The text on the page is not legible due to the quality of the image.
The only influence in the calculation being above the two operation operations with the parameters (\(\sigma_f, \sigma_s\)) of

\[ \text{Eqn. 1} \]

\[ \frac{\partial V}{\partial \sigma_f} \frac{\partial V}{\partial \sigma_s} \]

is to note that the two operations defined above the two operation operations with the parameters (\(\sigma_f, \sigma_s\)) of

\[ \text{Eqn. 2} \]

\[ \frac{\partial V}{\partial \sigma_f} \frac{\partial V}{\partial \sigma_s} \]

is to note that the two operations defined above the two operation operations with the parameters (\(\sigma_f, \sigma_s\)) of

\[ \text{Eqn. 3} \]

\[ \frac{\partial V}{\partial \sigma_f} \frac{\partial V}{\partial \sigma_s} \]

is to note that the two operations defined above the two operation operations with the parameters (\(\sigma_f, \sigma_s\)) of

\[ \text{Eqn. 4} \]

\[ \frac{\partial V}{\partial \sigma_f} \frac{\partial V}{\partial \sigma_s} \]

is to note that the two operations defined above the two operation operations with the parameters (\(\sigma_f, \sigma_s\)) of

\[ \text{Eqn. 5} \]

\[ \frac{\partial V}{\partial \sigma_f} \frac{\partial V}{\partial \sigma_s} \]

is to note that the two operations defined above the two operation operations with the parameters (\(\sigma_f, \sigma_s\)) of

\[ \text{Eqn. 6} \]

\[ \frac{\partial V}{\partial \sigma_f} \frac{\partial V}{\partial \sigma_s} \]

is to note that the two operations defined above the two operation operations with the parameters (\(\sigma_f, \sigma_s\)) of

\[ \text{Eqn. 7} \]

\[ \frac{\partial V}{\partial \sigma_f} \frac{\partial V}{\partial \sigma_s} \]

is to note that the two operations defined above the two operation operations with the parameters (\(\sigma_f, \sigma_s\)) of

\[ \text{Eqn. 8} \]

\[ \frac{\partial V}{\partial \sigma_f} \frac{\partial V}{\partial \sigma_s} \]

is to note that the two operations defined above the two operation operations with the parameters (\(\sigma_f, \sigma_s\)) of

\[ \text{Eqn. 9} \]

\[ \frac{\partial V}{\partial \sigma_f} \frac{\partial V}{\partial \sigma_s} \]

is to note that the two operations defined above the two operation operations with the parameters (\(\sigma_f, \sigma_s\)) of

\[ \text{Eqn. 10} \]

\[ \frac{\partial V}{\partial \sigma_f} \frac{\partial V}{\partial \sigma_s} \]
2.3. Tracking Minimizer

I implemented a variant of the Levenberg-Marquardt minimization algorithm. The minimizer computes and tracks the varying region of integration as the parameters of minimization change. The returned function values to the minimizer are normalized to the area of integration to perform a meaningful minimization.

The parameters cannot be allowed to change arbitrarily as the minimizer will shift the center of rotation out of the surface figure difference images with large rotation angles to get zero overlap and zero minimized value. A dynamic bound checking algorithm is linked to the minimizer to prevent it from "running out of the picture".

This method works very well and it is very fast. It takes a few minutes on a Sun Sparc 20 work station with minimal memory requirements to find the center and the two angles of rotation.

Fig. 1 and Fig. 2 show two double differenced surface figure error images (DS₁ and DS₂) with two different angles of rotation as measured by the surface metrology gauge in a particular set of data runs.

Fig. 3 shows a particular representation of the minimizer output. This is a plot of "goodness of fit" as a function of the starting coordinates of the center of rotation. The goodness of fit is zero for the absolute best fit in the absence of noise in the measurements. It is a minimum at the best fit values of the parameters. A four dimensional minimization was performed starting from the coordinates given and the goodness of fit thus obtained was plotted at these coordinates. The initial coordinates are used instead of the final coordinates of the minimum as the minimum can be reached from many values of the starting coordinates in general.

The true rotation center appears as the dip near the center of the figure. The only other possible fits place the starting values for the center of rotation coordinates very near the edges of the figure with very bad overlap. These are eliminated by comparing them against the marked initial images of the positions of the rotating mirror.

In this particular run, the minimizer gave the values: 157°, 118° as the center of rotation and -0.600 radians and 1.182 radians as the angles of rotation. Initially, the camera was adjusted to bring the center of rotation near the center of the image at (160,120). The crudely measured angles of rotation were -0.622 radians and 1.182 radians.

