The Influence of Spatial Variations of Diffusion Length on Charge Collected by Diffusion From Ion Tracks

Larry D. Edmonds

Jet Propulsion Laboratory[1]

California Institute of Technology

Pasadena, California 91109-8099

ABSTRACT

Charge collected by diffusion from ion tracks in a semiconductor substrate maybe influenced by the substrate diffusion length, which is related to recombination losses. A theoretical analysis shows that, excluding some extreme cases, charge collection is insensitive to spatial variations in the diffusion length function, so it is possible to define an effective diffusion length having the property that collected charge can be approximated by assuming a uniform diffusion length equal to this effective value. Extreme cases that must be excluded are those in which a large number of recombination centers are confined to a narrow region near the substrate boundary.

---

[1] The research described in this paper was carried out by the let Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration, and supported by NASA Code-Q RTOP funding.

1. Introduction

Measurements of charge collected by epi SRAMS from ion tracks were compared to a model prediction [1]. The model predicts collected charge to consist of the charge liberated in the epi layer, plus an additional contribution that diffuses to the epi from the heavily doped substrate below. The latter charge is controlled by the substrate diffusion length, which is related to losses from recombination centers (RCs), and the model calculates this charge from the "uniform approximation" which assumes a spatially uniform effective diffusion length. The model predictions fit the data very well for both virgin devices and devices that had a greatly reduced diffusion length as a result of extensive exposure to heavy ion irradiation. The excellent agreement may be somewhat surprising because the actual diffusion length function is likely to be spatially nonuniform (particularly in a heavily irradiated device having a reduced diffusion length) due to a nonuniform distribution of RCs, yet the same effective value accurately predicts collected charge from any ion track, long or short. The experimental data tend to validate the uniform approximation, but no theory was given to explain how this can be.

The present paper provides a theoretical explanation. The analysis will show that, excluding some extreme cases, charge collection is insensitive to spatial variations in the RC distribution, even when these variations are large. Different distributions that result in the same collected charge from tracks that are effectively infinitely long, also result in approximate cl y the same collected charge from any track, long or short, The uniform approximation using an appropriately chosen effective diffusion length will be shown to be a reasonably good approximation for all but the most extreme cases.

It should be noted that there are extreme theoretical cases such that the uniform approximation fails, Although such cases arc not relevant to the experimental observations discussed above, they may become relevant under some other circumstances, so it is important to note that the uniform approximation has limitations. Both types of cases (those in which the approximation works well, and those in which it does not) arc first discussed in Section 3 and then illustrated by examples in Section 4.

The conclusions to follow are derived from the assumption that the linear track density (charge density per unit distance) is uniform over a finite track length. ‘I’he actual density is not uniform, particularly when the track is short enough so that collected charge depends on track length. This is a weakness of the present analysis because variations in the track density will change quantitative results. It is postulated that such changes in quantitative results will not be large enough to change the qualitative conclusions that follow. This postulate is at least credible, and is consistent with the experimental observations discussed earlier.

2. Terminology and Statement of the Problem

The assumed physical arrangement consists of a device substrate containing an ion track which extends from the upper substrate surface to a depth equal to the track length. The upper substrate boundary is assumed to be an infinite plane which is a sink for minority carriers. The substrate is assumed to be infinitely thick, although the analysis in the appendix also treats finite thicknesses. The linear track density is assumed to be uniform over the track length, so the track is completely described by two parameters, which are the track length and the linear density. Charge that reaches the upper surface via diffusion from the track is determined by the two track parameters and by the substrate diffusion length function which describes recombination losses in the substrate. The diffusion length function is assumed to be laterally uniform, but Jnay be highly nonuniform in the vertical coordinate due to a nonuniform RC density.

The uniform approximation estimates collected charge by assuming some appropriately selected uniform diffusion length. The objective is to use diffusion theory to show that this approximation can, sometimes, provide a reasonably accurate estimate, even when the actual diffusion length function is highly nonuniform. The uniform approximation will obviously produce correct results if appropriate, and possibly different, uniform diffusion lengths are assumed for different ion tracks. 'l'he objective is to show that reasonably accurate estimates can be obtained for any ion track when the same effective diffusion length is assumed for all cases. Let Q(z) denote collected charge when the track length is z. The objective is to show that there is an effective diffusion length having the property that the uniform approximation produces a reasonably accurate estimate of Q(z) for any z between O and ∞.

The linear track density implicitly contained in Q(z) is superfluous when investigating the adequacy of the uniform approximation. It is convenient to define a normalized Q, which is denoted l(z) and defined to be Q(z) divided by the linear track density. Note that l(z) has the dimensions of distance. The quantity l(m) has a special significance because it has two interpretations. The first interpretation is immediately implied by its definition; it is the normalized charge collected from an infinitely long track. Note that for the special case of a uniform diffusion length, the normalized charge collected from an infinitely long track equals the diffusion length. Therefore the second interpretation of $l(\infty)$ is an effective diffusion length. It is the value that must be assigned to the effective diffusion length in order for the uniform approximation to correctly predict collected charge from an infinitely long track, in fact, $l(\infty)$ is the effective diffusion length that will be used with the uniform approximation in all discussions to follow, This choice for the effective diffusion length insures that the uniform approximation will be accurate whenever the track is sufficiently long. But it is still not clear how long is "sufficiently long", or how good the approximation is when the track is not sufficiently long. These questions are answered in the following sections.

3. Upper and Lower Bounds

Because $I(\infty)$ has a dual interpretation, it is sometimes a convenient unit for measuring both the dependent variable $I(z)$ and the independent variable z, i.e., it is sometimes convenient to plot the dimensionless parameter $I(z)/I(\infty)$ against the dimensionless parameter $z/I(\infty)$. The first parameter is interpreted as the charge collected from a track of length z divided by the charge collected from the infinitely long track, while the second is interpreted as the track length divided by the effective diffusion length. The uniform approximation is expressed in terms of these dimensionless parameters as

$$\frac{I(z)}{I(\infty)} = 1 - \exp\left(-\frac{z}{I(\infty)}\right) \quad (\textit{uniform approximation}). \tag{1}$$

It is shown in the appendix that, no matter what the actual diffusion length function is, an upper bound for the actual $I(z)/I(\infty)$ is given by

$$\frac{I(z)}{I(\infty)} \le \begin{cases} \dfrac{z}{I(\infty)} - \dfrac{1}{4}\left[\dfrac{z}{I(\infty)}\right]^2 & \textit{if} \quad \dfrac{z}{I(\infty)} \le 2 \\ 1 & \textit{if} \quad \dfrac{z}{I(\infty)} \ge 2 . \end{cases} \tag{2}$$

Unfortunately, there is no universal lower bound, except zero. To obtain a nontrivial lower bound, it is necessary to impose a constraint that limits the diffusion length functions that may be considered. The type of constraint that is convenient from the point of view of analysis is to stipulate that less than some specified fraction of collected charge may come from depths exceeding some specified multiple of the effective diffusion length. For example, we might consider the diffusion length functions satisfying the constraint

