

Transient Synthesis Using a Fourier Transform Envelope

An Alternative to Swept-Sine Vibration Testing

SUMMARY

Transient vibration tests offer an alternative to the conventional swept-sine vibration test, with controllable conservatism. A method of deriving a single vibration test transient from several spacecraft flight transients is described. These transients were from different physical locations on the spacecraft. The test transient was derived by taking the inverse Fourier transform of an envelope of the Fourier transforms of the flight transients. The inverse transformation was performed using the real and imaginary parts of that Fourier transform forming the envelope amplitude at each frequency. The test transient therefore has the same frequency content and maximum amplitudes as the flight transients. A generic test transient was therefore produced that duplicates many operating conditions at separate physical locations. It could be applied to many different structural components of a spacecraft, simplifying a vibration test program. The transient vibration test produces significantly less overttest than a conventional swept-sine vibration test.

DISCUSSION

In a complex spacecraft there is significant variation in the flight transient vibration environments occurring at different components and locations. Several transient vibration tests are therefore required to satisfy each of these vibration environments. A single generic transient vibration test, applied to each component, would be more desirable and cheaper. Over-test would vary, of course, between different locations. The conventional swept-sine vibration test produces excessive overttest regardless of the response characterization used [1]. This would be more pronounced with a single generic swept-sine vibration test. However, with a single generic transient test, this would be mitigated by the fact that transients, in general, provide significantly less overttest than conventional swept-sine tests [1]. The work described herein is an attempt to obtain such a singular generic test. The work is part of a larger study [2] to replace the conventional swept-sine vibration test with a transient vibration test having controllable conservatism. In reducing several transient vibration environments to a single one, a single characterization of the individual transient vibration environments is commonly used. A typical example is the enveloping of several shock response spectra (SRS) to produce a single representative SRS [3]. A test transient is then empirically produced using a sum of decaying sinusoids or wavelets [4], so that its SRS approximates that of the SRS-envelope. The SRS however, is not unique, and could be representative of many dissimilar environments. An SRS cannot therefore be analytically transformed back into the time domain. Other characterizations are available [5], but they too are not uniquely transformable back to the time domain. The Fourier Transform (FT) characterization, however, is unique and can be analytically transformed back into the time domain provided the real and imaginary parts of the FT are available.

The feasibility of generating a single test transient from the envelope of several flight transient Fourier transforms was examined. Ten flight transients representing two launch event conditions at five different locations on the Cassini spacecraft were used. These locations are correlated to the launch events according to the nomenclature of Table 1. The spacecraft "grid" point and finite

element model degree of freedom (DOF) is used to identify the location of the transient. The code is used herein as a simple transient identifier. One location is in the R coordinate axis for the high gain antenna (HGA) support point. Another location is in the R coordinate axis for the remote sensing platform attachment point. The remaining three locations are in the Y coordinate axis for each of the three radioactive thermoelectric generator (RTG) bases. Widely separated physical locations and vibration directions were deliberately used to add generality to the synthesized transient.

TABLE 1 - Transient locations

Code	Event	Grid	DOF	Description	Axis
0401	FU04	11502	R	HGA Support Point	+ X-Y
1401	FU14	11502	R	HGA Support Point	+ X+Y
0419	FU04	17011	R	RSP Attachment Point	Cylindrical
1419	FU14	17011	R	RSP Attachment Point	Cylindrical
0411	FU04	16314	Y	+ Y RTG Base	Horizontal
1411	FU14	16314	Y	+ Y RTG Base	Horizontal
0414	FU04	16324	Y	- Y RTG Base	Horizontal
1414	FU14	16324	Y	- Y RTG Base	Horizontal
0417	FU04	16334	Y	+ X RTG Base	Horizontal
1417	FU14	16334	Y	+ X RTG Base	Horizontal

The flight transients used were from two fuel depletion shutdown events of the Titan launch vehicle first stage operation. One shutdown event is called FU04 and the other FU14. These are considered to be the most critical, in terms of dynamic loads for the components considered. These transients are shown in Figures 1 and 2.

FOURIER TRANSFORM ANALYSIS

The discrete Fourier transform (DFT), $\ddot{X}(f)$, of an acceleration time history $\ddot{x}(t)$, is given [6] by:

$$\ddot{X}_m = \sum_{n=0}^{N-1} \ddot{x}_n e^{-j(2\pi mn/N)}; m = 0, 1, \dots, N-1 \quad (1)$$

where $j = \sqrt{-1}$

For transforming from the frequency to time domain, the inverse DFT is given by:

$$\ddot{x}_n = \frac{1}{N} \sum_{m=0}^{N-1} \ddot{X}_m e^{j2\pi mn/N}; n = 0, 1, \dots, N-1 \quad (2)$$

The following symbol definitions apply:

The expression \ddot{x}_n , is shorthand for $\ddot{x}(n, DT)$, where DT is the sampling interval used to describe N signal samples of the time history signal $\ddot{x}(t)$, beginning at time $t = 0$ and ending at time TD .

There are N real and imaginary frequency points (ω) in equation (1) defined by: $\omega = \frac{2\pi m}{N \cdot DT}$

The frequency increment, DF in the frequency domain transform (2) is given by:

$$DF = 1/Z = 1/(N \cdot DT)$$

The Fourier transform is normally displayed with the amplitude points \ddot{X}_m plotted against the real frequency $f = \omega / 2\pi$. In this format the plot is referred to as a Fourier spectrum. From the right hand side of equation (1), this magnitude is written in terms of its real and imaginary parts, respectively:

$$|\ddot{X}_m| = \left\{ \left| \sum_{n=0}^{N-1} \ddot{x}_n \cos \frac{2\pi mn}{N} \right|^2 + \left| \sum_{n=0}^{N-1} \ddot{x}_n \sin \frac{2\pi mn}{N} \right|^2 \right\}^{1/2} \quad (3)$$

The amplitude spectrum is therefore the quadrature sum of the real and imaginary components. In order to perform an inverse transform, both the real and imaginary components of a Fourier transform spectrum must be provided, as in equation (3), at each frequency increment. If the Fourier transform spectrum is artificial, as for an envelope of several spectra, then the real and imaginary spectrum components must be assumed. The assumption used herein was that the envelope phase was that associated with the FT forming the FT magnitude envelope at each frequency increment. The magnitude envelope is determined at each frequency, m , by taking the maximum of all the spectra being enveloped. The envelope spectrum amplitude $E\ddot{X}_m$, at the frequency increment m , would then be obtained from p spectra as:

$$E\ddot{X}_m = \text{Maximum} \left[\ddot{X}_{m,1}, \ddot{X}_{m,2}, \dots, \ddot{X}_{m,p} \right] \quad (4)$$