Fig. 4 shows the computed center of rotation in the field of view of the camera on the derived background picture. This picture shows the imperfections in the surface gauge initial beam independent of the test and the reference mirrors. The images are compensated by this background image before the surface figure differences are computed. A new background picture is generated every time a new data run is performed to prevent any changes.
Figure 2: (Goodness of fit) versus the moving center of rotation coordinates.

Figure 3: (Goodness of fit) versus the moving center of rotation coordinates.

Rotation Model (Test)

10/14/95 2:08
2.4. Separation of Figure Errors

Once the center and the angles of rotation are known, the surface figures can be solved using any pair of the equations $S_0 = R_1 T_1 R_f$, $S_1 = R_1 T_1 R_f$ and $S_2 = R_1 T_1 R_f$. However, any straightforward attempt to (10 S1) is very likely to result in many singularities that prevent these from being used directly in a linear equation solver like Gauss-Jordan elimination. One of the reasons for this is the small singular neighborhood around the center of rotation. Another reason is the existence of closed loops in the equations due to integer pixel locations and limited precision pixel values.

In fact, these closed loops can be taken advantage of by first solving for a small subset of points using all three (and more using another dataset) equations. The rest of the points which do not lie on a closed loop are solved subsequently. This part of the algorithm is currently being implemented. The results of this and the translation deconvolution will be reported in a subsequent paper.

3. ABSOLUTE METROLOGY GAUGE

The detailed operation of the absolute metrology gauge is described in a previous paper. In what follows, I will describe the measurements performed using this gauge. Currently, the gauge operates in still air. The speed of light in vacuum is used to compute the absolute length. This causes a small systematic error in the lengths themselves, but it does not present a problem as the gauge repeatability is within our specifications. In space, the gauge operates in vacuum eliminating this systematic error altogether. The demonstration will be moved into a vacuum envelope during the coming year.

3.1. Reference Cavity Length Measurements

Fig. 5 shows the optically contacted, high finesse, ULE reference cavity before it was installed in the cavity oven assembly. The vertical dark shade in the middle is produced by the small vent hole drilled into the body of the cavity. The ruler is marked in inches.
Figure 5: The reference cavity

Figure 6: The cavity oven
Figure 7: The reference cavity length during a day

**Absolute Metrology (05/16/95)**

Two lasers Heating, both locked

1. Fig. 6 shows the cavity oven assembly with its cover removed. The cavity is suspended inside the copper canister at the top. One of the heater pads on the canister is visible. The optical circulator, the MOO00001 (11") assembly and the matching lens holder occupy the middle section. The bottom section houses the five axis fiberlauncher assembly. A six inch ruler is resting along the edge of the 4" inner diameter bottom conflat flange.

Fig. 7 shows measurements of the reference cavity length at every 1.7 minutes (averaging time) for a period of nearly 20 hours. The cavity length is stable to 14 nm rms.

Fig. 8 shows measurements of the reference cavity length at every 15 minutes for 120 minutes in a quieter environment. The cavity length is stable to 3 nm rms.

3.2 Simultaneous Cavity Length and Absolute Distance Measurements

Fig. 9 shows measurements of the reference cavity length for three consecutive days. The lasers were maintaining lock during this time. The cavity length is stable to 18 nm rms.

Fig. 10 shows measurements of the absolute distance for three consecutive days. The measured length was servoed to constant using another relative metrology gauge between the same corner cubes. The distance is stable 18 microns rms. The actual distance measured was verified with a ruler to an accuracy of few millimeters.

Fig. 11 shows measurements of the absolute distance one after another with improved laser stabilization and absolute gauge code. The measured length was held constant using another relative metrology gauge between the same corner cubes. The distance measurement is repeatable to 2 microns rms.

New data indicate a repeatability down to 1 micron rms over the same absolute distance. These results together with a calibration tracking run (changing distance tracked by the absolute gauge) and absolute gauge vacuum results will be reported in a subsequent paper.

4. 3-D Metrology Gauge

The completed 3 dimensional metrology gauge is presented in the figures Fig. 12, Fig. 13, and Fig. 14. Five linear metrology heads with built in dithering are mounted on a 2" by 2" super invar breadboard. These heads monitor the distance between their internal corner cubes and one external corner cube. The external corner cube is mounted
Figure 8: The decrease cavity length for two hours
Figure 10: The absolute distance for three days

**Absolute Metrology (07/13/1995)**

Absolute length measurement

Figure 11: Improved repeatability

**Absolute Metrology (01/31/1995)**

Distance between corner cubes
on a five degree of freedom stage to simulate the angles of incidence encountered under realistic conditions. The breadboard acts as a thermally stable reference surface that holds the distances between the measurement heads constant.