Less than 10% of the charge collected from the infinitely long track is from depths exceeding four times the effective diffusion length. (3)

It is shown in the appendix that a lower bound for any I(z)/I(∞) consistent with this constraint is given by

$$\frac{I(z)}{I(\infty)} \geq \begin{cases} \frac{0.9}{4} \frac{z}{I(\infty)} & \text{if } \frac{z}{I(\infty)} \leq 4 \\ 0.9 & \text{if } \frac{z}{I(\infty)} \geq 4 \end{cases} \quad (4)$$

Plots of the right sides of(1), (2), and (4) are shown in Figure 1. This paper calls a 20% error "reasonably good", so agreement between the upper bound and the uniform approximate ion is reasonably good. This implies that agreement bet ween the uniform approximation and any actual curve that is above this approximation must also be seasonably good, because any such curve is bracketed between the uniform approximation and the upper bound. But agreement between the uniform approximation and the lower bound is not as good. If the constraint (3) is relaxed to include a larger class of diffusion length functions, the lower bound becomes lower and agreement becomes worse. 'l'he uniform approximation fails badly when the actual curve approximates the lower bound corresponding to a constraint that is more relaxed than (3).

It is unfortunate that there are cases such that the uniform approximation does not work well. It can be shown that such cases arc produced when a very large number of RCs arc confined to a very narrow region that is very close to the upper surface, But the approximation is reasonably good under all other conditions. For example, if a very large number of RCs are confined to a very narrow region, but this region is at a depth of at least three halves of the effective diffusion length, the actual curve will resemble the upper bound in Figure 1, which is fairly close to the uniform approximation. The approximation becomes even better for the less extreme cases in which the RC density is spread out to the extent that the density does not vary by more than a factor of a few (e.g., 5 or less). For these cases, the actual curve will look more like the uniform approximation than either bound shown in Figure 1. These statements arc illustrated by numerical examples in the next section.

4. Numerical Examples

The statements at the end of the previous section can be illustrated by numerical examples representing some actual measured data that were discussed at the beginning of Section 1. The heavily irradiated devices having reduced diffusion lengths are the most interesting because they arc likely to have the greatest nonuniformity in the RC density. Charge collection measurements found the effective diffusion length to be about 2 pm for these devices, The virgin devices were found to have an effective diffusion length of about 10 pm. 'J'here was sufficient evidence that this difference between diffusion lengths is not due to random part-to-part variations, so it is assumed that the irradiated devices had a 10 μm diffusion length prior to irradiation. Other than constraints imposed by this information, it is not known what the spatial distribution of the post-irradiated RC density is. We will consider a sampling of all possible distributions consistent with the measured data and compare the exact l(z), corresponding to an assumed distribution, to the uniform approximation for each case. The reciprocal of the diffusion length function, which is a measure of the RC density, is assumed to be a blip (possibly narrow or possibly broad) representing a (possibly localized or possibly spread-out) RC distribution produced by irradiation damage. The asymptotic (large depth) value of the diffusion length function is assumed to be the prc-irradiated value. The mathematical form of the diffusion length function, which was selected primarily on the basis of analytical tractability, is deferred to the appendix because the qualitative characteristics shown in the figures discussed later arc probably more relevant than mathematical expressions.

We consider a sampling of all possible diffusion length functions which are described by the equations in the appendix, have the asymptotic value of 10 μm, and are consistent with 1(m)= 2 μm. The sampling is worst-case from the point of view of demonstrating adequacy of the uniform approximation. This is accomplished by making the blip width as narrow as possible (within limits stated below), to obtain the greatest possible nonuniformity consistent with the stated conditions. Depending on the location of the blip center, an arbitrarily narrow blip may be mathematically compatible with the stated conditions if the blip amplitude is correspondingly large. This occurs when the blip is sufficiently close to the surface. When this is the case, the blip width is taken to be about 1 μm because this is sufficiently close to the mathematical limit (the blip approximates a Dirac delta function). But if the blip center is sufficiently deep, it is no longer true that an arbitrarily narrow blip can satisfy the condition that l(m) =- 2 μm. Some spread is required so that the RC density extends to higher locations. When this is the case, the blip width is selected to be the smallest value such that the condition can be satisfied. It can be shown that the demarcation between these cases occurs when the blip center depth is approximately twice the effective diffusion length (or 4 μm for the examples to follow). The approximation is accurate when recombination from the RCs outside and above the blip can be neglected. The sampling will usc a 1 μm blip width (approximately) if the blip center is at a depth less than 4 pm, and the minimum allowed blip width for larger depths.

Note that the shallow blips could be excluded on the basis of relevancy to the measured data, because a shallow and narrow blip could only be produced by damaging ions that stop near the top of the device. The damage in the parts considered was produced by ions having much longer ranges. Excluding the narrow and shallow blips will give a much more favor-able impression regarding the adequacy of the uniform approximation, But such cases may be relevant under other circumstances and it may be important to know that the uniform approximation does not always work well. Therefore the sampling includes such cases even though they are not relevant to the measured data.

The sampling is shown in Figures *2* through 6. Each figure presents an assumed RC distribution, measured in terms of the reciprocal of the diffusion length function L_D, and compares different predictions of the normalized collected charge l. The z in $L_D(z)$ is interpreted as depth, while the z in l(z) is interpreted as track length. Each l curve identified as "exact" was calculated from the exact equation describing the assumed L_D function.

The exact curve in Figure *2* resembles the uniform approximation when z is less than 1 pm because this places the track above the blip. But the exact curve is approximately linear and far below the uniform approximation for larger z. in fact, the exact curve in Figure 2 is even lower, at the larger values of z, than the lower bound shown in Figure 1. This is because the exact curve in Figure 2 violates the constraint (3) that applies to the lower bound in Figure 1. If the blip is moved higher than the 1 μm depth, the point where the exact curve and uniform approximation diverge in Figure 2 will move further to the left, and the exact curve will approximate the lower bound corresponding to a constraint that is much more relaxed than (3). Such small blip depths are the problem cases in which the uniform approximation fails badly. But the approximation becomes better as the blip is moved down. It is still not very good in Figure 3, but is reasonably good in Figures 4 and 5. Note that the exact curve in Figure 5 resembles the upper bound in Figure 1. The blip in Figure 5 is as deep as it can be without increasing the blip width. It is impossible for $l(\infty)$ to be as small as 2 μm unless the blip adds some RCs to the region above the 4 pm depth. A blip deeper than 4 μm implies that there must be some spread such as shown in Figure 6, Now that some spread is present, the uniform approximation becomes quite good. Although there is some spread, the RC density is still very nonuniform. It is therefore rather impressive that the uniform approximation works so well.

