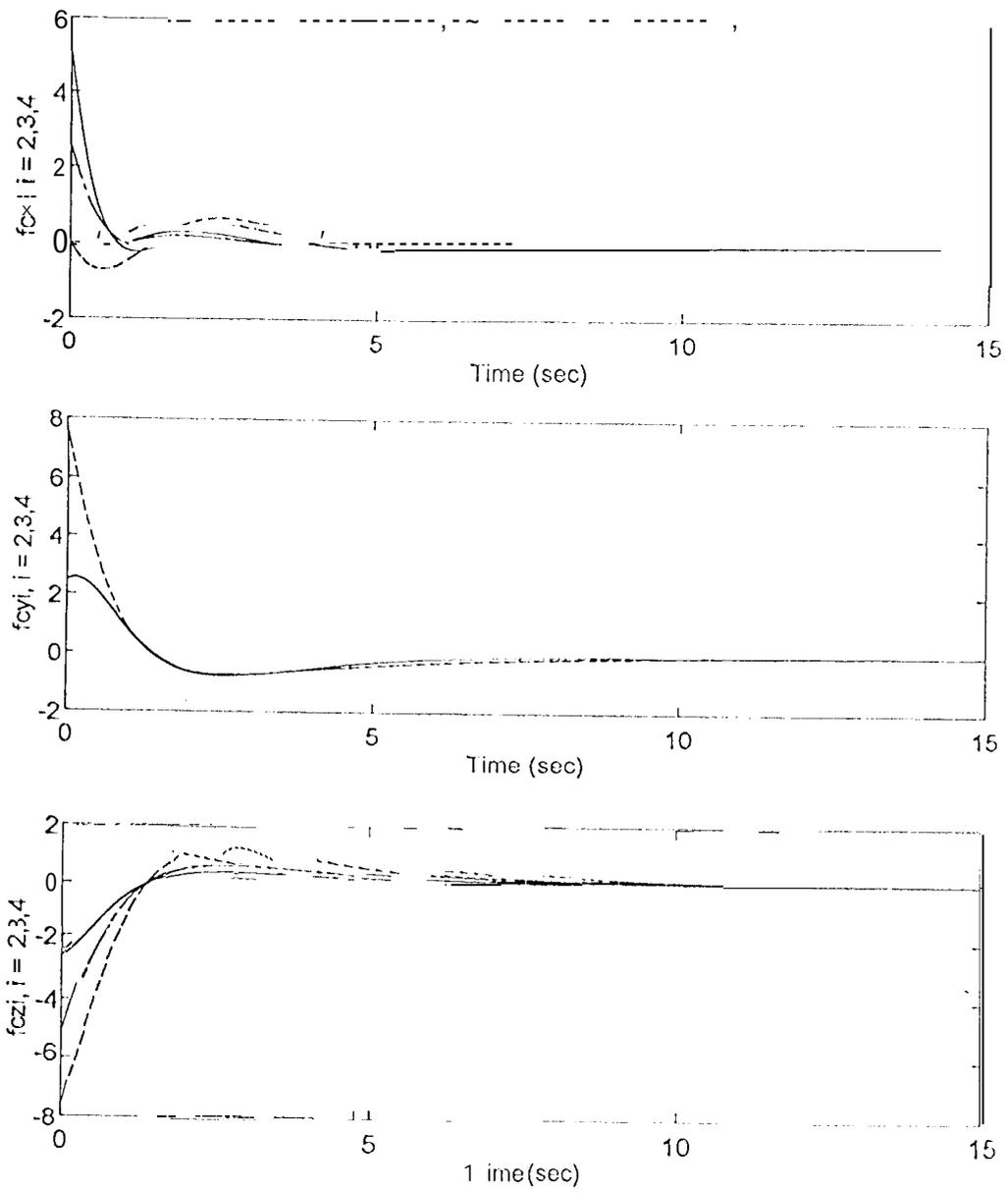


Fig.7

REFERENCE ORBITS

