Accuracy of the Residual Delay Absolute Phase Algorithm

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Abstract

An absolute phase algorithm is used in an interferometric radar mapper to determine the unknown constant number of cycles of phase difference after phase unwrapping, and thus the absolute topography. Using a model and a simulation I compute the accuracy of one such algorithm and compare it to the results for an actual system, The algorithm's accuracy for other systems is predicted.

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Introduction

Interferometric synthetic aperture radar (IFSAR) [5] has become a useful technology for producing high-quality topographic maps. Three measurements are required to reconstruct the three-dimensional coordinate of a point in an IFSAR image: the range, azimuth and elevation. The first is obtained by timing the return of the radar pulse, the second by measuring the doppler shift, and the last by measuring the phase difference between the signals received at two displaced antennae. For the elevation coordinate, one may write that the signal in channel 2 is related to that in channel 1 by a time delay and a phase shift:

$$s_2(t) = s_1(t - \Delta t)e^{2\pi i f_0 (t - \Delta t)} \tag{1}$$

where the s_i are the signals received in the two channels and $2\pi f_0 \Delta t$ is the phase difference between the two signals for a carrier frequency, f_0, and a corresponding time difference, Δt. This time difference is given by the interferometric baseline, $\mathbf{B}$, and the look direction, $\hat{\mathbf{n}}$: $c\Delta t \approx \hat{\mathbf{n}} \cdot \mathbf{B}$, where c is the speed of light and assuming the pulse is transmitted from one antenna and received simultaneously in both. For typical radar parameters, $|f_0 \Delta t| > 1$, so that phase difference between the two channels "wraps" and is only observed modulo 27r. After phase unwrapping [2] one obtains the relative phase difference between each point in the scene, so that the phase is known up to a number of cycles which inconstant for the entire image. This constant is determined by an "Absolute Phase" algorithm. One may use at least one known height in the scene, a "tie-point", to determine this constant. Alternatively, two algorithms proposed by Madsen [4] allow the estimation of this constant without recourse to tie-points. I evaluate the accuracy of one of these, the "Residual Delay" algorithm. Of the two algorithms proposed, this one has the advantage that it requires no alteration of the standard SAR-processing, and has thus become the absolute phase algorithm of choice for IFSAR processing.

The Residual Delay algorithm (RDA) employs the unwrapped interferogram to resample one of the signals to the other, removing the time shift except for a residual delay of $\lfloor f_0 \Delta t \rfloor / f_0$, which is constant over the entire image. ($\lfloor x \rfloor$ symbolizes the truncation operation on x.) This residual delay may be estimated by cross-correlating the two signals. The resampling step is accomplished with a 16-point Hanning interpolation filter [1] [3] with a precision of 8192. The location of the cross-correlation peak of tile two signals is obtained by the following procedure: 1) Divide the images into sub-patches of 32 x 32 samples each, 2) oversample each sub-patch by a factor of two, 3) FFT each signal, complex-multiply their spectra, and inverse FFT, 4) select a 16×16 sample region around the resulting correlation peak, and 5) oversample this peak by a factor of 16 to obtain an estimate of the delay to a precision of 1/32 bin. The residual delay is then computed by averaging over the estimates for each sub-patch. in this way, the delay of 0(1/100) bin maybe obtained; the accuracy with which this can be accomplished is the subject of this paper.

Model

The fraction of a range sample to which one must measure the delay due to one cycle of phase is given by the ratio of the sampling frequency, f_s, the the carrier frequency, f_0. The uncertainty with which this delay can be determined is reduced by the number of samples, N, over which the estimate is obtained, and also depends on the width of the correlation peak. One may model the accuracy of the residual delay algorithm, in cycles, as:

$$\sigma = \frac{f_0}{f_s} \frac{1}{\sqrt{N}} h(\gamma) \tag{2}$$

where γ is the total correlation Of the tWO signals, h is a function to be determined by simulation, and we nave assumed Gaussian statistics for the delay estimation errors- an assumption which must be verified by experiment. Note that this model does not account for phase-unwrapping errors, nor for systematic phase errors between the two signals such as those due to multipath and poor switch isolation.