If the trend started by Figures 2 through 6 is continued beyond Figure 6, the RC density becomes progressively more uniform and the uniform approximation becomes progressively better. Even Figure 6 probably did not carry the trend far enough to represent the actual test devices. The uniform approximation is probably even more accurate, for the RC distribution in the actual devices, than indicated in Figure 6. 'l'his explains the successful application of the model discussed at the beginning of Section 1

5. Conclusions

Accuracy of the uniform approximation was theoretically investigated for the case in which the effective diffusion length used in the approximation is set equal to the charge collected by the infinitely long track divided by the linear track density. It was found that the approximation does not always work well. It fails badly when a large number of RCs arc confined to a narrow region at a depth less than three halves of the effective diffusion length. But when such extreme cases are excluded, charge collection is insensitive to spatial variations in the RC distribution (subject to the important qualification that all distributions being compared produce the same effective diffusion length), and the uniform approximation ranges from reasonably good to excellent. The approximation is very good for the type of RC distribution that is expected in some actual devices, and this is the theoretical explanation for the successful application of the model discussed at the beginning of Section 1.

APPENDIX: MATHEMATICAL ANALYSIS

A 1. An Expression for I(z)

We eventually consider substrates that are effectively infinitely thick, but it is convenient to start with a finite thickness L and take a limit later. The substrate is imagined to lie between two infinite planes which arc both sinks for minority carriers, It was shown [2] that the amount of charge Q reaching the upper plane via diffusion from an ion track can be calculated from

$$Q = \int_0^L \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P_I(\zeta_1, \zeta_2, \zeta)\, \Omega(\zeta_1, \zeta_2, \zeta)\, d\zeta_1\, d\zeta_2\, d\zeta$$

where P_I is the initial track density (charge per unit volume) and Ω is the charge-collection efficiency. The two coordinates ζ_1 and ζ_2 are lateral coordinates, while ζ is the longitudinal coordinate. The charge-collection efficiency is calculated by solving the boundary value problem

$$\nabla^2 \Omega = f^2 \Omega \quad \textit{in substrate}, \quad \Omega = 1 \quad \textit{on upper plane}, \quad \Omega = 0 \quad \textit{on lower plane}$$

where f is the reciprocal of the diffusion length function. It is assumed that f depends only on the longitudinal coordinate, so Ω depends only on the longitudinal coordinate. Suppressing the superfluous coordinates, the equation for Ω becomes

$$\frac{d^2\Omega(\zeta)}{d\zeta^2} = f^2(\zeta)\, \Omega(\zeta) \textit{ for } 0 < \zeta < L,\ \Omega(0) = 1,\ \Omega(L) = 0. \qquad \text{(A1)}$$

integrating the equation for Q with respect to the lateral coordinates gives

$$Q = \int_0^L \rho(\zeta)\, \Omega(\zeta)\, d\zeta$$

where p is the linear track density (charge per unit length) We consider the case where this density is uniform over a track length $z \leq L$, so that p is a step function. It is zero when $\zeta > z$, and constant when $\zeta \leq z$. Dividing Q by this constant produces J which is calculated from

$$J(z) = \int_0^z \Omega(\zeta)\, d\zeta \qquad \text{(A2)}$$

A2. <u>An Upper Bound</u>

When f is specified and Ω is known from (A1), I(z) can be calculated from (A2). But the present objective is to obtain a bound for the ratio I(z)/I(L) which can be derived when f is not specified and Ω is not known. Such a bound can be obtained by replacing the unknown Ω on the right side of (A2) with an expression that still contains the unknown Ω, but has some properties of Ω built into it so that information can be extracted without requiring that Ω be solved. Such an expression can be obtained by converting (A1) into an integral equation. integrating (A1) twice and then using an integration by parts to change the appearance of the result gives

$$L\,\Omega(z) = L - z - \int_0^L G(z,\zeta)\, f^2(\zeta)\, \Omega(\zeta)\, d\zeta \qquad (A3)$$

where the Green's function G is given by

$$G(z,\zeta) = \begin{cases} (L - z)\zeta & \text{if } 0 \le \zeta \le z \le L \\ (L - \zeta) z & \text{if } 0 \le z \le \zeta \le L. \end{cases}$$

Substituting (A3) into (A2) and changing the order of integration gives

$$L\, I(z) = (L - z/2)\, z - \int_0^L H(z,\zeta)\, \Psi'^2(\zeta)\, d\zeta \qquad (A4)$$

where H and Ψ' are defined by

$$H(z,\zeta) = \begin{cases} (1/2)(z^2/\zeta)(L - \zeta) & \text{if } 0 \le z \le \zeta \le L \\ (L - z/2)\, z - L\zeta/2 & \text{if } 0 \le \zeta \le z \le L \end{cases} \qquad (A5)$$

$$\Psi'^2(\zeta) = \zeta\, f^2(\zeta)\, \Omega(\zeta)$$

Note that (Al) implies that Ω is not negative anywhere, so Ψ'^2 is not negative anywhere. Another property of Ψ'^2 can be derived by first differentiating (A3) and evaluating the derivative at $z = L$ to get

$$L \frac{d\Omega(z)}{dz}\bigg|_{z=L} = \int_0^L \Psi'^2(\zeta)\, d\zeta - 1 . \qquad (A6)$$

Note that $\Omega(L) = 0$ and Ω is not negative anywhere, so Ω cannot be increasing in any neighborhood of the point z = 1.. Therefore the derivative on the left side of (A6) cannot be positive. Combining this observation with (A6) gives

$$\int_0^L \Psi'^2(\zeta)\, d\zeta \le 1 . \qquad (A7)$$

Another equation containing Ψ'^2 is obtained by evaluating (A4) at $z = L$ while using (A5) to get

$$\int_0^L (L - \zeta)\, \Psi'^2(\zeta)\, d\zeta = L - 2J(L) . \qquad (A8)$$

An important property of $H(z,\zeta)$, implied by (AS), is the type of curvature that it has when plotted against ζ with z fixed. When $\zeta \in (0,z)$, H is linear in ζ. When $\zeta \in$ (x,].), H has the curvature illustrated in Figure A 1. The H curve is convex when viewed from the left, so it is bounded below by any tangent line, such as illustrated by the lower dashed line in the figure (the upper dashed line is needed in Section A3). Selecting any tangent line, which is tangent to the H curve at ζ = any $A \in [z,1,]$, we have the bound

$$H(z,\zeta) \ge \frac{z^2}{2A^2}\left[L(L-\zeta) - (L-A)^2\right] \quad if\ \zeta \in [0,L] \quad and\ A \in [z,L] \qquad (A9)$$

which applies to any $\zeta \in [0,1.]$ and any A $\in [z,L]$. The right side of (A9) is the equation of the tangent line, regarded as a function of ζ with z fixed. Multiplying (A9) by Ψ'^2 and integrating and then using (A4), (A7), and (A8) to substitute for the integrals gives

$$l(z) \leq z + z^2\left[\frac{I(L)}{A^2} - \frac{1}{A}\right] if \quad z \in [0, L] \; and \; A \in \; [z, \; L] \qquad (A10)$$

which applies for any A $\in$ [z,].]. In particular, (Al O) applies when A = z and we obtain the obvious result l(z) ≤ I(L). A stronger statement can be made when z ≤ 21(1.) because wc can then let A = 21(1.) and (A 10) becomes

$$I(z) \leq z - \frac{z^2}{4I(L)} \quad if \quad z \leq 2I(L)$$

When the above inequality does not apply, wc still have l(z) ≤ 1(1,), so the bound can be expressed as

$$\frac{I(z)}{1(1.)} \leq \begin{cases} \dfrac{z}{I(L)} - \dfrac{1}{4}\left[\dfrac{z}{I(L)}\right]^2 & if \quad \dfrac{z}{I(L)} \leq 2 \\ \\ 1 \quad if \quad \dfrac{z}{I(L)} \geq 2 \,. \end{cases} \qquad (Al\ 1)$$