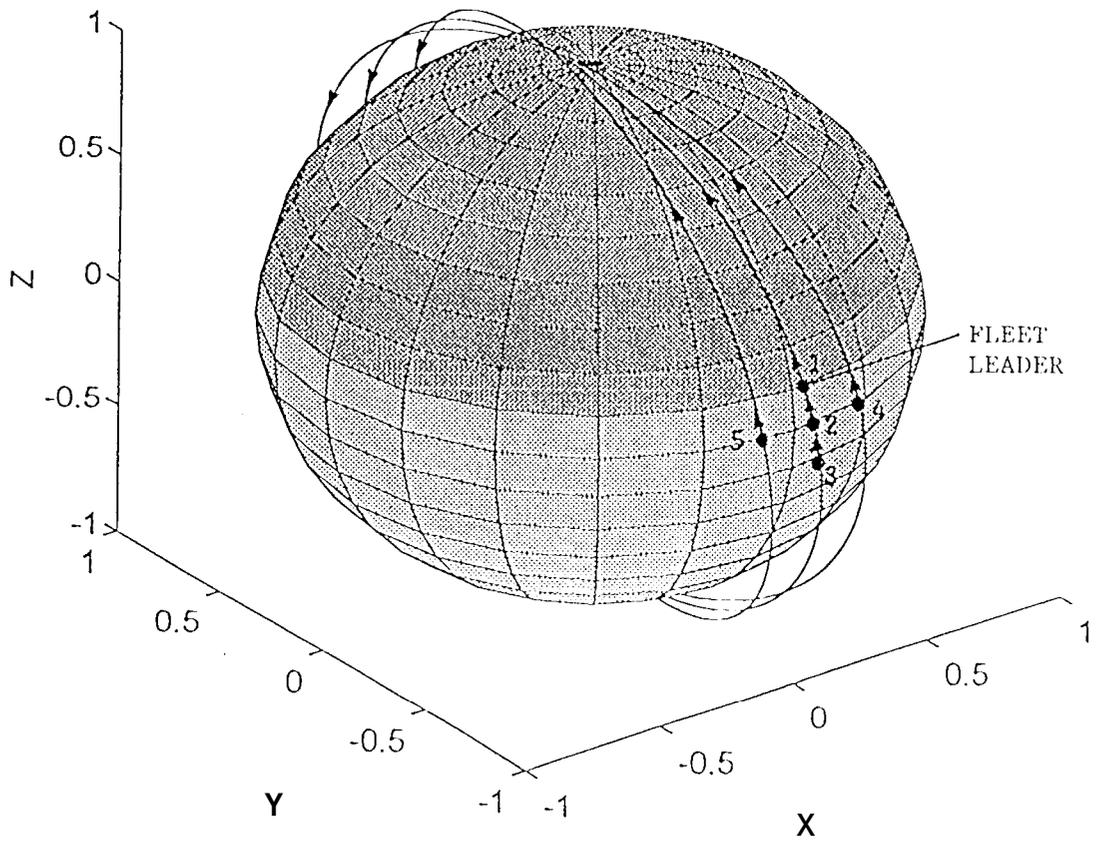


Fig.8

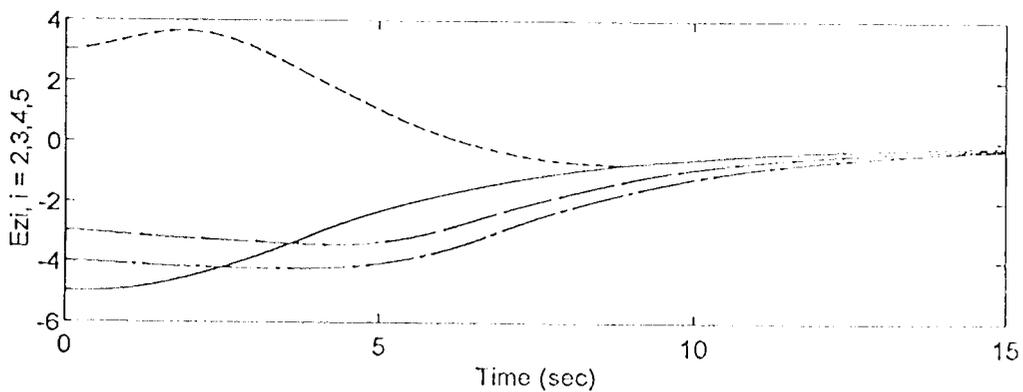
