Postglacial Rebound From VLBI Geodesy: On Establishing Vertical Reference

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Abstract. Difficulty in establishing a reference frame fixed to the earth's interior complicates the measurement of the vertical (radial) motions of the surface, I propose that a useful reference frame for vertical motions is that found by minimizing differences between vertical motions observed with VLBI [Ma and Ryan, 1995] and predictions from postglacial rebound predictions [Peltier, 1995]. The optimal translation of the geocenter is 1.7 mm/yr toward 36”N, 111 °E when determined from the motions of 10 VLBI sites. This translation gives a better fit of observations to predictions than does the VLBI reference frame used by Ma and Ryan [1995], but the improvement is statistically insignificant. The root mean square of differences decreases 20% to 0.73 mm/yr and the correlation coefficient increases from 0.76 to 0.87. Postglacial rebound is evident in the uplift of points in Sweden and Ontario that were beneath the ancient ice sheets of Fennoscandia and Canada, and in the subsidence of points in the northeastern U. S., Germany, and Alaska that were around the periphery of the ancient ice sheets.

introduction

Elevated beaches record uplift of the earth's surface in isostatic response to the inching of the ancient ice sheets. Radiocarbon dating of such Holocene shorelines forms the basis of postglacial rebound models built upon realistic assumptions about the thickness of the elastic lithosphere, the viscosity of the mantle, and the transformation of ice sheets into ocean water. In the model of Peltier [1995], areas beneath the ancient ice sheets of Canada, Fennoscandia, and Greenland today rise at 1 cm/yr and faster (Figure 1). Belts around the periphery of the ancient ice sheets now subside at several mm/yr. Rebound is minor far from the ice sheets.

Prior studies using VLBI to investigate postglacial rebound have focused on changes in baseline length, which mainly reflect horizontal motions. James and Lambert [1993] found that 2 of 3 baselines that should lengthen or shorten at significant rates do so. Mitrovica et al. [1993] use measured baseline lengths between 10 North American sites to limit combinations of the thickness of the elastic lithosphere and the viscosity of the upper and lower mantle.

I examine not baseline lengths but the vertical (radial) components of motion. Because vertical motions generated by rebound are faster than the horizontal motions, vertical measurements should be better able to detect rebound. Difficulty in establishing a reference frame fixed to the earth's interior complicates the measurement of surface velocities [Heki, 1994]. At the heart of this study is establishing the reference frame in which to measure vertical motions.

Figure 1

Methods

Ma and Ryan [1995, model GLB 1014j] estimate the velocities among 78 radio telescope sites using VLBI data from November 1979 to July 1995. Six values are needed to define the velocity reference frame, 3 to specify the rate of translation and 3 to specify the rate of rotation. Two years ago, Ma et al. [1994] defined the translation by fixing to zero the vertical motions of Westford (Massachusetts), Richmond (Florida), and Kauai (Hawaii); they defined the rotation by setting the horizontal velocity of Westford and the rate-of-change of the azimuth from Westford to Richmond to predictions from the NNR-NUVEL1 North American plate velocity [Argus and Gordon, 1991]. Ma and Ryan [1995] define a reference frame in a sophisticated manner: they minimize differences between NUVEL-1A predictions [DeMets et al., 1994] and the horizontal velocities of Westford, Richmond, Wettzell (Germany), Tidbinbilla (Australia), and Hobart (Tasmania) while imposing no net rotation of the 5 sites.