The upper bound given by (Al 1) is the smallest upper bound that applies when no restrictions arc imposed on the function f, This can be demonstrated by showing that the bound is approached by a limiting case. To demonstrate this, first note that the left side of (A8) is not negative, so 2I(L) ≤ L (it can be seen from (Al) and (A2) that the equality applies when f= O everywhere). The limiting case occurs when $f^2(\zeta)$ is a Dirac delta function centered at<=- 21(1,) and with an infinite coefficient selected so that a plot of $\Omega(\zeta)$ (satisfying (Al)) is a straight line connecting the point (ζ,Ω) = (O, 1) to the point

(21(1 .),0), and with $\Omega(\zeta)$ = O for $\zeta \geq$ *21(1,).* A direct evaluation of l(z) from (A2) using this Ω produces the right side of (Al 1).

The large 1, limit shown as (2) in Section 3 is obtained by simply replacing L with ∞ in (All).

A3. A Lower Bound

The curvature of 1 I discussed in Section A2 and illustrated in Figure A 1 implies that H is bounded by the chord connecting the end points (the upper dashed line in Figure Al). This gives the inequality

$$L\,H(z,\zeta) \leq z\,(L - z/2)(L - \zeta) \text{ for all } z \in [O, L] \text{ and all } \zeta \in [0, L].$$

Multiplying this inequality by Ψ'^2 and integrating and then using (A4) and (A8) to substitute for the integrals gives

$$\frac{I(z)}{I(L)} \geq 2\frac{z}{L} - \left[\frac{z}{L}\right]^2 \quad \text{for all } z \in [0, L]. \qquad (A12)$$

The lower bound given by (A 12) is the smallest lower bound that applies when no restrictions are imposed on the function f. This can be demonstrated by showing that the bound is approached by a limiting case. Such a case occurs when $f^2(\zeta)$ is a Dirac delta function centered at ζ =- 0+, and with a coefficient selected so that a plot of $\Omega(\zeta)$ (satisfying (Al)) first connects the point (ζ,Ω) = (O, 1) to the point (0+,21(1,)/1.), and then becomes a straight line connecting the point (0+,21 (1.)/1,) to the point (L,0). A direct evaluation of l(z) from (A2) using this Ω produces the right side of (Al 2).

Note that the upper bound in (Al 1) and the lower bound in (A 12) come together when the ratio 1./1(1.) is c1osc to the smallest allowed value (which is 2). This could have been anticipated from the fact that there is only onc possible f (zero) which can makc 21(1.) = I., so Ω is completely determined in this limit. Unfortunately, our concern is with the opposite extreme of a large 1,/1(1.). Replacing I. with ∞ in (Al 2) produces a lower bound of zero. To obtain a nontrivial lower bound, it is necessary to impose some constraint that restricts the set of functions that f may be selected from.

The type of constraint that is convenient for analysis is obtained by selecting some depth $Z \in (0,L)$ and some fraction $\alpha \in (0, 1)$, and stipulate that the fractional contribution to 1(1.), from charge collected from a depth exceeding Z, is not larger than α, i.e.,

$$[I(L) - I(Z)] / I(L) \leq a \,, \tag{A13}$$

A bound consistent with (A 13) can be derived by first evaluating (A4) at $z = Z$ and then usc (Al 3) to eliminate the I(Z) to get

$$\int_0^L H(Z,\zeta)\, \Psi'^2(\zeta)\, d\zeta \leq (L - Z/2)\, Z - L\,(1-\alpha)\, I(L) \,. \tag{A14}$$

If $z \geq Z$, we have the obvious bound $I(z) \geq I(Z) \geq (1-\alpha)\, I(L)$, Wc now consider the case where $z \leq Z$. Three possibilities to consider are $\zeta \in [0,z]$, $\zeta \in [z,Z]$, and $\zeta \in [Z,L]$. It can bc shown from (AS) that all three possibilities result in

$$H(z,\zeta) \leq \frac{(L - z/2)\, z}{(L - Z/2)\, Z} H(Z,\zeta)$$

with the equality applying when $\zeta = 0$. Multiplying the above inequality by Ψ'^2 and integrating and then using (A4) and (A14) to substitute for the integrals gives

$$I(z) \geq \frac{(1,-2/2)2}{(L-Z/2)Z}(1-\alpha)\ 1(1/)> (z/Z)(1-\alpha) I(L) \ if \ 0 < z \leq Z.$$

It is convenient to express Z as some multiple y of I(L). Using this notation, the bounds arc expressed as

$$\frac{I(Z)}{I(L)} \geq \begin{cases} \frac{(1-\alpha)}{\gamma}\frac{z}{I(L)} & if \ \frac{z}{I(L)} \leq \gamma \\ (1-a) \ if \ \frac{z}{I(L)} \geq \gamma \end{cases} \qquad (A15)$$

which provides a nontrivial result in the large L limit.

As an example, suppose the set of possible diffusion length functions, that the bound is to apply to, is restricted by the constraint that less than 10°/0 of the charge collected from the infinitely long track is from depths exceeding four times the effective diffusion length. We then let $L = \infty, \alpha = 0.1$, y =- 4, and (Al 5) becomes (4) in Section 3.

A4. Calculations Used for the Examples

The numerical examples in Section 4 were obtained by solving (Al) with f selected to represent some cases of interest. The easiest way to solve this problem is backwards. Instead of selecting an f representing a case of interest and attempting to solve (A 1) for Ω, it is easier to select an Ω and use (AI) to find out what f is. This is a trial and error method, If the f produced by a selected Ω does not approximate the function that wc would like it to be, it is necessary to try again with another Ω. The Ω used for the numerical examples is given by

$$\Omega(z) = B\exp(-k\,z) + AW\ln\left[1+\exp\left(\frac{z_0 - z}{W}\right)\right] \qquad (A16)$$

where k, A, W, and z_0 are constants satisfying

$$k > 0,\quad O < W < I/k,\quad A \geq 0,\ z_0 \geq 0,\ A\,W\ln[1+\exp(z_0/W)] < 1$$

but are otherwise arbitrary. The constant B is not arbitrary. It is calculated from

$$B = 1 - A\,W\ln[1+\exp(z_0/W)]\,.$$

The reciprocal of the diffusion length function used in the examples is f calculated from (Al) and (Al 6). The result is a blip, illustrated in the figures in Section 4, which has an asymptotic value equal to k. “l’he blip center depth is approximately z_0 when the blip is narrow. The relationship between blip depth and z_0 is more obscure with wider blips, but the depth increases when z_0 increases. The blip width is most strongly influenced by W and increases when W increases. The blip amplitude is most strongly influenced by A and increases when A increases.