The individual amplitudes $\ddot{X}_{m,p}$ are defined in terms of their real and imaginary components, obtained from the separate time-domain transients, as in equation (3). The inverse Fourier transform is then calculated using the real and imaginary components of the Fourier transform which had the maximum spectrum amplitude at each frequency, m . That is, the real and imaginary components of the FT which forms the FT magnitude envelope at the frequency, m , are used. Thus, the term \ddot{X}_m used in equation (2) to derive the inverse FT at each frequency is given by $\ddot{X}_{m,z}$ where:

$$\ddot{X}_{m,z} = \sum_{n=0}^{N-1} \ddot{x}_{n,z} \cos \frac{2\pi mn}{N} + \sum_{n=0}^{N-1} \ddot{x}_{n,z} \sin \frac{2\pi mn}{N}; m = 0, 1, \dots, N-1 \quad (5)$$

and the subscript z refers to the Fourier transform having the largest amplitude at the frequency increment m .

EXAMPLE

The DFT algorithm used here required A' to be equal to two raised to some power. The time history transients described here had 802 time points taken at a sampling rate of 500 points per second. Therefore the DFT analyses used 1024 for the value of N . The sampling rate yielded an upper frequency limit of 400 Hz and the 1024 sample points yielded a frequency resolution of 0.781 Hz.

The DFT's obtained from the transients of Figures 1 and 2 are shown in Figures 3 and 4. These are 2-sided Fourier spectra, with only the positive frequency axis shown. The negative frequency axis is the mirror image of the positive one and is not shown. The amplitude observed is therefore only half of what is normally displayed in a 1-sided Fourier-magnitude spectrum. No windowing was applied to the DFT analysis. These Fourier transforms were enveloped by the amplitude spectrum shown in Figure 5.

The time history at the top of Figure 6 was derived by taking the inverse Fourier transform of the spectrum of Figure 5. A small DC correction was applied to the transient to counteract the DC offset present. The associated velocity and displacement time histories are also shown in Figure 6.

The displacement change in Figure 6 is beyond the capability of most electrodynamic shakers. The displacement and velocity changes were brought within bounds by subtracting a low frequency compensating pulse from the transient time history [3]. This compensation pulse takes the form:

$$U(t) A e^{-\zeta \omega_p (t + \tau)} \sin \omega_p (t + \tau) \quad (6)$$

where $U(f)$ is a unit step function, i.e., $U(t) = 0$ for $t \leq 0$
 $= 1$ for $t \geq 0$.

ζ is the decay rate, τ is the time delay, A is the amplitude and ω_p is the pulse frequency.

The parameters of the compensation pulse were iteratively adjusted to minimize the residual velocity and displacement of the transient. This required a compensation pulse with a 9g amplitude (A) at 10 Hz with a time delay of 0.1 seconds and a decay rate of 0.5. The amplitude of the compensation pulse is not fully reflected in the displayed compensation pulse due to the exponential and sine terms in equation 6. As seen from Figure 7, this pulse does not adversely affect the format of the transient and minimizes the velocity and displacement changes.

CONSERVATISM

A transient has been derived that contains all the frequency content of the ten original transients. The conservatism obtained by using this single test transient at all ten original locations can be measured quantitatively [7]. The conservatism is the amount by which the test response exceeds (overtest) the flight response of the test article. Conservatism is measured by a so-called index of conservatism (*IOC*), which is defined by:

$$IOC = \frac{\bar{C}_T - \bar{C}_F}{\sqrt{\sigma_T^2 + \sigma_F^2}} = \frac{\bar{M}}{\sigma_M} \quad (7)$$

where \bar{M} is the mean margin of conservatism and \bar{C}_T and \bar{C}_F are the mean transient characterization values for the test (T) and flight (F) environments, and σ_M , σ_T and σ_F are the corresponding standard deviations. Positive and negative values of *IOC* indicate overtest and undertest respectively. The *IOC* actually measures the probability of achieving an over-test given the statistics of the test and flight environments. For instance *IOC* values of zero, one and two correspond to 50, 84.1 and 97.9 percent probabilities that an overtest will occur. The amount of overtest is quantified by the overtest factor (*OTF*) described in reference [7]:

$$OTF = \frac{\bar{C}_T}{\bar{C}_{T,I}} \quad (8)$$

The OTF defines how many times greater the actual mean test characterization, \bar{C}_T , is than the desired mean test characterization, $\bar{C}_{T,I}$, having an index of conservatism of I . This desired mean test characterization may be calculated by using equations 7 and 8, as described elsewhere [7]. The response of spacecraft components to the transients is described by the shock response spectrum (S1{S), which is a measure of the response of a single-degree-of-freedom system to a transient input. Therefore the response of an elastic system attached to the spacecraft structure at any location can be assessed by producing an SRS from the expected flight transient at that same location. The response of the same elastic system due to the synthesized transient can also be assessed using the SRS characterization. A comparison of these two (S1{S) spectra will determine the conservatism of the synthesized test transient relative to the expected flight transient. Figure 8 shows clearly how the SRS of the synthesized transient (**depctsr**s) envelopes the SRS's of all the flight transients. This is a good indication that the synthesized transient is valid. Figures 9 and 10 show the overttest factors (OTF) obtained using the synthesized transient at each location, as a function of frequency. An index of conservatism (IOC) of unity was used in the OTF calculation, An overall OTF may be calculated by averaging the OTF over the frequency axis. This simplifies the overttest to a single number as shown in Table 2.

Some locations show a large degree of overttest, as shown by a large OTF. This behavior is expected with a generic test transient that attempts to encompass several different spacecraft locations. The RTG locations show much larger degrees of overttest than the other locations. This is expected because the flight transients at these locations are much smaller than those at the non-RTG locations. The larger transients contained in the Fourier Transform envelope obviously dominate the response produced by the synthesized transient.

TABLE 2- Frequency Averaged OTF

Code	OTF
1401 I	1.21
1419	1.23
0401	1.39
0419	1.42
1414 I	3.18
1411	3.33
0411	3.48
0414	4.12
1417	4.91
0417	5.89

Another way of obtaining a time-domain signal from the Fourier transform amplitude envelope of Figure 5, is to assume a random phase distribution along the frequency axis. The phase is defined as the inverse tangent of the ratio of the imaginary to real FT components, This was done five times

with the phase bounded between 1 and 360 degrees. This produced the time histories shown in Figure 11, which appear more like portions of random signals. This is because the inverse Fourier transform of Fourier magnitudes with random phase will always produce a stationary segment of a Gaussian random signal and not a transient. This is the procedure used by random vibration test machine controllers to produce stationary random vibrations with a desired power spectrum [8].