I aim to determine vertical motions in a reference frame fixed to the earth's interior. The establishment of this frame must be indirect because there are no sites in the interior. I postulate that the interior moves with the geometric center of the surface, and estimate motion of this center by minimizing vertical components of surface velocity after adjusting for rebound: I minimize the root mean square of differences between VLBI vertical measurements [Ma and Ryan, 1995] and postglacial rebound predictions [Peltier, 1995]. This minimization defines the translation component of the reference frame. Because vertical motions are indifferent to a rotation of the interior, the rotational component of the reference frame is left unspecified. Postglacial rebound is assumed to be the sole cause of vertical motions,

I use measurements of the vertical rates of 10 radio telescope sites: Westford, Haystack (Massachusetts), Algonquin Park (Ontario), Green Bank (West Virginia), Richmond, Fairbanks (Alaska), Kauai, Onsala (Sweden), Wettzell (Germany), and Effelsberg (Germany). All 10 vertical rates are determined from observations over at least 8 years, and all have standard errors less than 0.8 mm/yr. Sites in deformation belts are generally excluded because tectonic processes—faulting and earthquake strain buildup—can cause uplift or subsidence. All but one of the sites are on plate interiors: Fairbanks, along the western margin of the North American plate, is included because VLBI measurements bound its motion relative to the plate interior to less than 3 mm/yr [Ma et al., 1994; Argus and Gordon, 1996], suggesting that tectonic processes do not cause fast vertical motions. Fort Davis (Texas) is excluded because of anomalous behavior: apparent shifts in position led Ma et al. [1994] to estimate its position every two months rather than solving for a single velocity. The measured subsidence of Fort Davis at –3.3 mm/yr [Ma and Ryan, 1995] is fit poorly by any rebound model.

Peltier [1995] determines rebound predictions from the ICE-4G model of the history of melting of the ancient ice sheets assuming an elastic lithosphere 120 km thick, an upper mantle of viscosity 10^{21} Pa s extending to 670 km depth, and a lower mantle of viscosity 2 x 10^{21} Pas extending to the core-mantle boundary.

The root mean square formulation weights the 10 vertical rates equally. Data decimation experiments [Ryan et al., 1993] and large residuals (described later) suggest that the formal errors of Ma and Ryan [1995] are unrealistically small [cf., Argus and Gordon, 1996].

Results

There exists a correlation between rebound predictions and VLBI vertical observations in the original reference frame of Ma and Ryan [1995] (Figure 2). If the measurements and predictions were everywhere equal, then all points in Figure 2 would fall along a line (solid) with slope 1 and intercept 0 mm/yr. The distribution of points begins to resemble this ideal. The root mean square of residuals is 0.92 mm/yr. The best-fitting line has slope 0.90 and intercept –0.17 mm/yr. The linear-correlation coefficient is 0.76: the probability of exceeding this value in a random sample from an uncorrelated parent population is 1 % [Bevington, 1969, Equation 7.S, Table C-3]. High correlation coefficients occur with best-fitting lines having slopes other than 1; thus, the probability of obtaining both such a high correlation coefficient and a line with the proper orientation is much less than 1%.

Figure 2

Minimizing the root mean square of differences between measurements and predictions requires a translation of all the VLBI velocities by -0.S mm/yr in x, 1.3 mm/yr in y, and 1.0 mm/yr in z, where x, y, and z define a Cartesian coordinate system with axes parallel to the geocentric vectors to 0°N, 0°E; 0°N, 90°E; and 90°N, respectively. This optimal translation amounts to a total of 1.7 mm/yr in the direction of a geocentric vector to 36°N, 111°E. By definition, the translation changes all velocities by the same amount. But the vertical components of velocity change by different amounts: -0.8 mm/yr (Richmond), -0.5 mm/yr (Green Bank), -0.4 mm/yr (Haystack, Westford), -0,2 mm/yr (Algonquin Park), +0.4 mm/yr (Kauai), +0.6 mm/yr (Effelsberg, Wettzell), + 0.7 mm/yr (Onsala), and + 0.8 mm/yr (Fairbanks).

The translation improves the agreement between VLBI vertical measurements and rebound predictions (Figure 3). Although the misfit at Onsala increases, the misfits at Wettzell, Effelsberg, Fairbanks, and Haystack decrease. The root mean square of residuals decreases 20% to 0.73 mm/yr. The best-fitting line, with slope 1.06 and intercept -0.01 mm/yr, is nearer the ideal. The correlation coefficient increases to 0.87. If a translation had not been fit, the probability of obtaining such a high correlation coefficient would be 0.190.