The normalized charge l(z) is calculated from (A2) and (Al 6). To evaluate the integral in (A2), it is necessary to evaluate the integral S defined by

$$S(X) = \int_0^X \ln[1+\exp(-t)]\,dt$$

Special values arc given by S(0) = O and S(m)== n^2/12. The argument X can have either sign. When X > 0, S can be evaluated from the series

$$S(X) = \frac{\pi^2}{12} - \sum_{m=1}^{\infty}\frac{(-1)^{m-1}}{m^2}\exp(-mX) \quad if \quad X \geq 0$$

'l'he series converges very slowly when $X \approx O$, although convergence is faster for larger X. Calculations used for the examples approximated the series with the first 100 terms for all $X > 0$. Truncation errors are reduced by adding the terms in reverse order. The series diverges if $X < 0$. This case can be treated by first converting the argument to a positive number using the identity

$$s'(x) =- -s(-x) - X^2 /2$$

and then using the series to evaluate the term containing the positive argument.

REFERENCES

[1] L. Edmonds, G. Swift, and A. Johnston, "Non-Destructive Measurements for CMOS Devices Using Charge-Collection Techniques," *IEEE Trans. Nucl. Sci.*, December 1996.

[2] L. Edmonds, "A Graphical Method for Estimating Charge Collected by Diffusion from an Ion Track," *IEEE Trans. Nucl. Sci.*, vol.43, no.4, pp.2346-2357, August 1996.

FIGURE CAPTIONS

Figure 1: Plots of the uniform approximation, the upper bound (no restrictions), and the lower bound (subject to (3)).

Figure 2: The assumed L_D in (a) produces the exact 1 in (b), which is compared to the uniform approximation in (b).

Figure 3: The assumed L_D in (a) produces the exact 1 in (b), which is compared to the uniform approximation in (b).

Figure 4: The assumed L_D in (a) produces the exact 1 in (b), which is compared to the uniform approximation in (b).

Figure 5: The assumed L_D in (a) produces the exact I in (b), which is compared to the uniform approximation in (b).

Figure 6: The assumed L_D in (a) produces the exact 1 in (b), which is compared to the uniform approximation in (b).

Figure Al: Illustration of the curvature of $H(z,\zeta)$ when plotted against ζ with z fixed. The H curve (solid) is bounded above by the chord connecting the end points (upper dashed line), and bounded below by any tangent line, such as the lower dashed line shown,

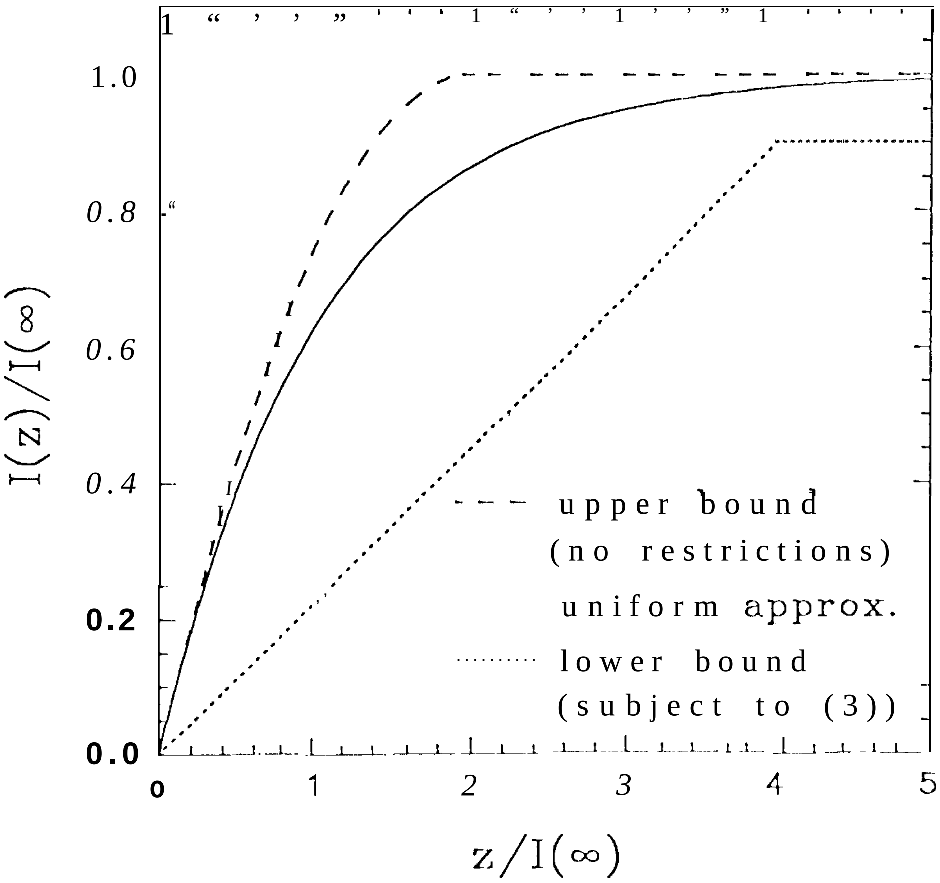

Figure 1: Plots of the uniform approximation, the upper bound (no restrictions), and the lower bound (subject to (3)).