In order to compute the accuracy of the RDA, I employed a simulation to generate controlled sample data sets to which the RDA was applied. The simulation consisted of the followings steps: 1) generate a simulated pair of SAR-processed radar images, 2) apply the RDA to determine the absolute phase, as detailed in the Introduction, 3) compute the mean error in absolute phase over the image and 4) repeat the first three steps many times for each set of simulation parameters to determine the standard deviation of the mean error, whit.1) is interpreted as the accuracy of the algorithm. For the first step, noise was added to the signal for cad] channel, $s_{jk} = (u_{jk} + iv_{jk})/\sqrt{2}$, to generate the required decorrelation:

$$n_{jk} = \frac{1}{\sqrt{2}} \sqrt{\gamma^{-1} - 1}(u'_{jk} + iv'_{jk}) \tag{3}$$

where u aud v are Gaussian random variables with zero mean and variance of 1, independent for each sample $\{jk\}$.

Results

The simulation was run with several parameter sets of differing f_s/f_0 and N values. The best fit to all of the simulation results (Figure 1) was obtained with a quadratic dependence on the correlation:

$_h = \sum_{n=0}^{2} a_n(1-\gamma)^n$, where $a_0 = 0.5723$ cycles, $a_1 = 8.3703$ cycles, and $a_2 = 38.5,589$ cycles. (This only applies over values of $\gamma \in [0.85, 1.0]$. For lower values of -y, phase unwrapping errors become significant, and the algorithm will fail if not suitably modified to handle these errors.) In order to verify this result, I compared the model prediction for the RDA accuracy to that obtained in the processing of JPL TOPSAR [6] data ($f_0 = 5.3$ GHz, $f_s = 45$ MHz). For each pate.11 in a given data take, the TOPSAR processor records the average correlation as well as a series of RDA absolute phase estimates over sub-patches of 1024 x 256 samples distributed both across range and along-track. One of the comparisons used to verify the simulation result is shown in Figure 2. This is for a scene which contains relatively rugged terrain, as well as urban areas and a region over the ocean towards the end of the run. The laid-over slopes in the rugged terrain reduce the average correlation of the scene, and the SNR is worse over the ocean. The latter effect is evident in the figure as a worsening in the accuracy both predicted by the model and realized in the data. '1'here arc differences between the two arising both from the systematic phase variations in the TOPSAR system not accounted for by the model and from using only the average correlation for each patch to drive the model. Nevertheless, the agreement between the two gives one confidence in predicting the accuracy for other systems.

Discussion

The distribution of the mean error from the simulation is observed to be Gaussian, justifying the assumption of the model. Therefore one may use the model value for the accuracy to compute the confidence level that a given measurement of the absolute phase will get the right answer, i.e., the correct value to within ± 0.5 cycles:

$$CL(\%) = \int_{-0.5}^{0.5} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}} \, dy = \mathrm{erf}\left[\frac{0.5}{\sigma\sqrt{2}}\right] \tag{4}$$

In Figure 3 the accuracy scaling coefficient, $\frac{f_0}{f_s}\frac{1}{\sqrt{N}}$, is plotted versus decorrelation for several error rates (1 - CL) as an aid to determining whether the RDA will be successful for a particular IFSAR system. Five example systems arc noted on this plot, showing the predicted RDA performance both for current and for proposed systems. (See the figure caption for more detail.) It is clear from the figure that it is desirable to achieve

$$\frac{f_0}{f_s}\frac{1}{\sqrt{N}} < 0“, \tag{5}$$

in order to ensure that the RDA performs accurately. The SRTM mission does not appear to meet this criterion, nor does the C-band TOPSAR system. however the L-band TOPSAR system dots, implying that some sort of absolute phase "bootstrapping" of the C-band system by the L-band system may be possible. Finally, the RDA should perform very accurately for the proposed High Resolution Global Topography Mapper.

