Modeling and Optimization of a 1 Joint-Tc 1 Jot-Electron Superconducting Mixer for Terahertz Applications


Center for Space Microelectronics Technology, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109

Abstract

The development of a YBa2Cu3O7-δ (YBCO) hot-electron bolometer (HEB) quasioptical mixer for a 2.5 THz heterodyne receiver is discussed. The modeled device is a submicron bridge made from a 10 nm thick film on a high thermal conductance substrate. The mixer performance expected for this device is analyzed in the framework of a two-temperature model which includes heating both of the electrons and the lattice. Also, the contribution of heat diffusion from the film through the substrate and from the film to the normal metal contacts is evaluated. The intrinsic conversion gain and the noise temperature have been calculated as functions of the device size, substrate material, and ambient temperature. Assuming energy fluctuations and Johnson noise to be the main sources of noise, a single sideband (SSB) mixer noise temperature of less than 2000 K is predicted. For our modeled device, the conversion efficiency at an IF of 2.5 GHz is > 10 dB or better and the required local oscillator (LO) power is less than 5 μW.

1. Introduction

The superconductive HEB is presently considered the most promising heterodyne mixer device for the terahertz frequency range. Recent experiments with a Nb HEB mixer demonstrated a 560 K DSB noise temperature at 533 GHz and should remain relatively low for rf up to ~10 THz [1]. This type of mixer can
be especially useful in space-borne applications for atmospheric research if operated at elevated temperatures where low-power mechanical cryocoolers are readily available and where requirements for a low LO power are critical. For such an application, a HEB device made from a thin YBCO film can be used. The fabrication technology for such films has been significantly improved since the discovery of high-T_c superconductivity. Now, ultrathin films having a thickness d down to a few unit cells have been successfully fabricated [2, 6]. The critical temperature \( T_c > 85 \) K and superconducting transition width \( \Delta T_c = 1 \) – 2 K are typical for films with \( d > 10 \) nm, and a critical current density \( j_c = 8 \times 10^6 \) A/cm\(^2\) was observed in 10 nm thick films at \( 77 \) K [6].

Fabrication of superconducting structures made from YBCO with in-plane sizes 100–500 nm has also been demonstrated [7–11]. Critical current densities as large as \( 5 \times 10^6 \) A/cm\(^2\) have been measured in 200 nm wide superconducting lines [12]. A variety of materials (e.g., MgO, LaAlO\(_3\), NdGaO\(_3\), YSZ) have been found to provide a moderate dielectric constant and epitaxial YBCO film growth. Also, the use of buffer layers allows growth of YBCO films on silicon and sapphire (YSZ buffer layer for Si and CeO\(_2\) for sapphire). With such promising film growth technology, it becomes important for us to now examine the theoretical issues involved in designing optimum devices.

In contrast to slow bulk bolometric detectors, a HEB mixer can operate with a high intermediate frequency (IF) of the order of several gigahertz, and under appropriate LO power (typically of order of \( \mu \)W for submicron devices). This sets quite different from detector device criteria for mixer device optimization. In this paper we give a detailed analysis of the thermal processes important for good HEB mixer performance. Within the framework of a model which includes the temperature of both the electrons and phonons, expressions for mixer conversion efficiency, and IF impedance have been derived and analyzed at \( f_{IF} = 2.5 \) GHz. The contributions of both electron temperature fluctuations and Johnson noise in the mixer noise temperature have been investigated as functions of dc and LO power. Also, the requirements for the substrate thermal conductivity in relationship to the device in-plane size have been determined. A SSB noise temperature \( \leq 2000 \) K should be achievable for an optimized device.

The model developed here is required for optimization of a YBCO HEB mixer for potential use in a heterodyne receiver to observe OH at 2512 GHz in the upper atmosphere. This receiver is part of NASA’s Earth Observing System Microwave Limb Sounder instrument. An IF of 2.5 GHz is desired to simultaneously observe the doublet OH lines. The mixer will employ a planar twin-slot antenna on an elliptical silicon lens. A complete description of the experimental details will be given at a later date.
The non-equilibrium phonons leave the thin otherwise thin substrate boundary at the interface with the device.