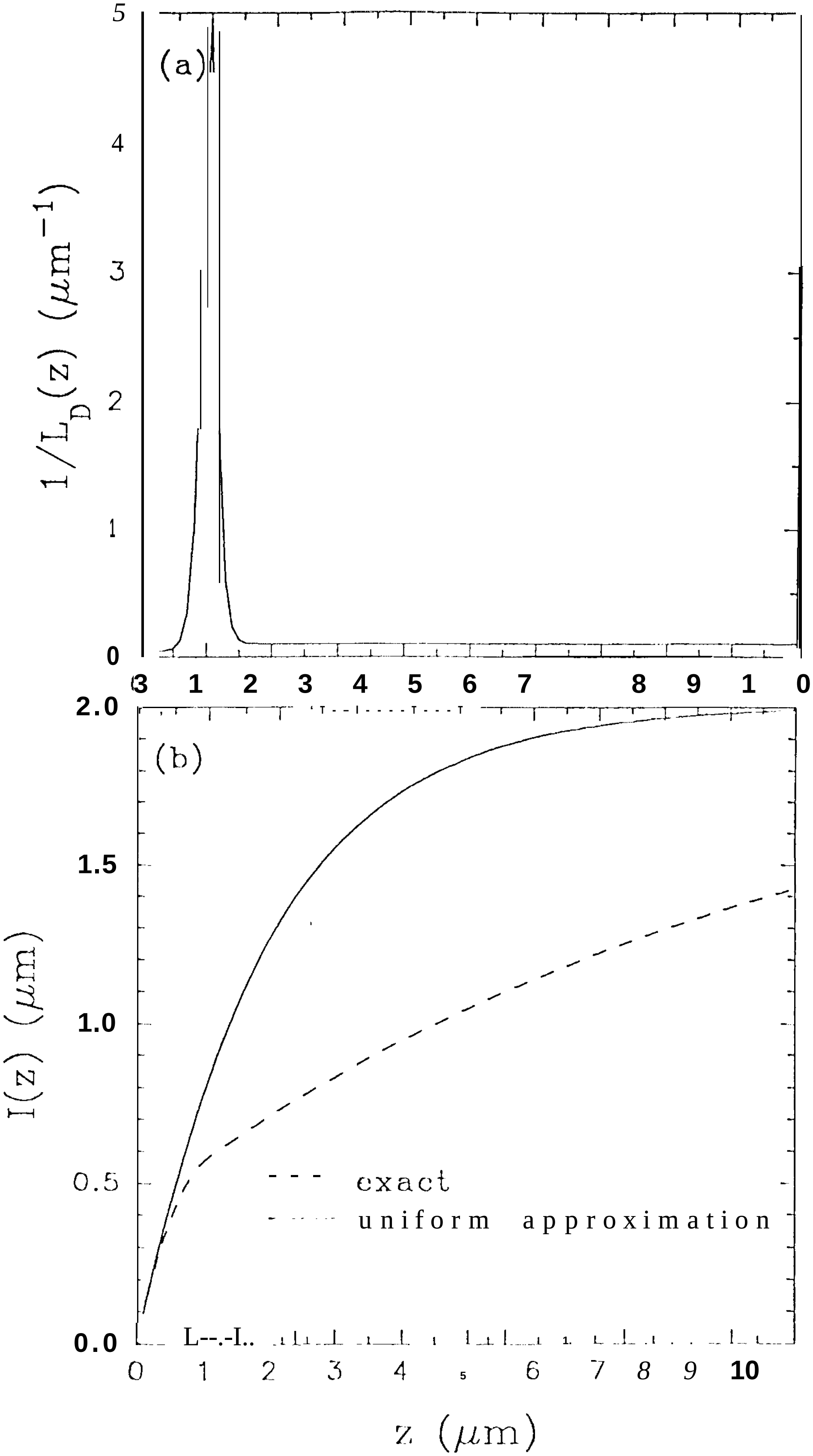

Figure 2: The assumed L_D in (a) produces the exact I in (b), which is compared to the uniform approximation in (b).

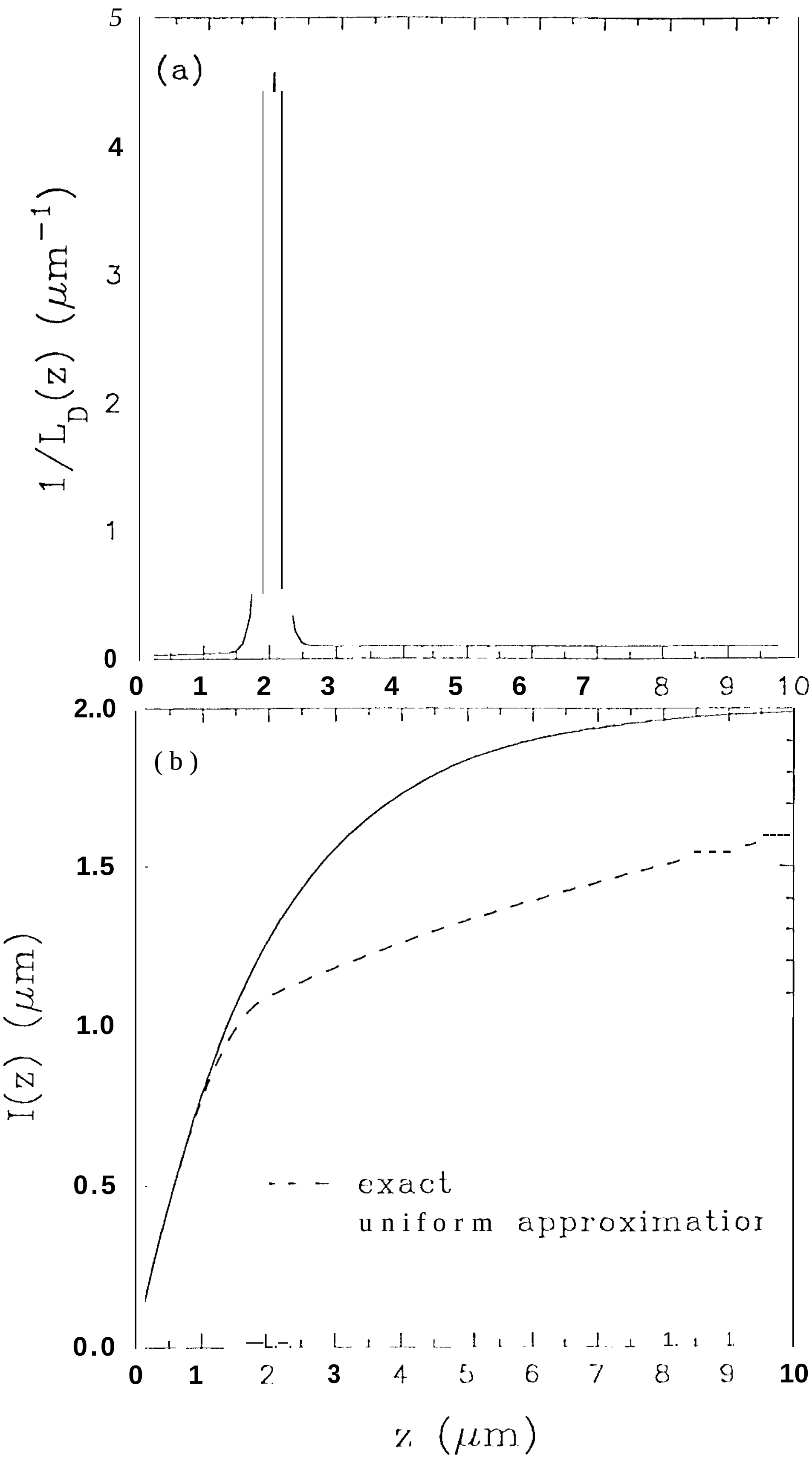

Figure 3: The assumed L_D in (a) produces the exact 1 in (b), which is compared to the uniform approximation in (b).