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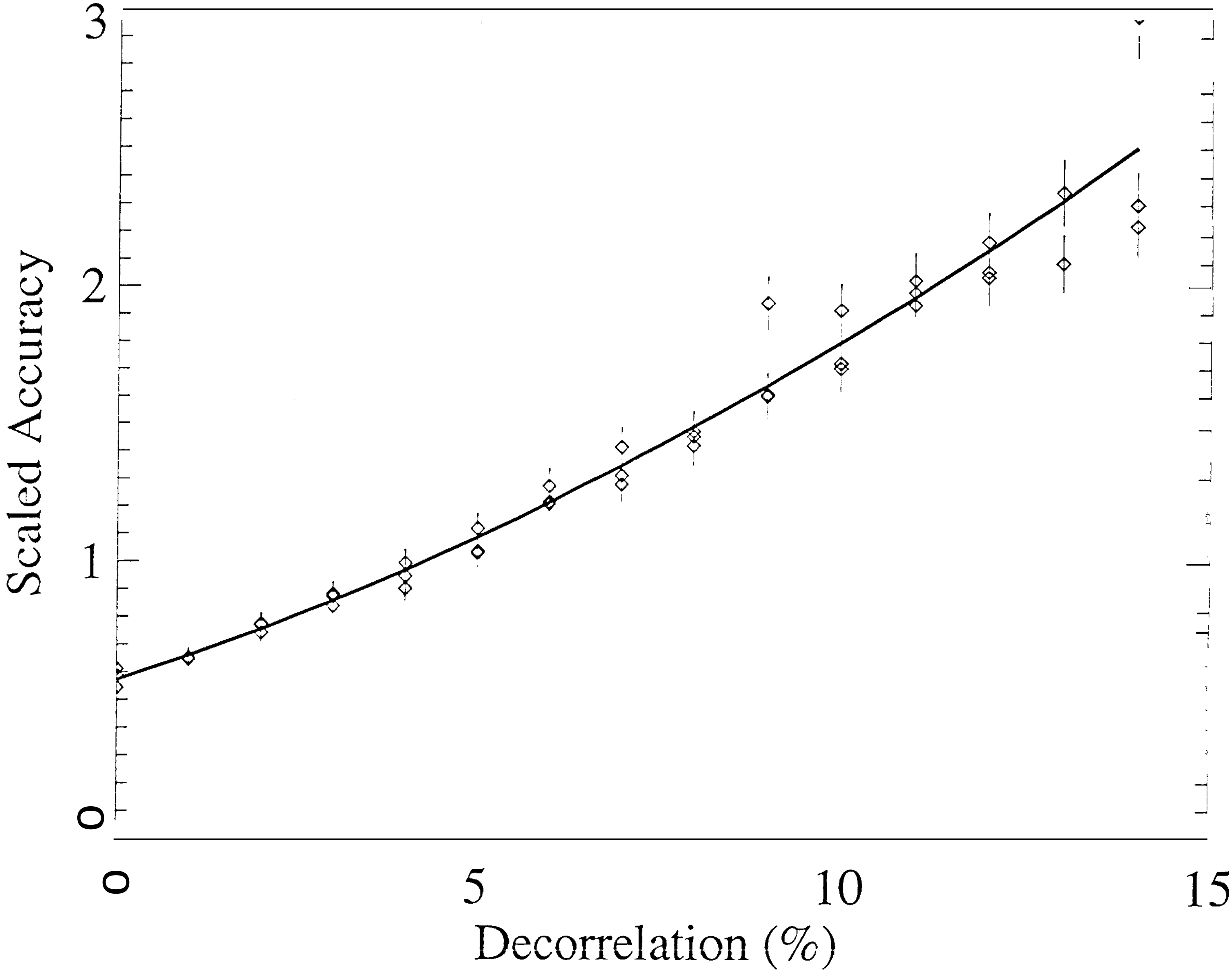
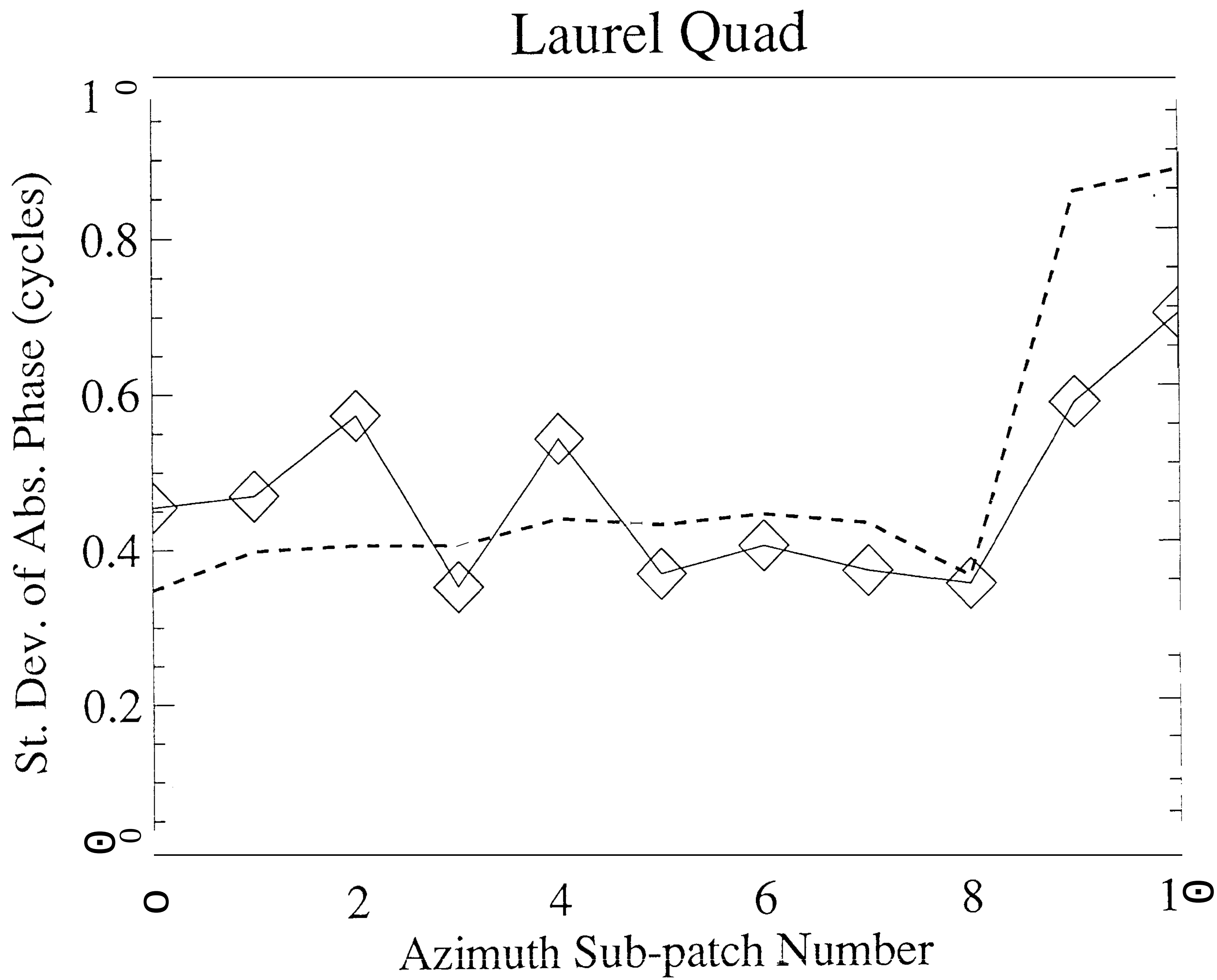
Figure Captions