We note that the corresponding diffusion length for a phonon in a plane parallel to the device is much smaller than any phonon in YbCO films. The electron diffusion length is given by $\lambda_e = \frac{\hbar}{m_e e B}$, where $B$ is the magnetic field. The electron diffusion length is much less than the phonon diffusion length, making non-equilibrium phonons a very short characteristic of non-equilibrium phonons.

The principle of non-equilibrium phonon transport is briefly shown in Figure 1. The phonons leave the thin substrate boundary with a high speed, where $v_p = \frac{\hbar}{m_p k_B}$, and can be considered a phonon without the magnetic field. The phonons leave the thin substrate boundary with a characteristic time of $t = \frac{\lambda_e}{v_p}$, and can be considered a phonon with a characteristic time of $t = \frac{\lambda_e}{v_p}$. The non-equilibrium phonons can be described in terms of interaction of the electron phonon coupling in the non-equilibrium phonons. The matrix of $\xi$, the phonon-phonon interaction matrix, is diagonal, and can be described as $\xi = \xi^{\text{eff}}$. The non-equilibrium phonons in high-$T_c$ YbCO films have been studied for a variety of years.
\[ R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \]

where \( R \) is the thermal conductivty of the substrate, \( R_1 \) is the effective thermal conductivity of the device area, and \( R_2 \) is the thermal resistance between the substrate and the heat source.

And different areas:

"Phonons (ph) that escape the substrate heat the contacts, which in turn heat the substrate. The substrate's temperature is measured, and the new thermal conductivty is used to calculate the new effective thermal resistance."

[Diagram showing heat transfer process involving phonons and substrate]
The optimization of the total thermal resistance between an electron subsystem and a heat sink as well as its modulation frequency dependence is quite important for the HEB mixer operation. A rule of thumb is that the thermal resistance should be made as low as possible and its frequency dependence should be as flat as possible. The minimum thermal resistance determines the maximum LO power contributing to the IF signal. A pronounced frequency dependence of the thermal resistance generally yields a loss of power at the IF of interest with respect to the maximum attainable conversion efficiency at a zero IF. The contributions from thermal boundary resistance and from the substrate will be compared in the following section.

III. Conversion Gain and IF Impedance in a Two-Temperature Model.

In contrast to a low-\( T_e \) HEB mixer, a high-\( T_e \) HEB mixer cannot be described in terms of the electron temperature only. This is because at temperatures \( < 90 \text{ K} \) the phonon heat capacity is always much larger than that of the electrons. A more appropriate approach [21] makes use of a "two-temperature" model describing the dynamics of the electron and phonon temperatures which are both different from the temperature of the heat sink. This is the approach we use here.

The coupled differential equations for the electron and phonon temperatures are given in [27]. The following spectrum of the electron temperature was obtained:

\[
\Delta T_e = \alpha P_{\text{rf}} \frac{\tau_e}{c_e} \left( \frac{c_e/c_p}{T_e} \right) \frac{1 + (\omega \tau_0)^2}{\sqrt{\left[ 1 + (\omega \tau_1)^2 \right] \left[ 1 + (\omega \tau_2)^2 \right]}}
\]

where \( \alpha \) is the rf coupling factor, \( P_{\text{rf}} \) is the amplitude of the incident rf power, and \( c_e \) is the electron specific heat of the film. \( \tau_{\text{rf}}, \tau_f, \) and \( \tau_2 \) are given by the following formulas:

\[
\tau_{1,2} = \tau_{1,2}^{-1} = \frac{1}{2\tau} \left( 1 + \frac{\tau_1^2}{\tau_e \tau_{es}} \right), \quad \tau^{-1} = \tau_{es}^{-1} + \tau_{e}^{-1} + \tau_{p}^{-1}, \quad \tau_{\text{rf}}^{-1} = \tau_{es}^{-1} + \tau_{p}^{-1}, \quad \tau_p = \tau_e c_p/c_e.
\]
In YBCO $\tau_e$ is so short that $\tau_p \approx \tau_{es}$. This condition, along with $\tau_p \approx \tau_e$, allows one to simplify Eqn's. 3:

$$\tau_1 = \tau_e, \tau_2 = \tau_{es}, \tau_{p} = \tau_p$$

(4)

and obtain the following spectrum of the electron temperature:

$$N \tau_e = \alpha N_{mol} \tau_e \left( \frac{c_e}{c_p} \right) \tau_{es} \left[ \frac{1}{1 + \left( \omega \tau_p \right)^2} \right] \left( \frac{1}{1 + \left( \omega \tau_{es} \right)^2} \right)^{\frac{1}{2}} \frac{1}{1 + \left( \omega \tau_{es} \right)^2}$$