Figure 3

I performed Monte Carlo simulations to find the likelihood of obtaining such characteristics after fitting a translation to a random sample. No matter what the size of error assumed to generate Gaussian deviates, there is less than 0.5% probability of obtaining the following: (1) a correlation coefficient formally significant at the 0.1 % risk level, (2) a best-fitting line with slope and intercept differing by less than 0.2 from the ideal, (3) and a root mean square of residuals less than 0.8 mm/yr.

The decrease in misfit is insignificant: an F-ratio test indicates that adding 3 parameters has a 20% chance of decreasing misfit by more than that obtained.

Residuals are large relative to errors determined using linear propagation of the errors of Ma and Ryan [1995]. Seven data are misfit by more than the standard error, and 4 are misfit by more than 2σ (Figure 3). The normalized standard deviation is 4.0. Although the root mean square of residuals is just 0.73 mm/yr, the. standard errors are minuscule, with the median standard error being just 0.27 mm/yr. Either the errors are underestimated, errors in the rebound predictions are significant, or other phenomena [Kukal, 1990] generate vertical motion. Errors in atmosphere delay models could bias the vertical measurements, or water table changes could alter surface levels. Incorporating a systematic error of 0.65 mm/yr would be sufficient to increase the normalized standard deviation to 1; standard errors would increase to between 0,7 mm/yr (Westford) and 0.8 mm/yr (Effelsberg).

Interpretation

Postglacial rebound is evident in the measured uplift of Onsala (+2.7 ±0.2 mm/yr) and Algonquin Park (+1.6 ±0.5 mm/yr), which were beneath the ancient ice sheets of Fennoscandia and Canada, respectively. (Uncertainties quoted are 1σ errors determined from linear propagation of the errors of Ma and Ryan [1995].) The isostatic response of the lithosphere in response to ice sheet melting is also evident in the measured subsidence of sites that were around the periphery of the ancient ice sheets: Green Bank (–2.6 ±0.4 mm/yr), Westford (-1.2 ±0.2 mm/yr), Haystack (–0.7 ±0.4 mm/yr), Effelsberg (–1.5 ±0.5 mm/yr), Wettzell (–1. 0 ±0.2 mm/yr), and Fairbanks (–0.9 ±0.2 mm/yr). The minor vertical motions measured at Richmond (-0.9 ±0.3 mm/yr) and Kauai (-0.2 ±0.2 mm/yr) support the premise that rebound is minor far from the ice sheets. The small difference between the measured subsidence of Haystack and Westford, which are just 1.2 km apart, must reflect measurement error or anomalous local motion.

Residuals with respect to the rebound predictions are: +1.4 mm/yr (Onsala), +1.0 mm/yr (Haystack), -0.9 mm/yr (Effelsberg), -0.8 mm/yr (Wettzell), –0.5 mm/yr (Green Bank), -0.5 mm/yr (Richmond), +0.5 mm/yr (Westford), -0.2 mm/yr (Algonquin Park), –0.2 mm/yr (Fairbanks), –O. 1 mm/yr (Kauai). Three of the 4 largest residuals involve European sites, and could point to modification of the rebound model. One possibility is that the isolines of equal vertical motion are, near Denmark, farther from the ice sheet center than in the model. The gradient in vertical rates at Onsala is steep; movement of the isolines away from the ice sheet by 100 km is sufficient to fit the measured uplift of Onsala. Outward movement of the isolines also improves the fit of Effelsberg and Wettzell, placing them nearer the axis of maximum subsidence in the peripheral belt. On the other hand, radiocarbon dating of ancient shorelines around the Baltic and North Seas provides precise measurements of Holocene uplift rates there [Pirazzoli, 1991; Tushingham and Peltier, Appendix B, 1991]. Although the European misfits may result from measurement error or other phenomena, they stimulate interest in the uncertainties in rebound predictions. For example, can both the Holocene and VLBI observations be fit by a model with assumptions different than Peltier's [1995] about the history of ice sheet melting, the thickness of the elastic lithosphere, or the viscosity of the lower and upper mantle?