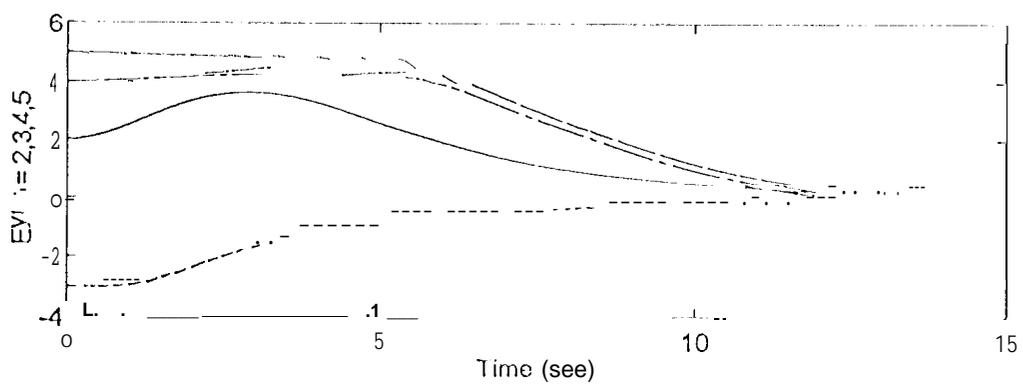
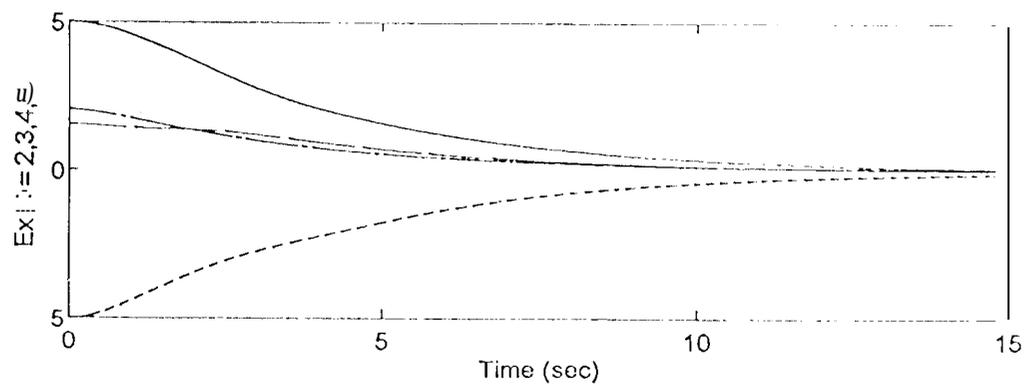