CONCLUSIONS

A unique method of producing a test transient time history from the Fourier transform envelope of several flight transients has been demonstrated. The overttest associated with using such a transient has been quantified using the overttest factor. By using the real and imaginary components of the Fourier transform constituting the spectrum envelope, a valid synthesized test transient can be produced.

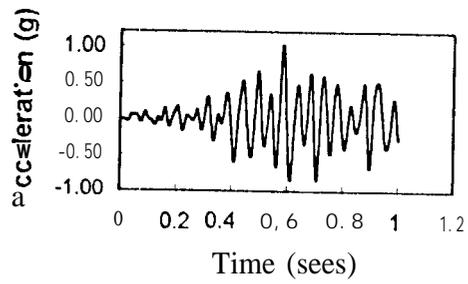
ACKNOWLEDGMENTS

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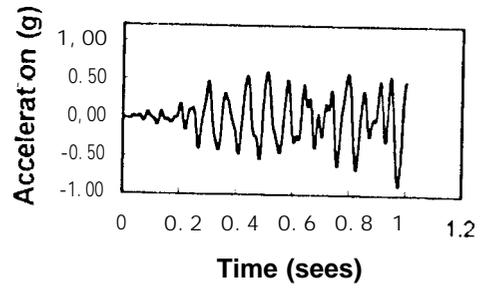
REFERENCES

1. "Swept Sine Vibration Test Conservatism". Journal of the Institute of Environmental Sciences, November/December, 1995, pp. 13-17.
2. "Controlling Conservatism in Transient Vibration Tests". Proceedings of the 15th. Aerospace Testing Seminar, pp. 231-237, Oct. 11-13, 1994, Manhattan Beach, California.
3. D. O. Smallwood & A. R. Nerd, "Matching Shock Spectra with Sums of Decaying Sinusoids Compensated for Shaker Velocity and Displacement Limitations", The Shock and Vibration Bulletin 44, (part 3 of 5 parts), August 1974.
4. D. Nelson, personal communication, Sandia National Laboratories, Livermore, California, August 1995.
5. J. J. Baca, "Alternative Shock Characterizations for Consistent Shock Test Specification". The Shock and Vibration Bulletin S4, (part 2), June 1984.
6. S. D. Stearns, "Digital Signal Analysis". Hayden Book Company, inc., 1975.
7. T. J. Baca and T. D. Blacker, "Relative Conservatism of Drop Table and Shaker Shock Tests". The Shock and Vibration Bulletin S6, (part 1), August 1986.
8. A. G. Piersol, personal communication, February 27, 1996

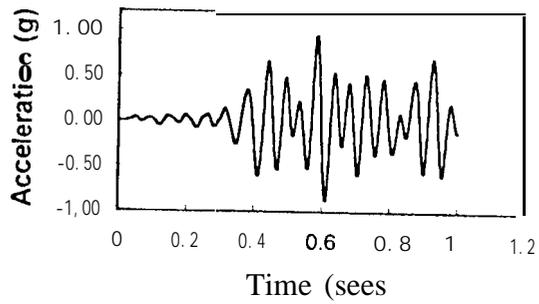
0401 Transient



1401 Transient



0419 Transient



1419 Transient

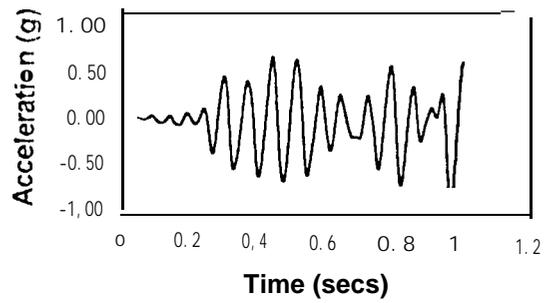


Figure 1- Transient Time Histories

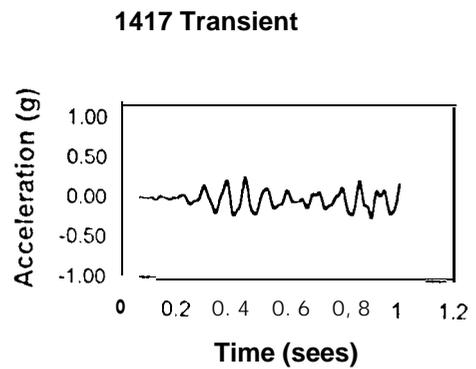
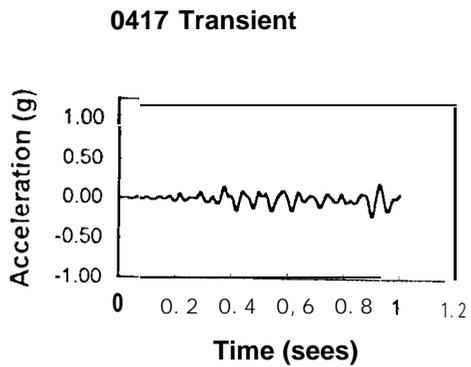
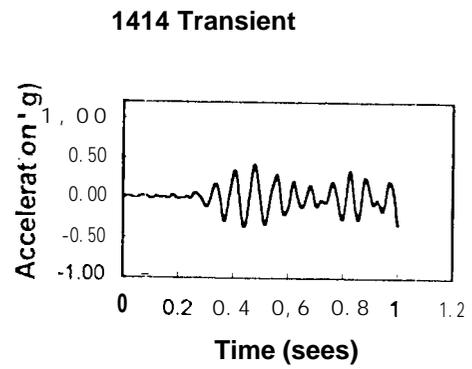
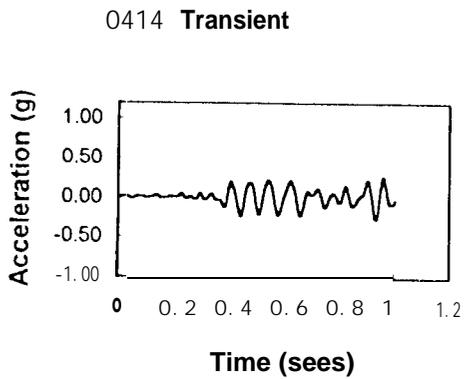
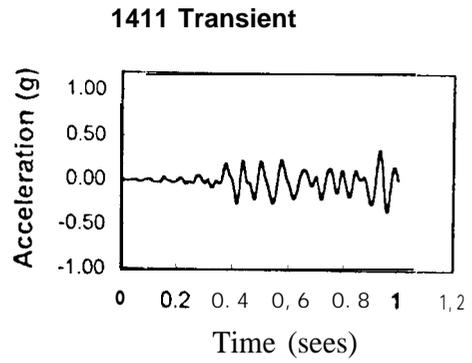
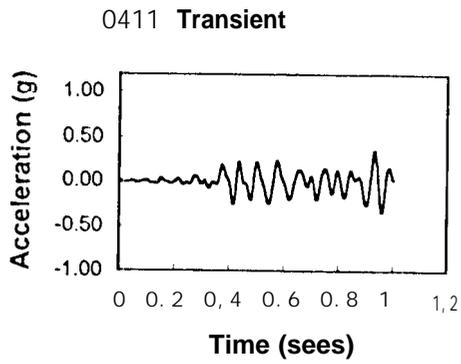