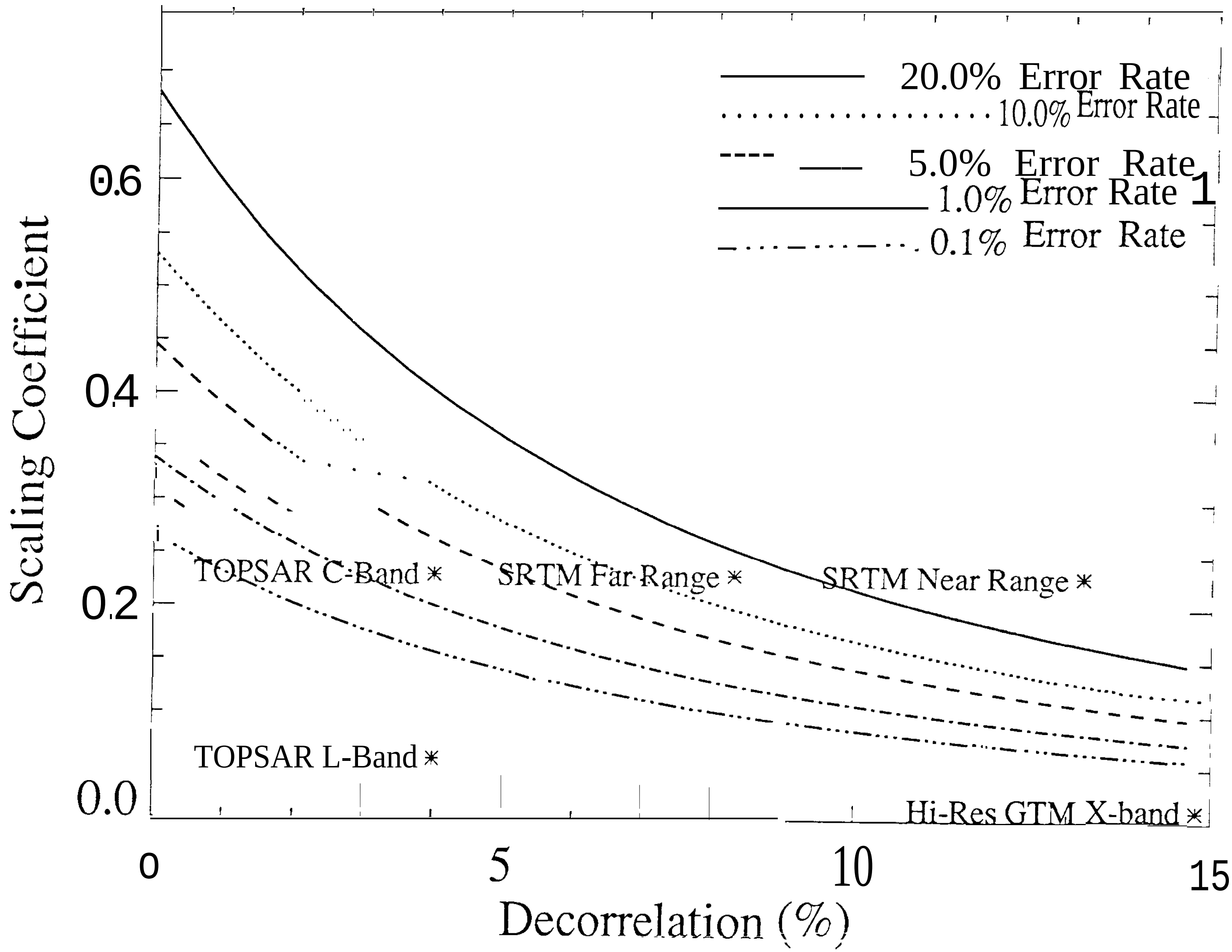
1. Scaled accuracies $f_s \sigma \sqrt{N}/f_0$ of the RDA as a function of decorrelation for simulated data sets. The solid line is a fit to the data using a quadratic in the decorrelation.