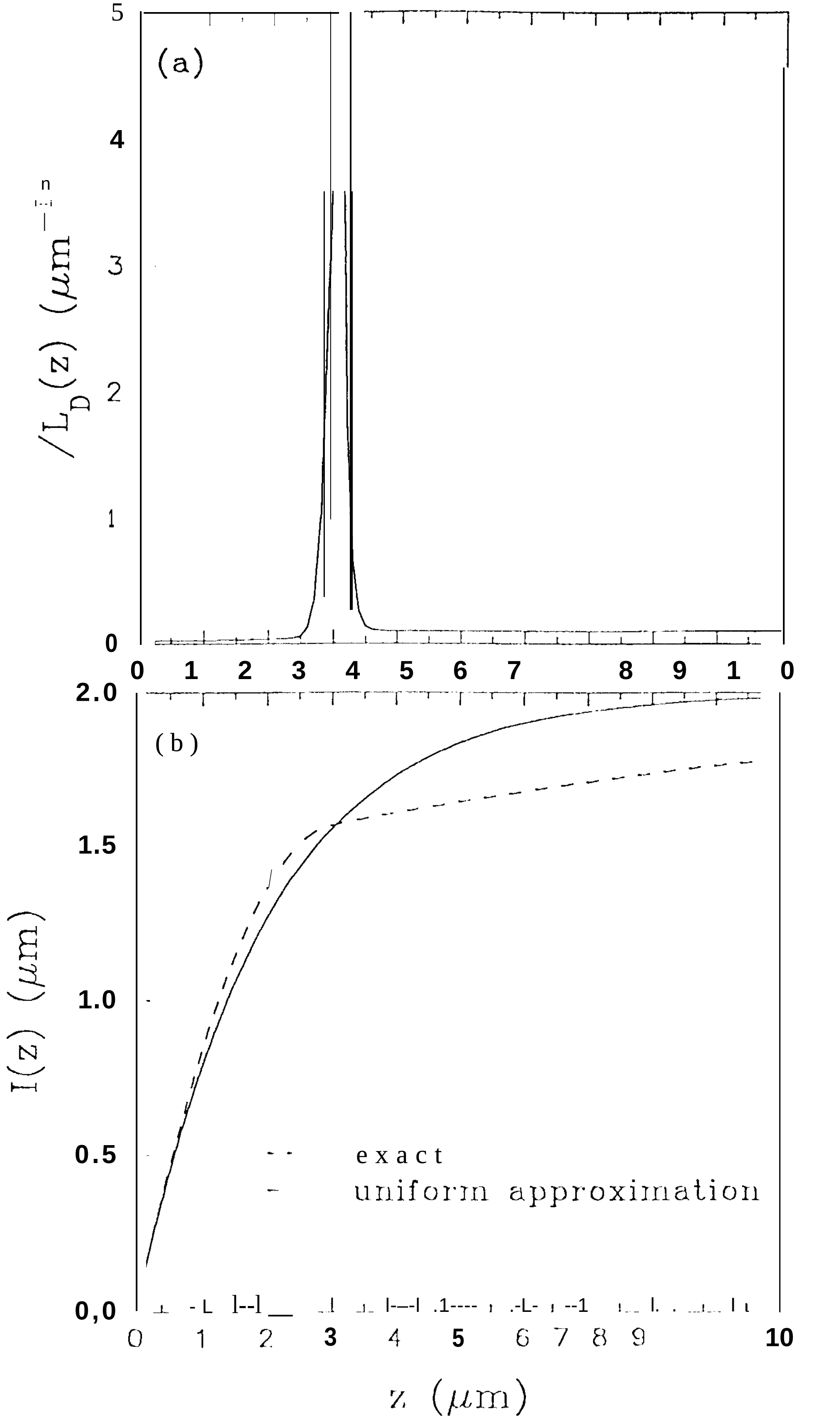

Figure 4: **The** assumed I_D in (a) produces the exact I in (b), which is compared to the uniform approximation in (b).

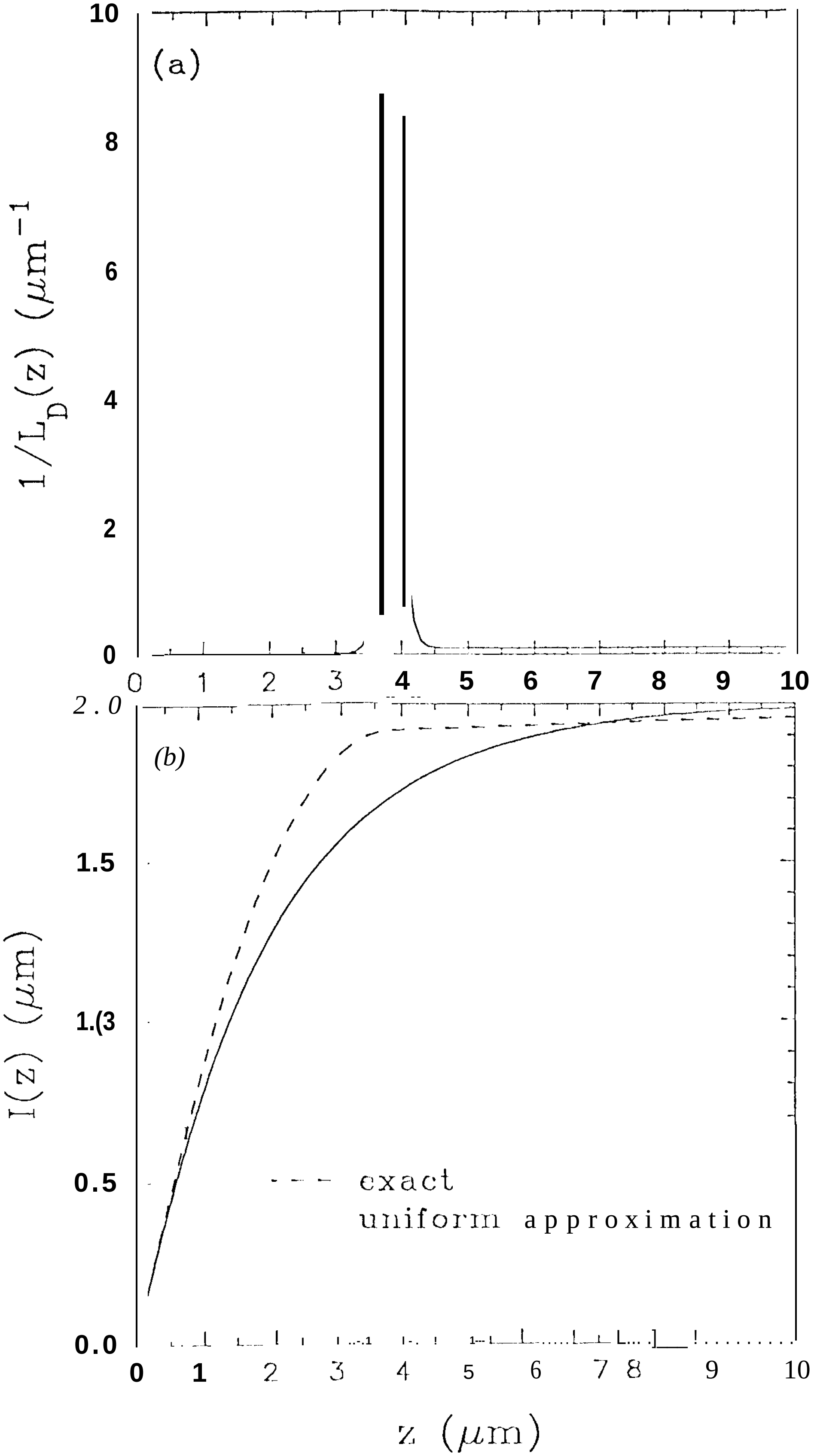

Figure 5: The assumed L_D in (a) produces the exact I in (b), which is compared to the uniform approximation in (b).