Figure 2- Transient Time Histories

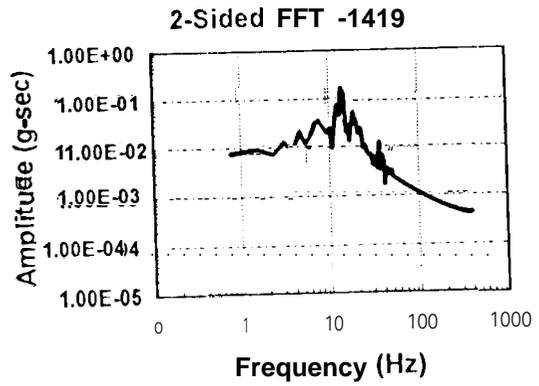
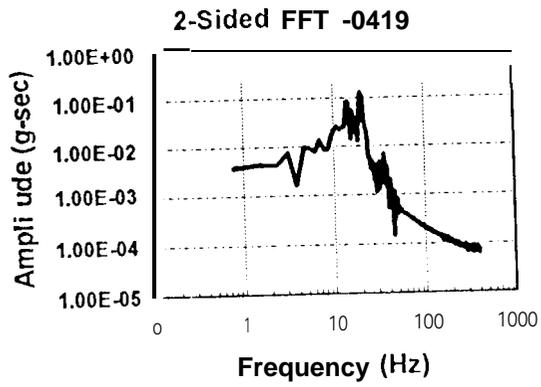
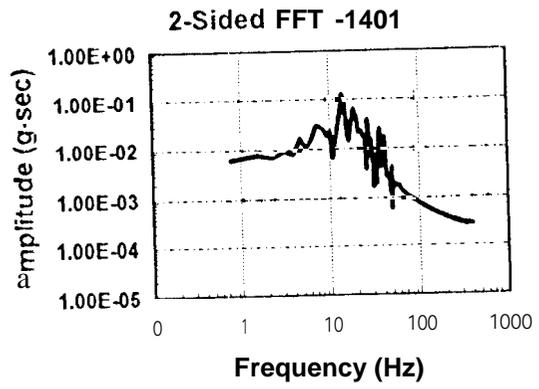
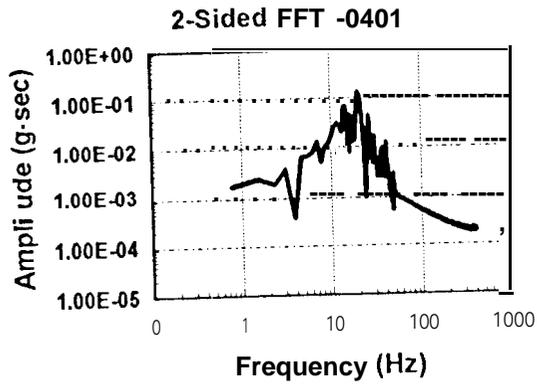


Figure 3- Fourier Transforms

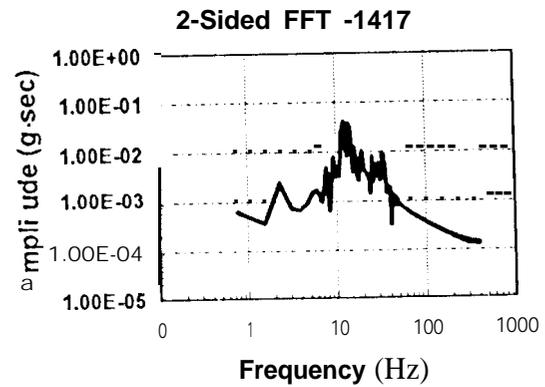
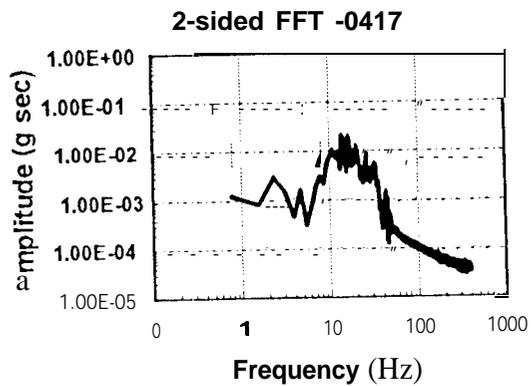
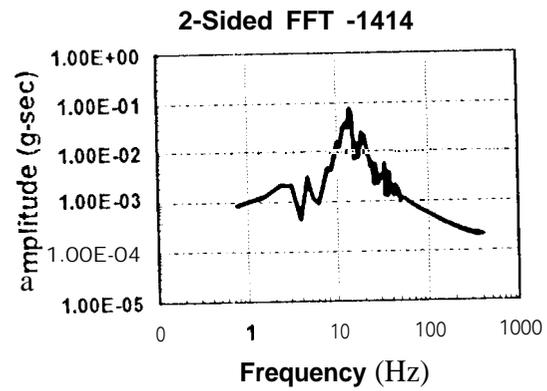
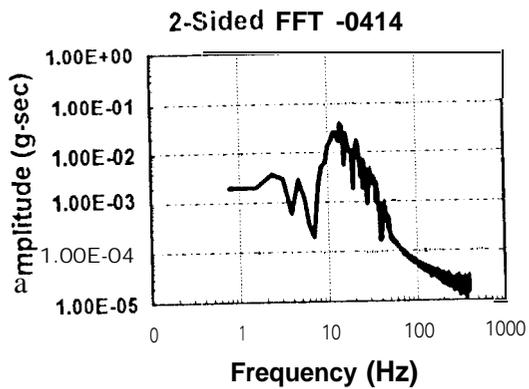
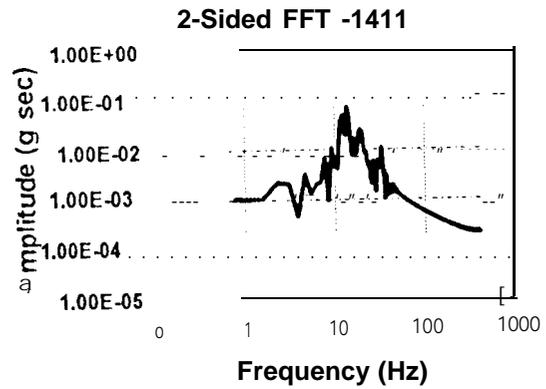
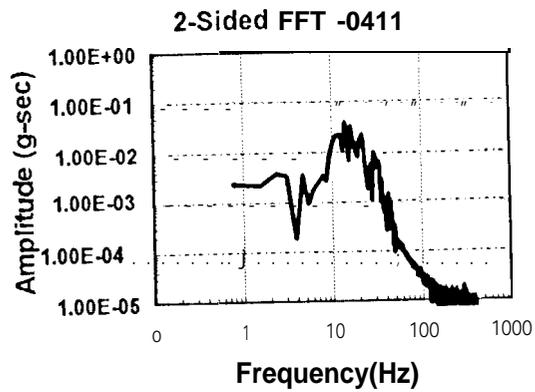