(5)

This frequency dependence for a response in thin YBCO films was observed in recent optical mixing experiments at $\lambda = 1.54 \mu m$ [21] and $\lambda = 9.6 \mu m$ [23].

From Eq. 5 one can obtain the effective thermal resistance between electrons and substrate:

$$R_{e,s} = \frac{\tau_e}{c_e V \left[ 1 + \left( \omega \tau_e \right)^2 \right] \left[ 1 + \left( \omega \tau_{es} \right)^2 \right]} \left[ \frac{1}{1 + \left( \omega \tau_p \right)^2} \right] \left( \frac{1}{1 + \left( \omega \tau_{es} \right)^2} \right)^{\frac{1}{2}} \frac{1}{1 + \left( \omega \tau_{es} \right)^2}$$

(6)

and the total thermal resistance to the bath is

$$R_{tot} = R_{e,s} + R_s$$

(7)

Figure 2 shows the behavior of $R_s$, $R_{e,s}$ and $R_{tot}$ for two widely used substrates (MgO and LaAlO$_3$) and two device sizes ($L = 10 \mu m$ and $L = 1 \mu m$). The YBCO film thickness is 10 nm in all cases. $R_s$ dominates for poor thermal conducting substrate, large device sizes, and low IF (e.g. LaAlO$_3$ for $L = 10 \mu m$). We should point out that Eq. 7 underestimates the total thermal resistance since the reverse flow of phonons from the substrate to the YBCO film is not taken into account. The effect should be larger for larger device areas and lower substrate thermal conductivity. Nevertheless, it is believed that MgO substrates, where $R_s \approx R_{e,s}$ are nearly ideal, $R_s$ becomes negligible for submicron-size devices and will not be considered in the following analysis. We will also not consider the heat diffusion to the contacts. This
process can reduce the total thermal resistance of a 0.1 μm long device. However, even without this mechanism good mixer performance is predicted.

Table I. Physical parameters of YBCO and some substrates at ~ 90 K.

<table>
<thead>
<tr>
<th>Material</th>
<th>YBCO</th>
<th>MgO</th>
<th>LaAlO₃</th>
<th>YSZ</th>
<th>Sapphire</th>
<th>YAlO₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_v$, JK$^2$ cm$^{-3}$</td>
<td>0.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_p$, JK$^2$ cm$^{-3}$</td>
<td>0.64</td>
<td>0.53</td>
<td>0.40</td>
<td>0.70</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>$\kappa$, $\omega$ cm$^{-1}$</td>
<td>0.015</td>
<td>3.4</td>
<td>0.35</td>
<td>0.015</td>
<td>0.4</td>
<td>0.2-0.4 (^a)</td>
</tr>
<tr>
<td>$R_{\text{sh}}$, K cm$^{-2}$ W$^{-1}$</td>
<td>5.0x10$^{-2}$</td>
<td>1.0x10$^{-3}$</td>
<td>N/A</td>
<td>1.1x10$^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>10</td>
<td>24</td>
<td>28</td>
<td>11</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>$\tan \delta$</td>
<td>7x10$^{-6}$</td>
<td>5x10$^{-7}$</td>
<td>4x10$^{-4}$</td>
<td>8x10$^{-6}$</td>
<td>1x10$^{-5}$</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) this work

For use in a practical 2.5 THz receiver, the substrate must also exhibit low rf absorption. We have measured MgO, YAlO₃ and sapphire at 77 K in a Fourier transform spectrometer and found them to have acceptable rf transparency. The physical constants for YBCO and a number of useful substrates at 90 K are given in Table 1.