Fig. 9a

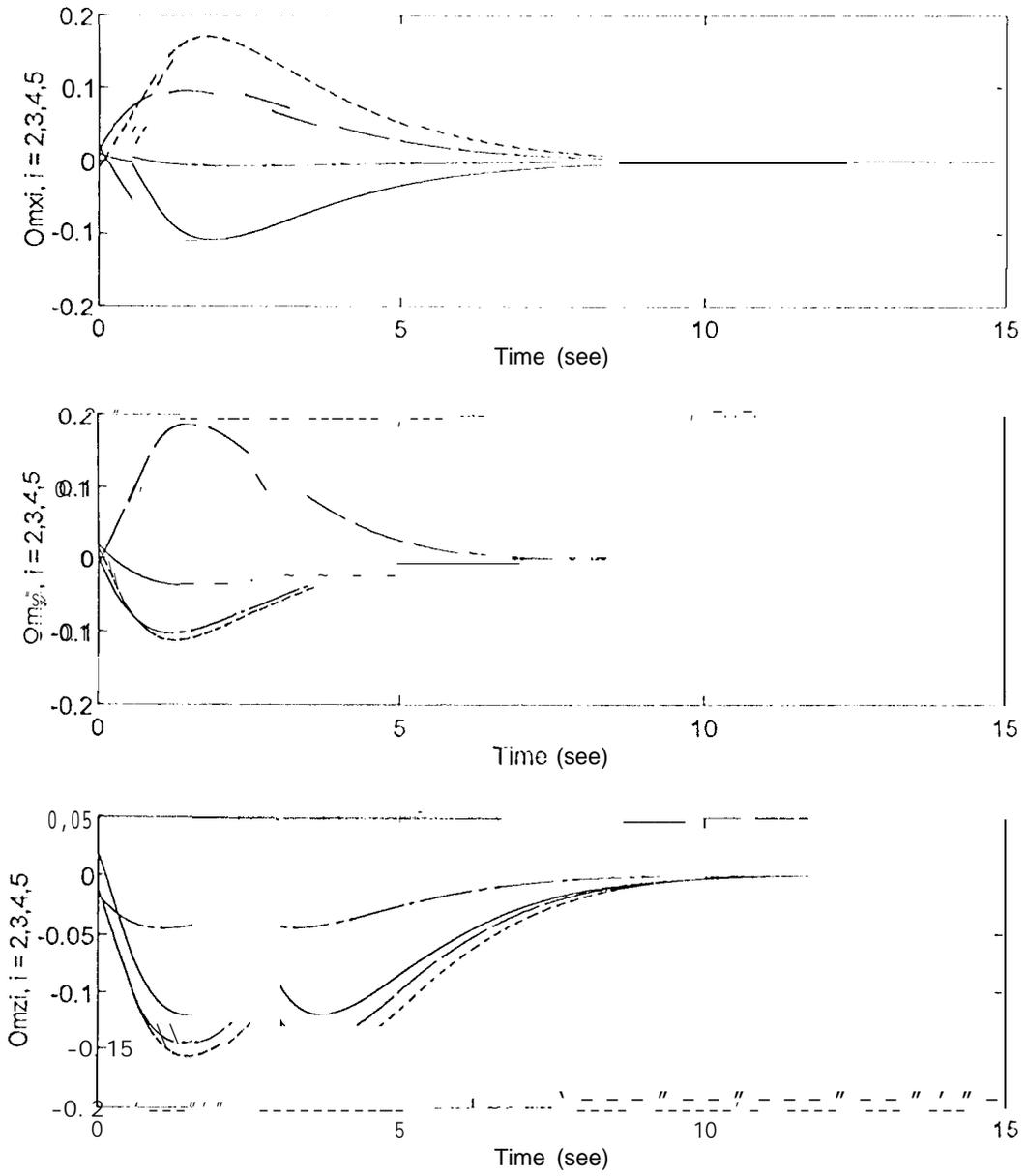


Fig.9b

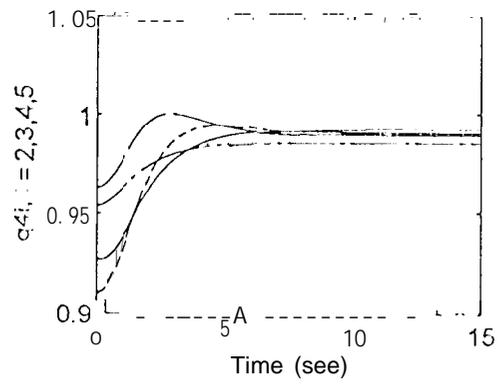
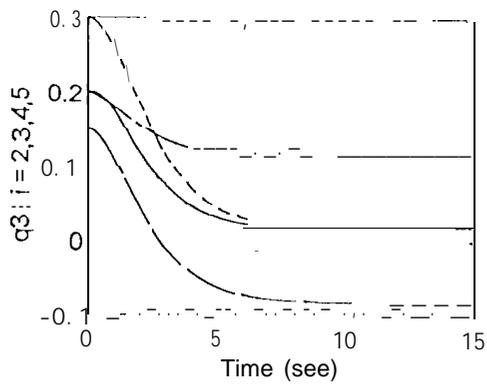
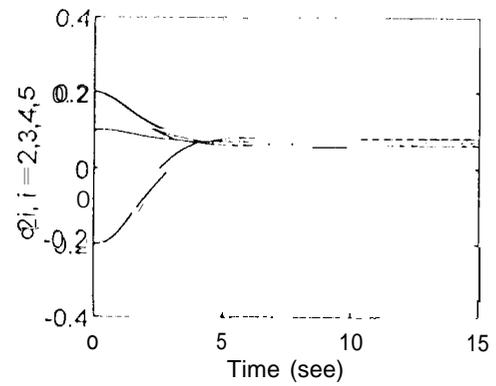
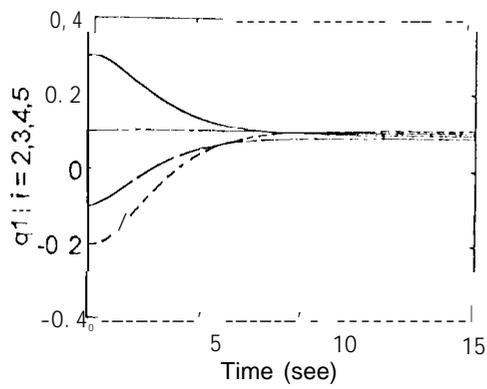