Figure 4- Fourier Transforms

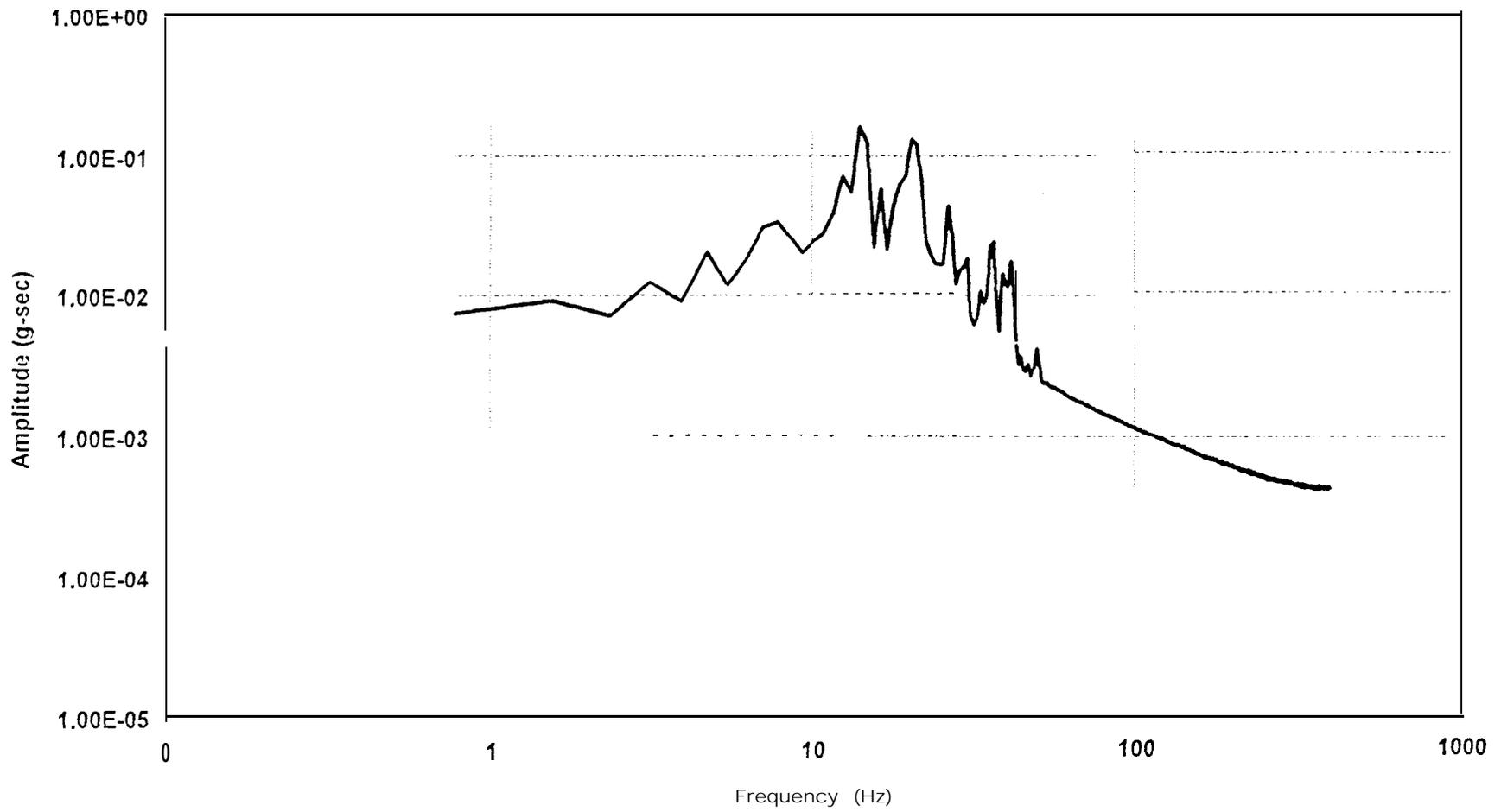


Figure 5- Fourier Transform Envelope

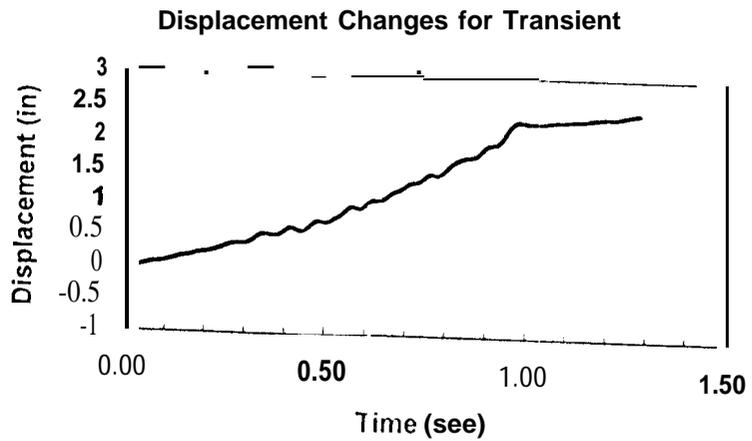
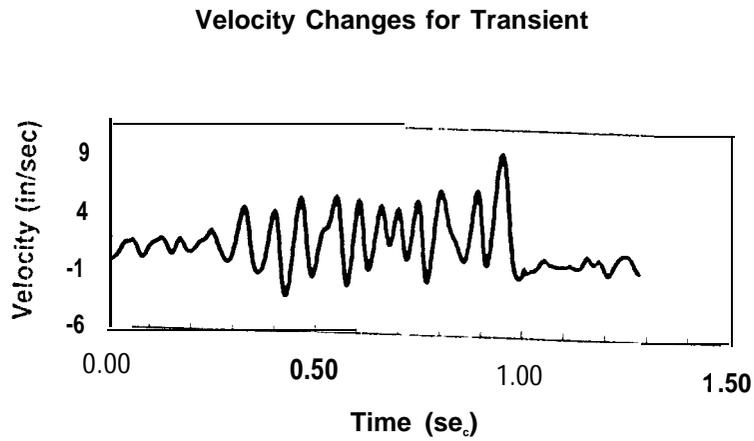
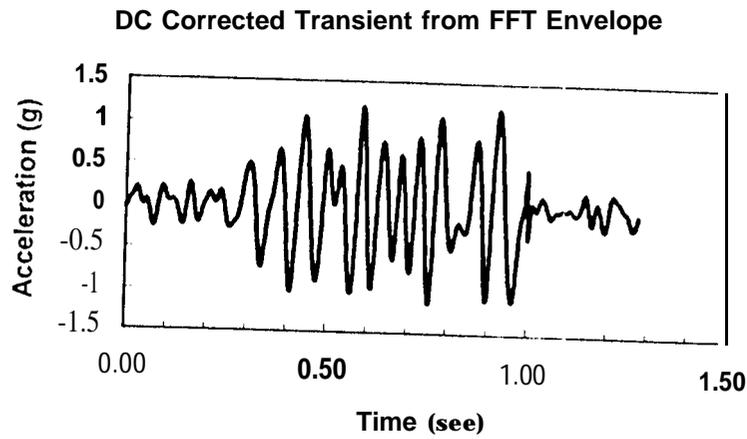