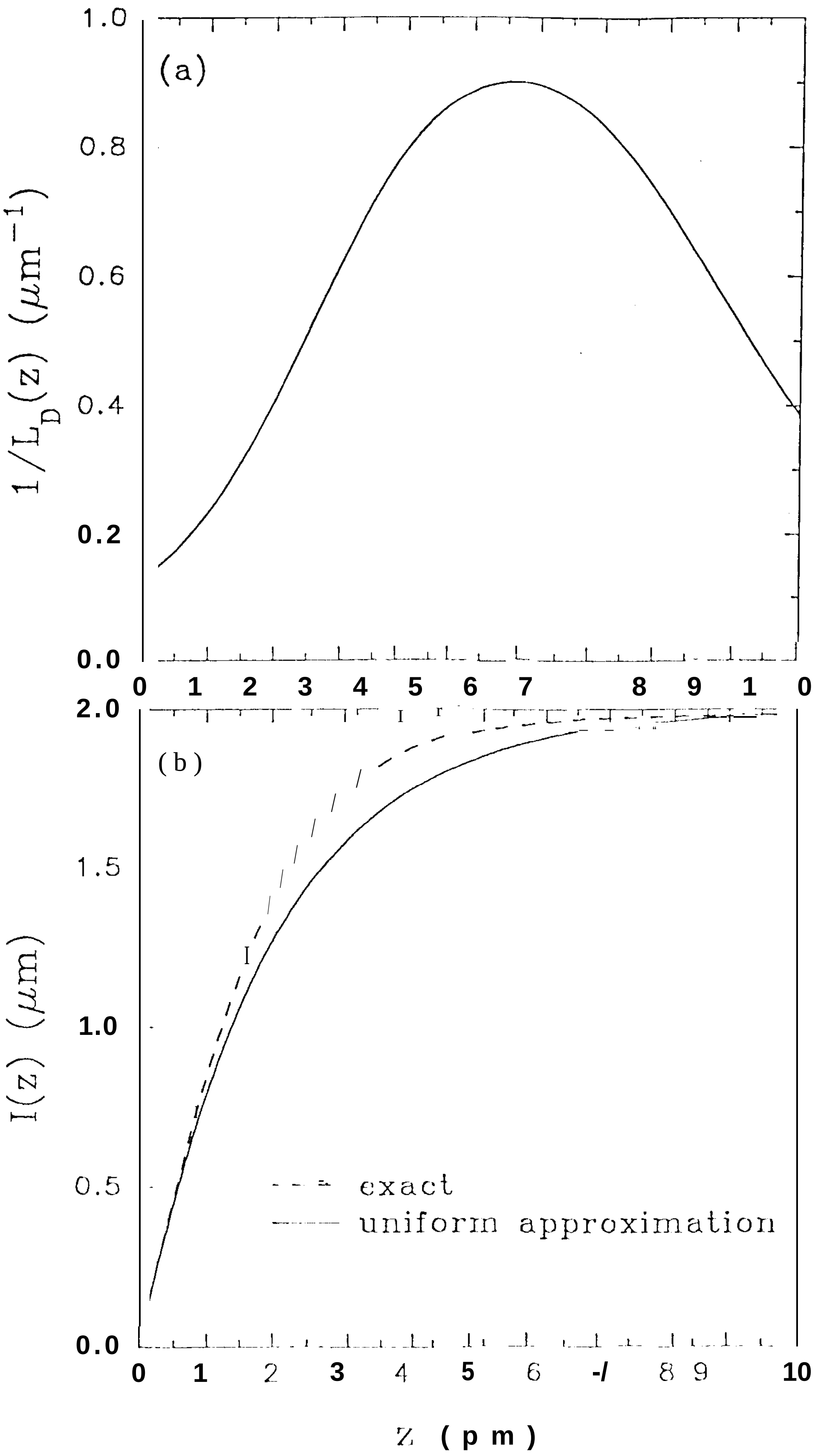

Figure 6: The assumed L_D in (a) produces the exact I in (b), which is compared to the uniform approximation in (b).

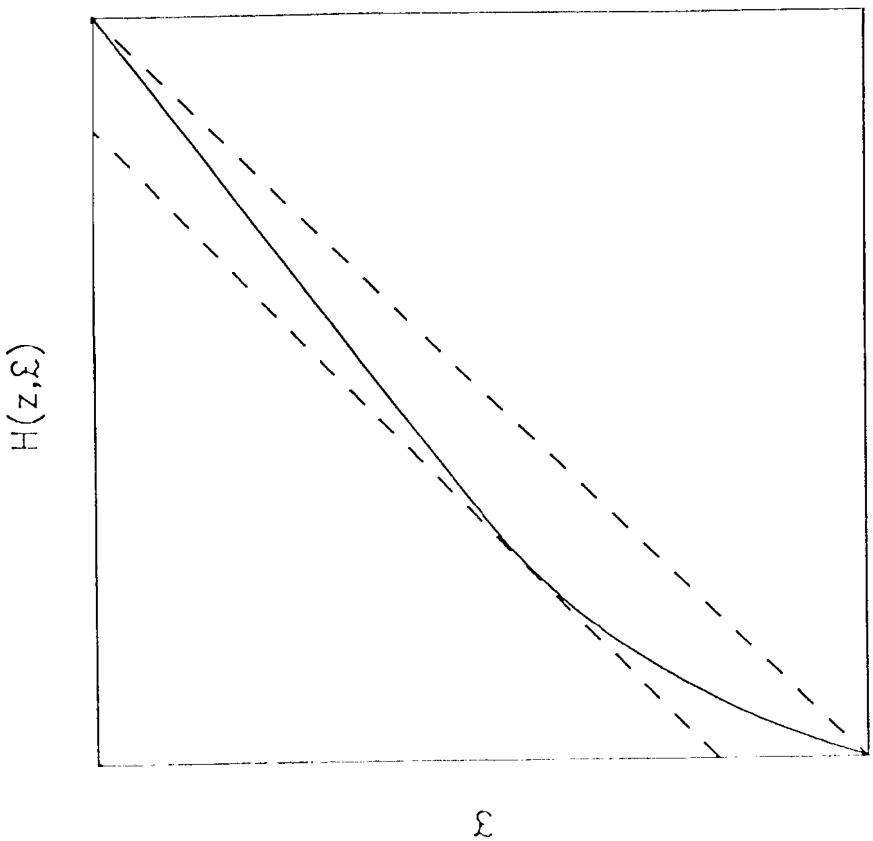

Figure A1: illustration of the curvature of $H(z,\zeta)$ when plotted against ζ with z fixed. The H curve (solid) is bounded above by the chord connecting the end points (upper dashed line), and bounded below by any tangent line, such as the lower dashed line shown,