Equations 2 and 5 were obtained assuming no self-heating effects in the superconducting film (small dc current) and a simple, linear (with respect to the electron temperature shift) dependence for the heat flow from electrons to phonons. The latter assumption is applicable for only small differences between $T_e$ and $T_p$. It has been found experimentally that in YBCO films, $\tau_e \sim T_e^{-1}$ [24], hence the heat flow from electrons to phonons is actually proportional to $(T_e^{-1} - T_p^{-1})$.

Here we discuss the more realistic situation where the device is so far from equilibrium that one can neglect neither the non-linearity in the heat conductance (strong pumping), nor the self-heating caused by transport current. We also include in the model the feedback effect from the load influencing the conversion mixer gain and modifying the mixer bandwidth [25].
can be made

Here is the volume of the molecule A is the constant characterizing the smooth of the electron.

\[
\begin{align*}
A^p = (\mathbf{q_L} - \mathbf{q_C})^{p}_{\mathbf{AV}} - (\mathbf{q_L} - \mathbf{q_C})^{p}_{\mathbf{IP}} \cdot \mathbf{A}^{p,2} \\
A^p + q_A^{p} + \left(\mathbf{q_L} - \mathbf{q_C}\right)^{p}_{\mathbf{AV}} - (\mathbf{q_L} - \mathbf{q_C})^{p}_{\mathbf{IP}} \cdot \mathbf{A}^{p,2}
\end{align*}
\]

We start with the following dynamic equations:

\[ L = 1.0 L \]
\[
(z + (\frac{L}{\Lambda})) \left( \frac{d}{d\Lambda} \right) + \left( \frac{0}{L} \right) \frac{d}{d\Lambda} \right) \left( \frac{0/df}{\varepsilon L \tan \theta} \right) = \sum L A V E
\]

Substituting from Eq. 100 into Eq. 101 we obtain:

\[
\lambda \bigg|_{\frac{d}{d\Lambda}} = \left( \frac{L}{\Lambda} \right) + \left( \frac{0}{L} \right) + \left( \frac{0/df}{\varepsilon L \tan \theta} \right) = \sum L A V E
\]

and for the \( \pm \) harmonics:

\[
0 = \bigg|_{\frac{d}{d\Lambda}} \left( \frac{L}{\Lambda} \right) + \left( \frac{0/df}{\varepsilon L \tan \theta} \right) = \sum L A V E
\]
One can verify that in the low temperature limit, when $\tau_{\text{c}} = \tau_{\text{p}} = \tau_{\text{e}}$, the frequency dependent term in the product given by Eq. 13 is reduced to $\left| 1 + j \omega \tau_{\text{e}} \right| \left| \frac{1}{4} C \left( R_0 + R_T \right) \right|^2$, giving the expression previously obtained in [25].

Postulating that the IF impedance $Z(\omega) = R_0 + \left( d R/d I_{\text{e}} \right) I_{\text{e}}$, and using Eqn's. 8, one can obtain

$$
\tilde{Z}(\omega) = R_0 \left[ \frac{1}{\tau_{\text{c}} - \omega^2 \tau_{\text{e}} \tau_{\text{c}} + j \omega (\tau_{\text{e}} + \tau_{\text{c}})} \right] \frac{1}{\left( 1 + j \omega \tau_{\text{c}} \right)} \left( 1 - j \omega \tau_{\text{c}} \right),
$$

which coincides in the low-temperature limit with the following expression from [26]:

$$
\tilde{Z}(\omega) = R_0 \left[ \frac{1}{\tau_{\text{c}} - \omega^2 \tau_{\text{e}} \tau_{\text{c}} + j \omega (\tau_{\text{e}} + \tau_{\text{c}})} \right] \frac{1}{\left( 1 + j \omega \tau_{\text{c}} \right)} \left( 1 - j \omega \tau_{\text{c}} \right)
$$

### IV. Noise Temperature.

The expression for the noise temperature due to the electron temperature fluctuations of a low-$T_{\text{c}}$ HEB mixer was given in [26]. It was also shown that this quantity does not depend on the conversion gain, i.e., it is fairly universal. We believe it is applicable for a high-$T_{\text{c}}$ HEB mixer, and the corresponding SSB noise temperature contribution is given by:

$$
T_{\text{ni}}^{\text{TV}} = \frac{2 T_{\text{e}}^2 G_e}{\alpha^2 P_{\perp 0}},
$$

where $G_e = 3 \alpha V / \partial T_{\text{e}}^2$ is the thermal conductance between electrons and phonons.