Figure 6- Characteristics of DC Corrected Transient

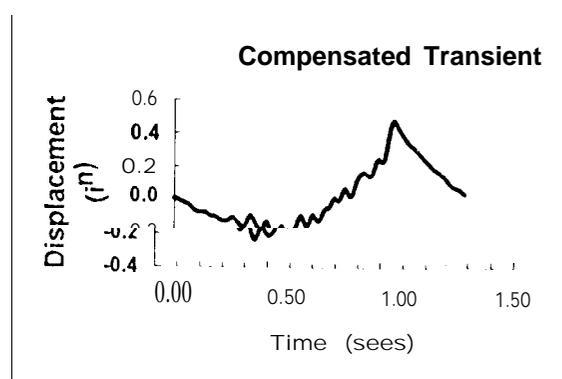
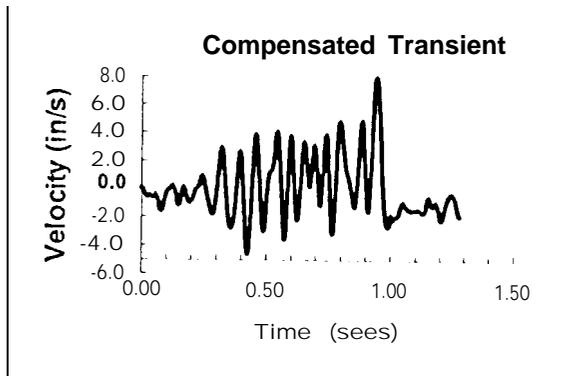
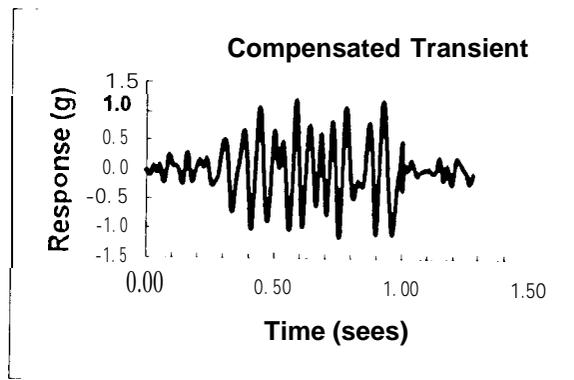
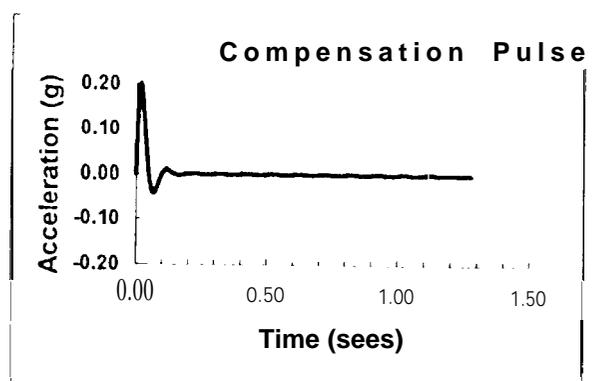
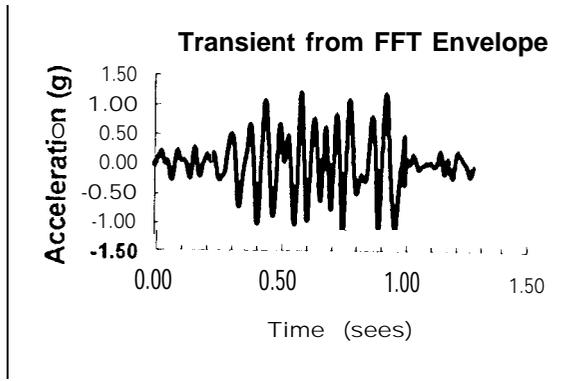