Conclusion

The reference frame definition presented in this article depends only on the vertical measurements and rebound predictions. It is not subject to biases arising when horizontal velocities are set equal to NUVEL-1A [DeMets et al., 1994] predictions if there are in fact differences between the 3-million-year average and current velocities. The definition is subject to errors in the rebound model.

The Lageos and Global Positioning System satellites rotate about the earth's center of mass, providing a potential measure of the motion of earth's interior. Thus, satellite laser ranging and GPS geodesy open the possibility of using the earth's mass center as a reference with which to measure vertical motions. So far, however, neither technique has convincingly measured postglacial rebound.

A translation changes the horizontal as well as the vertical components of velocity. For example, a translation of 1 mm/yr in the direction of the North Pole not only increases the uplift rate at the North Pole by 1 mm/yr, it also increases the north components of velocity of all points along the equator by 1 mm/yr. Such a translation produces tiny changes in the relative horizontal within a region less than 10OC- km in dimension, but it can substantially change. estimates of the angular velocities of the major plates.

In summary, VLBI observations over 15 years have detected vertical motions produced by postglacial rebound. Estimates of the vertical and horizontal components of surface velocity depend on the reference frame chosen to represent the earth's interior. and attention needs to be paid to defining and assessing uncertainty in this reference frame. Extending the time duration and spatial distribution of observations should, over the next decade, bring useful constraints to the study of the uplift and subsidence produced by postglacial rebound,

Appendix

I determine the optimal translation by minimizing the root mean square of **measurements** of vertical components of velocity after adjusting for postglacial rebound; I minimize:

$$\sum_{i=1}^{m}\left[(\mathbf{u}_i + 0, \text{“} \hat{\mathbf{x}}_i)\right]^2 \qquad (1)$$

where u, is the measured velocity of the *i*th site less the rebound prediction, $\hat{\mathbf{x}}_i$ is the unit geocentric vector to the *i*th site, o is the optimal translation, and *m* is the number of measurements. The $\mathbf{u}_i$, $\hat{\mathbf{x}}_i$, and o arc **3x** 1 vectors of Cartesian components:

$$\mathbf{x}_i = \begin{vmatrix} x_i \\ y_i \\ z_i \end{vmatrix} \qquad \mathbf{u}_i = \begin{vmatrix} u_i \\ v_i \\ w_i \end{vmatrix} \qquad o = \begin{vmatrix} u_0 \\ v\text{“} \\ w_0 \end{vmatrix}$$

Taking the. first derivative with respect to each of the 3 Cartesian components of the optimal translation, setting them equal to zero, and solving the resulting set of 3 equations in 3 unknowns, I find the optimal translation is:

$$\mathbf{o} = -\mathbf{R}^{-1}\mathbf{S}\,\mathbf{u} \qquad (2)$$

where $\tilde{\mathbf{u}}$ is a 3*m*x 1 vector consisting of the *m* $\mathbf{u}_i$'s

$$\underline{\mathbf{u}} = \begin{vmatrix} u, \\ : \\ u\text{“}, \end{vmatrix}$$

R is a 3x3 matrix

$$\mathbf{R} = \begin{vmatrix} \sum_{i=1}^{m} x_i x_i & \sum_{i=1}^{m} x_i y_i & \sum_{i=1}^{m} x_i z_i \\ \sum_{i=1}^{m} y_i x_i & \sum_{i=1}^{m} y_i y_i & \sum_{i=1}^{m} y_i z_i \\ \sum_{i=1}^{m} z_i x_i & \sum_{i=1}^{m} z_i y_i & \sum_{i=1}^{m} z_i z_i \end{vmatrix}$$