The contribution of Johnson noise should be evaluated by taking into account the enhancement of the noise due to the self-heating in a bolometer. Simply, one can use the equivalent noise circuit introduced in [27] (see Fig. 3). Following [26,27], we assume that the classical Johnson noise source $e_j = \sqrt{4k_B R T_{\text{e}}}$ must appear twice in the bolometer equivalent circuit. Source $e_{\parallel} = e_j$ acts simply as a voltage source in
series with the bolometer impedance $Z(\omega)$. The source $E_2 = e_f/2$ is placed to take into account the output noise enhancement caused by the self-detection of the Johnson noise in the bolometer. The impedance $Z_\lambda$ represents the bolometer reactance due to its thermal input and self-feeding contribution. $Z_\lambda$ is chosen to agree with Eq. 14 for the bolometer IF impedance. After passing the frequency dependent impedance $Z_\lambda$ a "white" noise $e_f$ becomes frequency dependent at the load $R_l$. The corresponding expression for the noise temperature is obtained by dividing the noise power dissipated in the load by the conversion gain given in Eq. 13. A relatively simple expression has been obtained for a low-$T_c$ HEB mixer [26]. However, for the high-$T_c$ case the expression turns to be very cumbersome, therefore we just calculate the noise temperature numerically.

![Bolometer Circuit](image)

*Fig. 3. Equivalent circuit for calculations of the Johnson noise temperature in a bolometer.*

V. Numerical Results.

Contour plots in Fig. 4 represent the results of simulations of the HEB mixer SSB noise temperature, $T_M$ (Fig. 4a), and its components: due to electron temperature fluctuations, $T_{M}\gamma$ (Fig. 4b), and due to Johnson noise $T_{M}\eta$ (Fig. 4c) at $f_M = 2.5$ GHz. Parameters are chosen which represent realistic estimates for a device to be used in practical cryocooled mixer applications: an area of $0.1 \times 0.1 \mu m^2$, a thickness of 10 nm, $T_\gamma = 85$ K, $\delta T = 2$ K, normal resistance $R_n = 400 \Omega$, and an operating temperature of $T = 66$ K. A coupling factor $\alpha$ was chosen to be 1 for simplicity ($\alpha$ will depend on the details of the
planar antenna and optics), so $P_{LO}$ designates the absorbed LO power. Figure 4d shows the SSB mixer conversion efficiency under the same conditions, and Fig. 4e shows the device dc resistance. The contours in Figs. 4 are plotted versus dc and LO power since these are two important and experimentally variable parameters for a planar mixer. With the given dc and LO power scales, the top right corner of all plots corresponds to the normal state, the bottom left corner corresponds to a nearly superconducting state. At both of these edges the noise temperature is very high. In the superconducting state, where the LO power is low, $T_M^H$ is high. In the normal state, where the conversion loss is very high, $T_M^L$ is high. Just at the middle of the resistive transition, the noise temperature reaches its minimum value (~1000 K). Figure 5 shows the behavior of the noise temperature in the vicinity of the minimum. Figure 6a shows the IF spectra of the conversion gain and the IF impedance. It is interesting to point out that a negative differential dc resistance $Z_0$ turns into positive real impedance of about 230 $\Omega$ at 2.5 GHz. The conversion efficiency at this frequency is still high (+0.36 dB). The combination of the parameters at the optimum point is given in Table 2 (point 1). Negative resistance is not a necessary condition for high conversion efficiency or low noise temperature. For example, the operating point at slightly higher LO power (just above the middle of the superconducting transition) yields a 1300 K noise temperature (Table 2, point 2) and a positive real part of the IF impedance at all frequencies (see Fig. 6b).