Figure 7- Characteristics of DC and Pulse Compensated Transient

Absolute Maximax Shock Response Spectrum

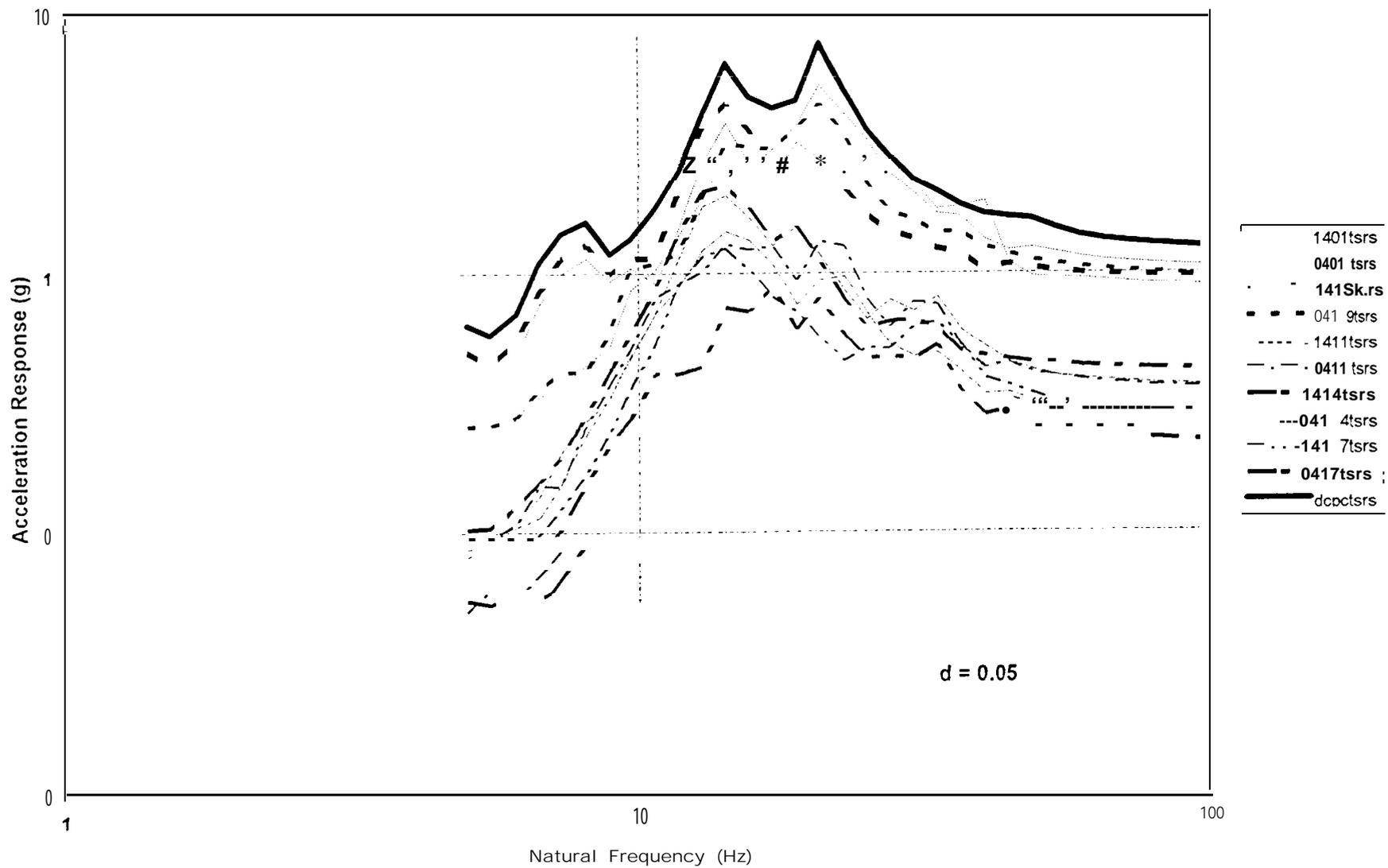


Figure 8- SRS of Transients

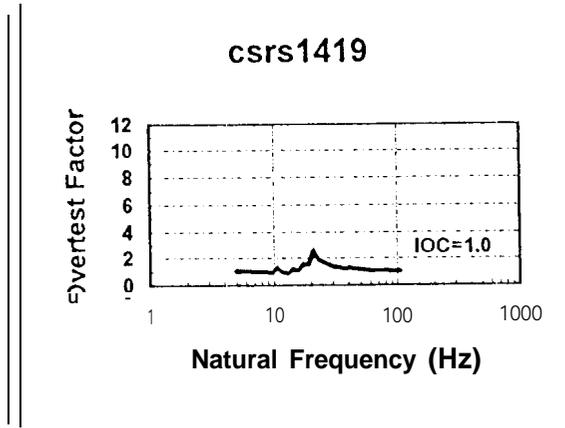
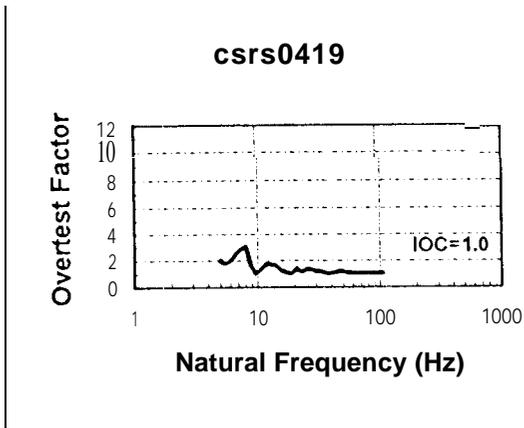
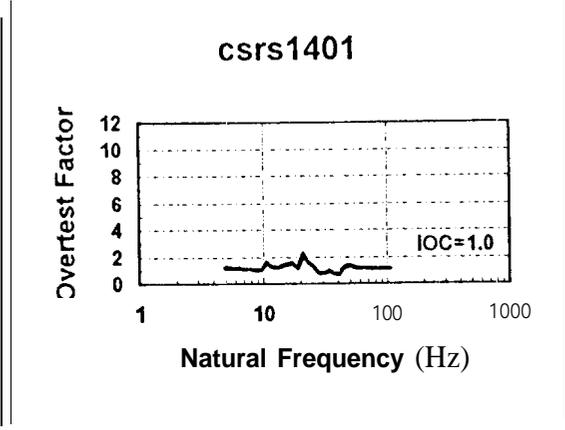
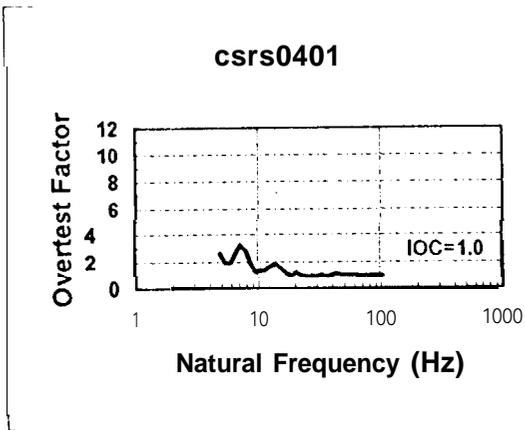