and S is a 3x3*m* matrix

$$\mathbf{S} = \begin{vmatrix} x,x, & x_1 y_1 & x_1 z_1 & \ldots & x_m x_m & x_m y_m & x_m z_m \\ y_1 x_1 & y_1 y_1 & y_1 z_1 & ,., & y_m x_m & y_m y_m & y_m z_m \\ z_1 x_1 & z_1 y_1 & z_1 z_1 & \ldots \text{?.”}, & xn, & 7.\text{”}, y_m & 7.\text{”},7.\text{”}, \end{vmatrix}$$

The transformed velocity of a site equals the sum of its original velocity and the translation:

$$\mathbf{u}_i' = \mathbf{u}_i + \mathbf{o} \qquad (3)$$

The transformed velocities in terms of the originals arc:

$$\underline{\mathbf{u}}' = \mathbf{A}\,\underline{\mathbf{u}} \qquad (4)$$

where A is a 3*m*x3*m* matrix

$$A = \left(I - \begin{vmatrix} R^{-1} S \\ : \\ R\text{-l } S \end{vmatrix} \right)$$

I is the 3*m*x3*m* identity matrix, and the 3*m*x3*m* matrix to its right consists of *m* identical 3x3*m* matrices piled upon each other. The transformed covariance matrix in terms of the original is:

$$\underline{\mathbf{M}}' = \mathbf{A}\,\underline{\mathbf{M}}\,\mathbf{A}^{T} \qquad (5)$$

I add back the rebound prediction to obtain the final velocity.

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Figure 1. Northern hemisphere map showing radio telescope site locations and predictions of vertical rates from the postglacial rebound model of Peltier [1995]. Equal-area projection.

Figure 2. Predictions of postglacial vertical rates [Peltier, 1995] (abscissas) versus VLBI measurements in the reference frame of Ma and Ryan [1995] (ordinates). The ideal line (solid) has slope I and intercept O mm/yr. The best-fitting line (dashed) is from an unweighted fit. Standard errors (vertical bars) are from Ma and Ryan [1995].

Figure 3. Predictions of postglacial vertical rates [Peltier, 1995] (abscissas) versus VLBI measurements in the reference frame defined in this article (ordinates). The ideal line (solid) has slope 1 and intercept O mm/yr. The best-fitting line (dashed) is from an unweighted fit. Standard errors (vertical bars) are determined from linear propagation of the errors of Ma and Ryan [1995].

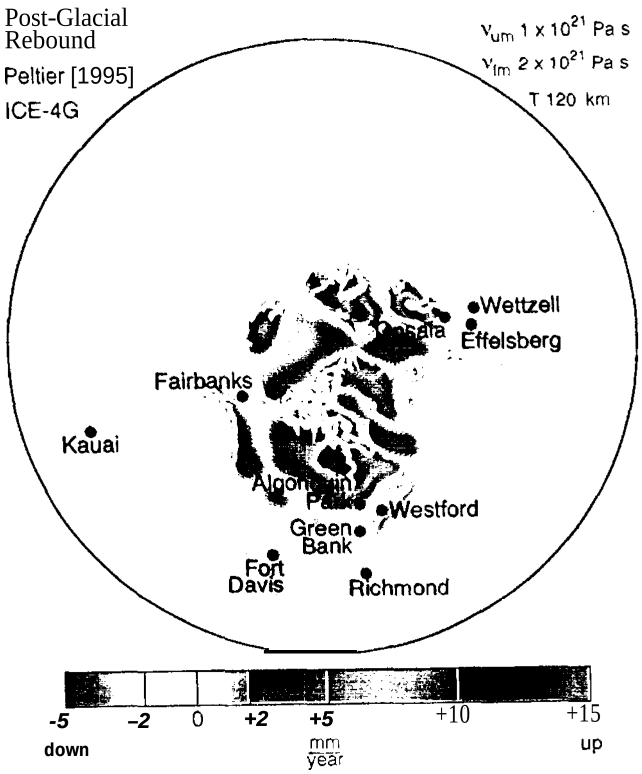
Post-Glacial Rebound
Peltier [1995]
ICE-4G
ν_um 1 x 10^21 Pa s
ν_lm 2 x 10^21 Pa s
T 120 km
Wettzell
Onsala
Effelsberg
Fairbanks
Kauai
Algonquin Park
Westford
Green Bank
Fort Davis
Richmond
-5
-2
0
+2
+5
+10
+15
down
mm/year
up