Our model does not predict any degradation of the device noise performance if the device size $L$ is made larger. The only parameters which change are LO power and dc power. They simply scale as the device area, i.e. $P_{LO} \propto L^2$ and $P_{DC} \propto L$. The effect of the device thickness is more complicated, since not only the dissipated power, but also the total thermal resistance is being changed with the thickness. Figure 7 shows the dependence of the minimum noise temperature vs device thickness. The device area is 0.1x0.1 $\mu$m$^2$ and the normal resistance increases as $1/d$. One can see that $T_M$ is an almost linear function of $d$, whereas $P_{LO}$ saturates with thickness. This is because $T_c$ $T_p$ becomes smaller for larger $d$ (see Eq. 8b), and, therefore, the total thermal resistance is dominated by phonon escape (i.e. by $R_p$). It means that for large thickness, an optimal mixing electron temperature is reached with the same power dissipated in the device. In this case the shift of electron temperature caused by LO power is very small and the optimal LO power is determined by only thermal boundary resistance $R_p$ and does not depend on $d$.
Fig. 4a. Total mixer noise temperature $T_M (K)$. Waviness is an artifact of numerical precision in the simulations.

Fig. 4b. Noise temperature contribution due to electron temperature fluctuations $T_{M}^{TF} (K)$. 

Fig. 4c. Noise temperature contribution due to Johnson noise $T_M (K)$. Waviness is an artifact of numerical precision in the simulations.

Fig. 4d. Mixer conversion efficiency (dB).
Fig. 4c. Mixer device dc resistance (Ω)

Fig. 5. Noise temperature as a function of LO power in the vicinity of the optimum point.
Fig. 6. Conversion efficiency vs IF and the IF impedance at two operating points:

a) minimum noise temperature (1100 K)
b) slightly higher LO power and noise temperature (1700 K). *

An increase of the operating temperature T causes a degradation of the mixer noise temperature because of a decrease of the optimal LO power. A set of parameters for T = 77 K is given in Table 2 (point 3). One can see that the minimum noise temperature is 3 times higher than for T = 66 K. An advantage, however, is that the differential resistance at the optimum point is positive.

Table 2. Parameters of the mixer at different operating points.

<table>
<thead>
<tr>
<th>Point</th>
<th>$\gamma$, K</th>
<th>$P_{LOP}$</th>
<th>$P_{dc}$, mW</th>
<th>$R$, $\Omega$</th>
<th>$\eta$, dB</th>
<th>$Z_0$, $\Omega$</th>
<th>$Z(2.5\text{GHz})$, $\Omega$</th>
<th>$T_c$, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1100</td>
<td>1.9</td>
<td>20</td>
<td>187</td>
<td>-0.30</td>
<td>-1000</td>
<td>234-126j</td>
<td>84.9</td>
</tr>
<tr>
<td>2</td>
<td>1300*</td>
<td>2.0</td>
<td>20</td>
<td>329</td>
<td>-9.3</td>
<td>936</td>
<td>367-60j</td>
<td>85.8</td>
</tr>
<tr>
<td>3</td>
<td>3000</td>
<td>0.7</td>
<td>10</td>
<td>198</td>
<td>-4.7</td>
<td>1000</td>
<td>225-61j</td>
<td>85</td>
</tr>
</tbody>
</table>
Fig. 7. Thickness dependence of the mixer noise temperature and optimal LO power.

VI. Conclusion.

We have developed a comprehensive model of thermal processes in a high-\(T_c\) HEB mixer. It was shown that the heat conductance from electron to phonons and escape of phonons through the film/substrate interface are the most critical processes in determining the device response and sensitivity. Using a two-temperature model, all important mixer parameters were calculated and studied for a practical range of conditions needed for a 2.5 THz heterodyne receiver for OI measurements. The effects of device size and of heat sink temperature have been evaluated. It was demonstrated that a submicron-size device made of a 10 nm thick film can have a very low noise temperature (\(-1000\) K) and require only microwatts of LO power. These combination of parameters are very favorable for space-borne heterodyne instruments operating at terahertz frequencies.

The research described in this paper was performed by the Center for Space Microelectronics Technology, Jet Propulsion Laboratory, California Institute of Technology, and was sponsored by the National Aeronautics and Space Administration, Office of Mission to Planet Earth, and the Office of Space Access and Technology.
References.