Figure 9- Overtest Factors for Transient Test

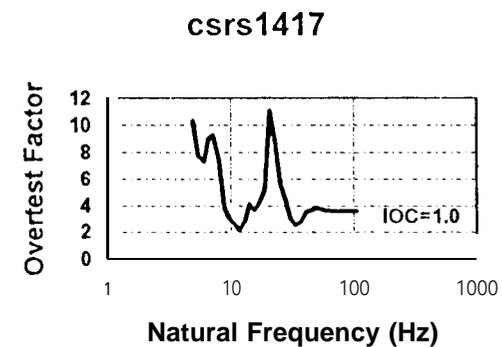
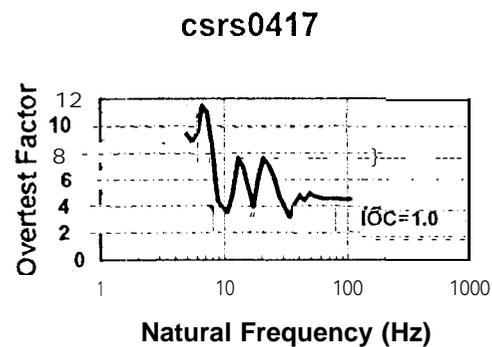
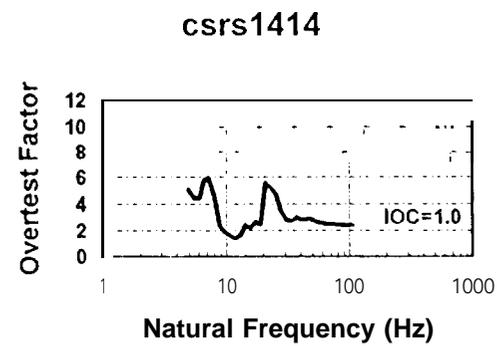
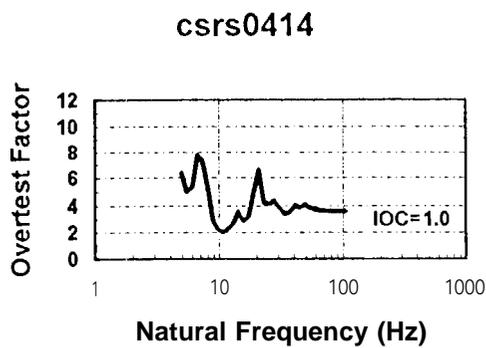
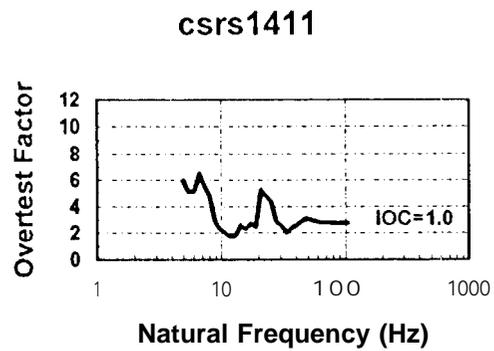
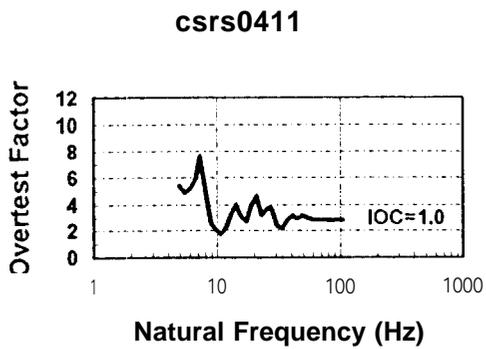


Figure 10 -Overtest Factors for Transient Test

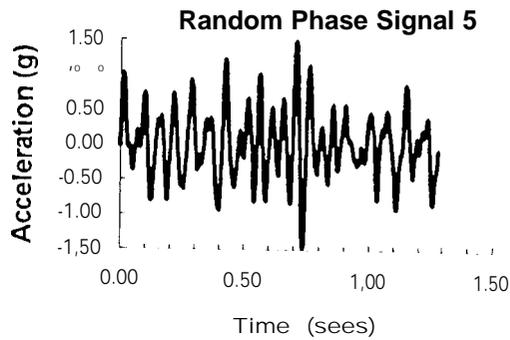
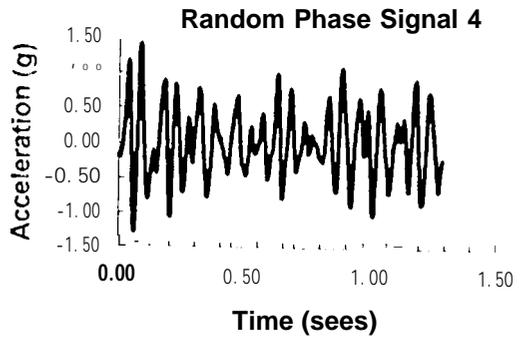
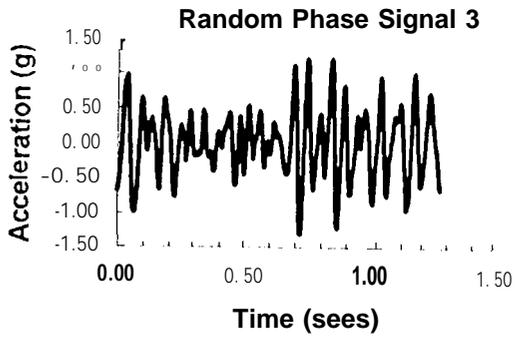
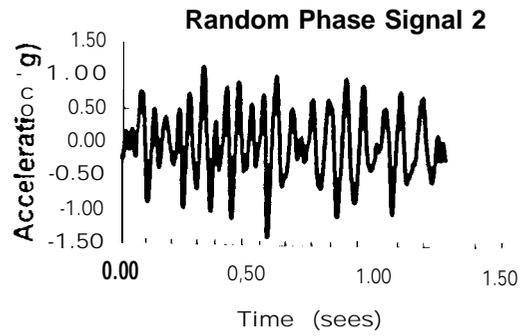
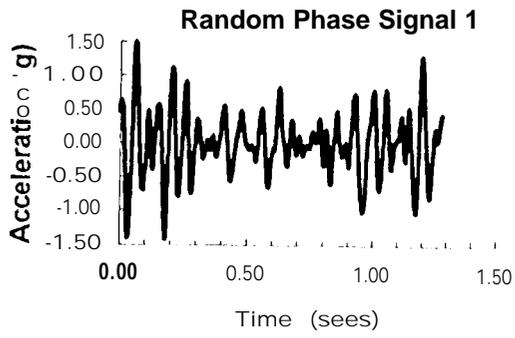


Figure 11. Random-Phase Signals