Fig. 9c

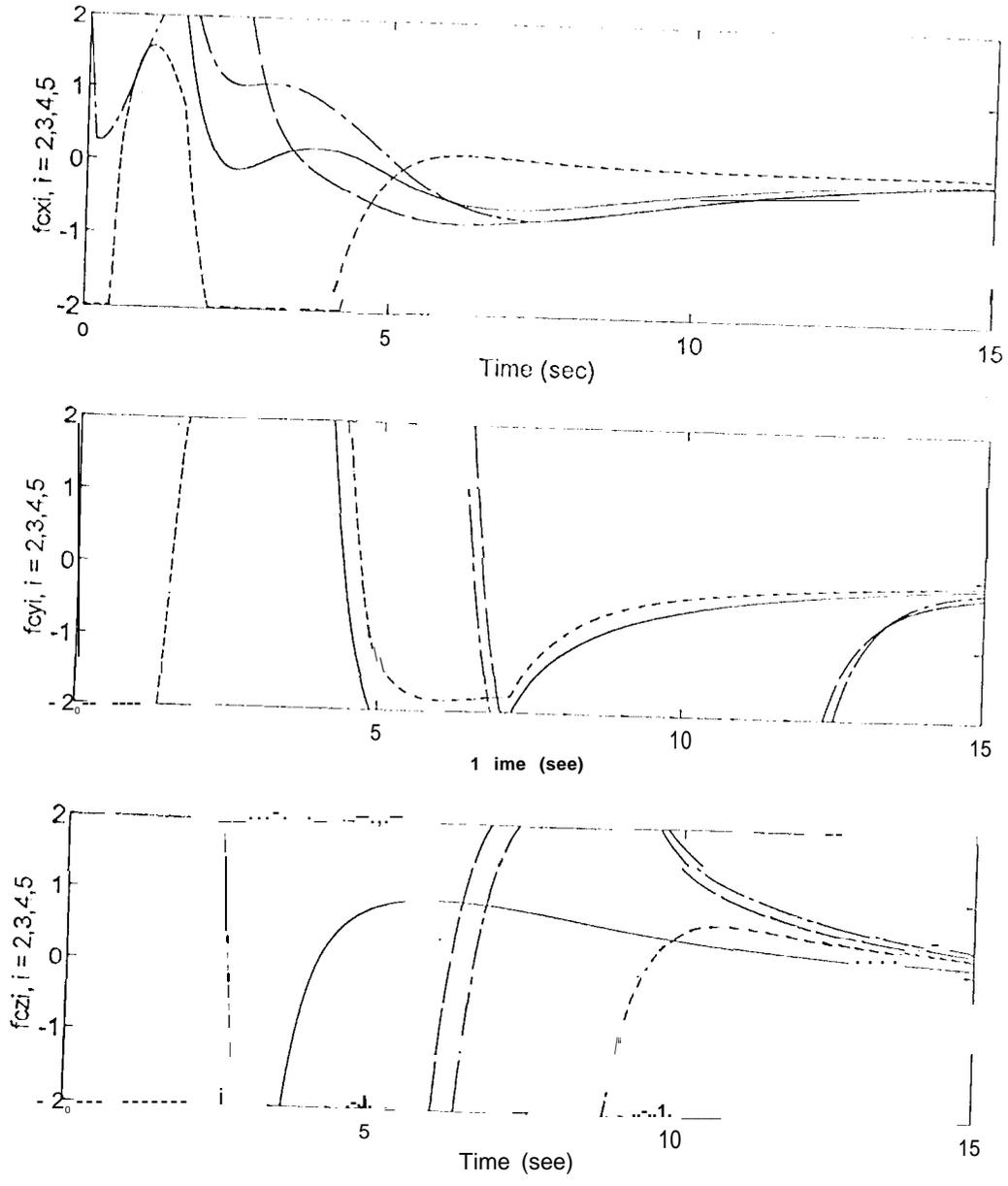