Figure 1

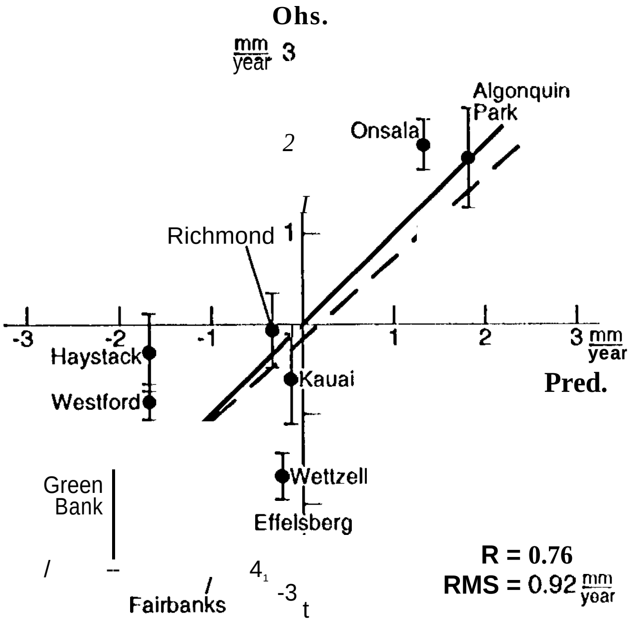
Ohs.
mm/year 3
2
1
-3
-2
-1
1
2
3 mm/year
Pred.
Algonquin Park
Onsala
Richmond
Haystack
Westford
Kauai
Wettzell
Effelsberg
Green Bank
Fairbanks
-3
R = 0.76
RMS = 0.92 mm/year

Figure 2

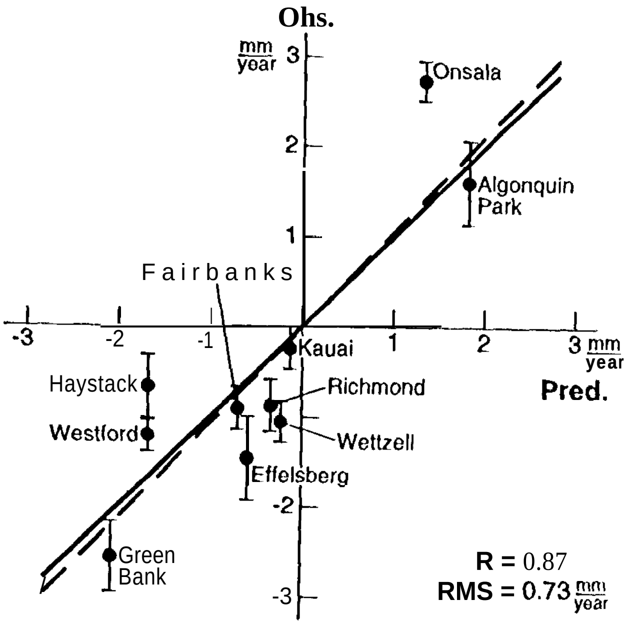
Ohs.
mm/year
3
2
1
-3
-2
-1
1
2
3 mm/year
Pred.
-2
-3
Onsala
Algonquin Park
Fairbanks
Kauai
Richmond
Haystack
Westford
Wettzell
Effelsberg
Green Bank
R = 0.87
RMS = 0.73 mm/year

Figure 3