Precise Measurements of the Density and Critical Phenomena near the Phase Transitions in Helium using High-Q Niobium Microwave Cavities*

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The experimental approach of using high-Q niobium microwave resonators (Q~10^8) and high resolution thermometry to achieve precise measurements of the density and critical phenomena in liquid helium near phase transitions is described. The numerical verifications of the experimental feasibility as well as the applications of the precision density measurement techniques are discussed.

1. INTRODUCTION

The Lambda Point Experiment (LPE) recently conducted in space[1] has demonstrated the importance of microgravity and the high-resolution thermometry (HRT) in advancing fundamental knowledge of continuous phase transitions and in confirming the renormalization group (RG) theory [2]. In this work, we report a new experimental approach which can yield state-of-the-art density and critical phenomena measurements at the phase transitions of helium. The principle of this approach is to utilize the HRT technique and the capability of high-resolution and stability in the resonance frequencies of superconducting cavities to detect small changes in the dielectric constant of liquid helium near the lambda transition, thereby providing precision measurements for the specific heat critical exponent $\alpha$ and the amplitude coefficients.

2. THE PHYSICAL CONCEPTS FOR PRECISION DENSITY MEASUREMENTS

The derivation of precise density $\rho$ information from the dielectric constant $\varepsilon$ is based on the Clausius-Mossotti relation $(e-1)/(e+1) = (\Delta \alpha_0 \rho)/(2 |P|)$, where $M$ is the molecular weight, and the polarizability is assumed to be constant near the lambda transition. The resonant frequency shift $(\Delta f)$ of a microwave cavity containing liquid helium is related to the change in the dielectric constant of the liquid helium:

$$
\Delta f = f - f_0 = f_0 \left( \int_{0}^{\Omega_0} \frac{\varepsilon - \varepsilon_0}{\varepsilon_0 |P|} d\Omega \right) \left( \int_{0}^{\Omega_0} \frac{\varepsilon_0 |P|}{\varepsilon_0 |P|} d\Omega \right)
$$

where the integration is performed over the entire maximum $\Omega_0$ of the microwave cavity, and $P$ is the electric field of the resonant mode.

3. THE EXPERIMENTAL APPROACH

Our experimental techniques involve using high-Q niobium microwave resonators (Q~10^8) to achieve high frequency resolution (a few parts in 10^8) for precise measurements of the dielectric constant $\varepsilon$ in liquid helium near the lambda transition. The temperature resolution and stability of 10^-4 K are achieved with the use of HRT. The state-of-the-art density measurements of the liquid He near the lambda transition can provide best ever values for the critical exponent $\alpha$ and the amplitude coefficients for the thermal expansion coefficient $\beta_T$ (under constant pressure $P$ and for given reduced temperature of $(T - T_{\lambda})/T_{\lambda}$):

$$
\beta_T = -\frac{1}{\rho} \frac{\partial \rho}{\partial T} = A|T|^{-\alpha} + B
$$

In addition, obtaining more precise values of $\alpha$, $A$ and $B$, we can investigate the universality of the lambda transition by precise density measurements near $T_{\lambda}(P)$ at various pressures far away from the vapor pressure, as well as near $T_{\lambda}(X)$ for different He concentrations $(X)$ in the $^3$He-$^4$He mixtures. The pressure variations can be achieved by using the height of the liquid helium column in a capacitor lever sensor which is connected to a microwave cavity, and the pressure control and readout can be achieved by using the tunnel diode oscillator and the capacitor-inductor circuit developed by C. J. Y. Degen[3]. A precision of one part in 10^8 can be achieved for the helium height readout, and since it is well known that $T_{\lambda}(P)$ shifts downward with the height $y$ by $T_{\lambda}(Z) - T_{\lambda}(0)$, with $y = 1.273 \mu m/cm$, the 10 cm column height allows universality to be treated for temperature shifts up to 13 K. The verification of

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universality near $T_s(X)$ for different $^3$He concentrations can be achieved by fine-tuning $X$ values using the enhanced flush technique[4].

4. NUMERICAL VERIFICATIONS & APPLICATIONS OF THE TECHNIQUE

To test whether the density profile $\rho(z, T)$ can be derived from the measurements of the resonant frequency shift, we consider the microwave cavity at a constant temperature between $T_s(0)$ and $T_s(d)$, where $T_s(d) = T_s(0) + d$, and $d$ is the height of the cavity. Both the density and the dielectric constant are functions of the position, as shown in Fig. 1. We assume that the expression fitted to the $\rho(T)$ data by Donnelly et al.[5] can be interpolated into the sub-microkelvin region, and use them to generate the corresponding $\varepsilon(T)$ and $\left[\Delta f(T)/f_0\right]$, where $f_0$ denotes the resonant frequency at $T_s(T)$. These “experimental values” of $\left[\Delta f(T)/f_0\right]$ are used to deconvolute the density profile $\rho(z, T)$, and the results are compared with the original curves fitted to $\rho(T)$ by Donnelly et al., for verification of the feasibility of our approach. The representative results from a microwave cavity of 1.0 cm height, temperature stability of $\delta T/T \sim 10^{-5}$ and frequency resolution of $\delta f/f_0 \sim 10^{-12}$, are shown in Figs. 2(a) and 2(b) for Earth-bound measurements and for a microgravity environment, respectively. We note that, although our frequency resolution can be up to $\delta f/f_0 \sim 10^{-16}$, the density resolution is limited by the temperature stability $\delta T/T \sim 10^{-5}$. As the HHT techniques advance further, higher frequency resolutions will become relevant. In addition, we note that the critical exponent $\nu$ and the amplitude coefficients can be determined to unprecedented precision, approximately to one part in $10^4$.

Another application of our technique is the investigation of the gravity-induced superfluid/normal fluid interface. For a given temperature between $T_s(0)$ and $T_s(d)$, the interface thickness $\delta(T)$ (where the correlation length is given by $\xi(T) \sim \xi_0(\nu)$) is approximately equal to

$$\xi_0 \delta(T) = \xi_0 \left[\frac{T_s(0)}{T_s(0) - T_s(d)}\right]^{1 - \nu} \left[\frac{T_s(d)}{T_s(0)}\right]^{1 - \nu} \left[\frac{T_s(0) - T_s(d)}{T_s(0)}\right].$$

Consequently, by measuring the abrupt changes in the $\Delta f/f_0$ as the interface enters the microwave cavity from the bottom, we can determine the thickness of the interface regime as a function of the temperature, thereby estimating the critical exponent $\nu$. Since, for a three-dimensional system, the scaling relation $\nu = 2 - d$ holds, the accuracy of the critical exponent $\nu$ can be independently verified via the measurement of $\nu$.

Finally, our microwave techniques can also be used to study the losses associated with the critical fluctuations by measuring the $Q$-values of the cavity under the presence of ultrasonic waves on liquid helium. The losses due to critical fluctuations can provide direct information for the dynamic exponent $\nu$ near the phase transitions of the $^3$He-$^4$He mixtures.

References