The design uses the previously developed\(^1\) \(^2\) linear relative metrology gauge. The improvements to the linear gauge include an all-fiber distribution system and built-in dithering on a thermally stable base. The modulation system is the same as the one used in the absolute metrology gauge. The entire gauge once again is constructed on a seismically isolated optical breadboard inside the four feet vacuum chamber.

The gauge is driven by one of the stabilized lasers that also functions in the absolute gauge. The fibers bring the heterodyne beams to the 3-dimensional gauge base table. A 2 by 5 fiber splitter generates all the laser beams needed for the five heads.

The metrology launching head consists of two fiber collimators, a polarizing beam combiner, a beam launcher cube, a reference corner cube, reference and unknown photodetector circuits and the dithering piezo electric tip-tilt stage. Five heads are arranged in such a way that all of their beams intersect at the corner of the measurement corner cube.

The measurement corner cube is mounted on a piezo electric flexure mount approximately 33 inches away from the super mirror table. This mount is used to hold the distance between the central head and the measurement corner cube constant. It is also used to change this distance predictably and accurately.

The initial tests of the system consist of measuring the ('10S' (1 loop performance of the measurement corner cube servo system and determining the tracking error between any pair of heads in air, without seismic isolation) a current damper is self-interference cancellation to get a baseline set of data before moving it into the vacuum envelope.

Fig. 15 shows one-second averages of the servo error signal of the measurement corner cube servo system under the conditions described above. The residual jitter is about 39 pprms. The unity gain oscillation frequency of this servo is 444 Hz. The servo error signal is sampled at 4096 Hz and feedback is applied through a simple integrator.

Fig. 16 shows the tracking error between two heads when the measurement corner cube is moved in a rapid cycle to cancel the effects of air turbulence and ground vibrations. The servo is made to oscillate at its unity-gain frequency. The peak-to-peak amplitudes of the motions detected at each head at every half cycle of the oscillation is compared to each other. The ratio of these peak-to-peak amplitudes is constant and it is a function of the angle.
Figure 15: The 3-dimensional gauge servo error signal

![Graph](Graph1.png)

Figure 16: Tracking error between two heads

![Graph](Graph2.png)
between the heads. This particular data set indicated an angle of 17.3 degrees ± 1 degree between these two heads. A crude measurement using a ruler gave an approximate angle of 20 degrees with an error of ± 2 degrees between the same heads. The difference between the motions detected at these heads after accounting for the derived angle difference is about 8 um rms.

4.1. Corner cube induced aberrations

Fig. 17 shows a 10mm visual diameter, nearly Gaussian laser beam with a wavelength of 6331111 impinging on a 2° clean aperture, few arc-seconds orthogonality,  λ/10 each surface figure, open-faced corner cube.

Fig. 18 shows the returned beam from the same corner cube nearly 1.5 meters away. The beam looks like anything but a Gaussian. In the 3-dimensional metrology gauge, an output beam, after being reflected by two corner cubes, is made to interfere with a beam that looks like the input beam as shown in Fig. 17. When the input beam moves to dither the deep diffraction pattern moves with it potentially causing large phase errors. The 3 dimensional gauge will solve for and eliminate these types of systematic problems as well.

The in-vacuum results from this gauge with dithering will be reported in the subsequent paper.

5. SUMMARY

The surface gauge absolute calibration is in progress. The deconvolution of the rotation from the surface figure difference images is complete. The solution for the separated surface figures is being implemented. These results and the translation deconvolution will be reported in a subsequent paper.

The absolute gauge has reached a cavity length measurement accuracy of 3 um rms for a cavity length of nearly 5 cm. The absolute distance repeatability is down to 2 micron rms for a one-way distance of nearly 1 meter. New data indicate down to 1 micron repeatability in air. The calibration of this gauge by tracking a changing distance and the gauge operation in vacuum will be reported in the subsequent paper.

The construction of the 3-dimensional metrology gauge is complete. The gauge is being tested in air before it is placed in vacuum. The results from this gauge will be presented in the subsequent paper.
6. ACKNOWLEDGEMENTS

I would like to thank M. Shao and J. Yu for many fruitful discussions. I would also like to thank D. Moore for his design of the mechanical structure of gauge heads. The research described was performed at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

7. REFERENCES