Fig. 9d

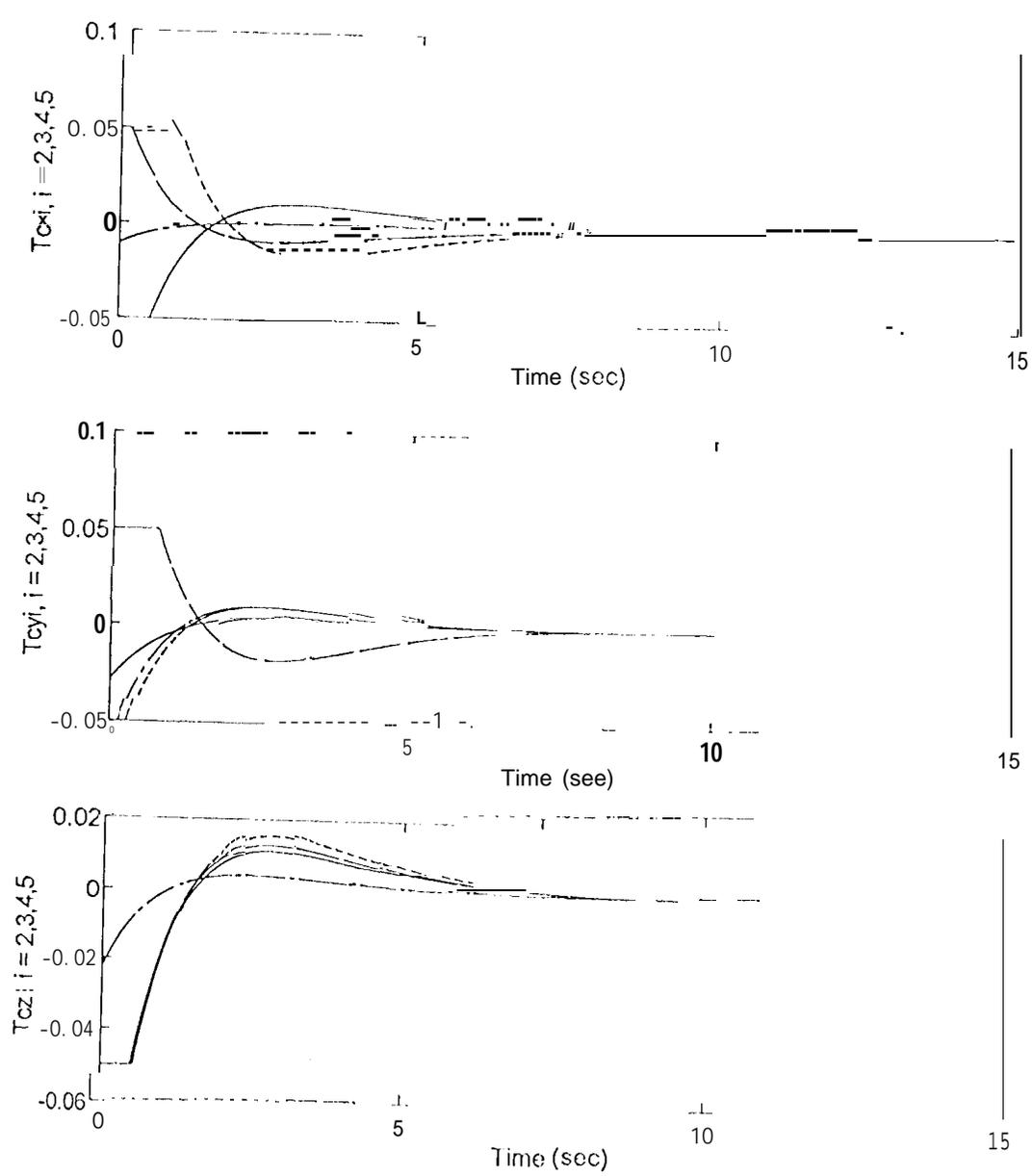


Fig. 9e