2. Comparison of the measured accuracy (diamonds) of the absolute phase algorithm to the predicted accuracy (dashed line) for the Laurel Quad scene. Note the transition around azimuth sub-patch 8 from land to ocean.

3. RDA accuracy scaling coefficients, $f_0/(f_s\sqrt{N})$, required to obtain selected error rates as a function of decorrelation. Five parameter sets have been noted on this plot: TOPSAR (f_s = 45 MHz, $\sqrt{N}$ = 512) L-band (f_0 = 1.3 GHz) and C-band (f_0 = 5.3 GHz); the High-Resolution Global Topography Mapper (f_0 = 10.0 GHz, f_s = 900 MHz, $\sqrt{N}$ = 1024); and SRTM (f_0 = 5.3 GHz, f_s = 11.25 MHz, $\sqrt{N}$ = 2048) Near Range Swath (γ = 87%) and Far Range Swath (γ = 92%). Only the SRTM values of the decorrelation are purposeful here; the other systems may have decorrelations very different from those where they are placed on this figure. Of course, the correlation depends upon the characteristics of each scene.


Scaled Accuracy
0
1
2
3
0
5
10
15
Decorrelation (%)


Laurel Quad
St. Dev. of Abs. Phase (cycles)
1 0
0.8
0.6
0.4
0.2
0 0
0
2
4
6
8
10
Azimuth Sub-patch Number


20.0% Error Rate
10.0% Error Rate
5.0% Error Rate
1.0% Error Rate
0.1% Error Rate
Scaling Coefficient
0.6
0.4
0.2
0.0
TOPSAR C-Band
SRTM Far Range
SRTM Near Range
TOPSAR L-Band
Hi-Res GTM X-band
0
5
10
15
Decorrelation (%)