A 360-Degree and -Order Model of Venus Topography

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This report presents the most recent spherical harmonic topography model of Venus developed at Jet Propulsion Laboratory. It was produced by a spherical harmonic analysis of the most complete set of Magellan altimetry data, augmented by Pioneer Venus and Venera data. The harmonic coefficients of the topography were computed to degree and order 360. Compared to previous topography models, this one has the highest correlation with the gravity field of Venus. © 1994 Academic Press, Inc.

1. INTRODUCTION

This report provides the scientific community with the latest spherical harmonic topographic model of Venus produced at the Jet Propulsion Laboratory (JPL). The data set that was used is described in Section II. Section III presents the harmonic analysis of Venus' topography. Section IV presents scientific implications. Section V contains a summary.

II. DATA

The starting point for building the set of data that were analyzed was the GTDR files, which contain maps of Magellan altimetry data, produced by the Massachusetts Institute of Technology for the Magellan Project (Ford and Pettengill 1992). These data cover the planet by means of three maps: a Mercator projection map and two polar maps. These maps provide the most internally consistent version of the Magellan altimetry data set (Professor Ford, personal communication). The maps were projected and averaged into a cylindrical grid of $0.25^\circ \times 0.25^\circ$. The original data have a pixel size of $\sim 5$ km. Hence, the reprojection process reduces the resolution of the data set, especially at low latitudes. Overall, every sample of our topography model is based on a number of Magellan altimetry data points.

We follow Bills and Kobrick (1985) and Konopliv et al. (1993) and use a rectangular grid. This is intuitively justified from the behavior of the harmonic functions: each harmonic function has the same number of zeroes on each parallel. Furthermore, because of the method that we use to compute the harmonic coefficients, each data point is effectively weighted by the area of the corresponding cell.

The pre-Magellan topography model (TOPODR 4.0; Yewell 1993), which consisted of data from Pioneer Venus Orbiter (PVO) altimetry merged with Venera 15/16 altimetry, was used to fill gaps.

At this point, three gaps remained in the data. The two main gaps are located near the south pole, and a smaller gap is present near the north pole. These gaps have two separate origins: orbits during superior conjunction and orbits affected by the thermal hide strategy, in which the High Gain Antenna was used to shadow the spacecraft, precluding altimetry observations.

Our first topography model of Venus used a less complete set of Magellan anti PVO altimetry data and a least squares method to compute the harmonic coefficients of the topography to order and degree 21 (McNamee et al. 1993). The least squares method did not require that the gaps be filled. Our second topography model used a more complete set of data and a computation by quadrature to obtain the coefficients to degree and order 20 (Konopliv et al. 1993). This new method required that no gap be present in the data. Therefore, we filled tile gaps by using topographical heights computed from the $21 \times 21$ model. The same quadrature method is used in this paper. The gaps were filled by using the $120 \times 120$ model. Finally, we obtained a complete set of data referenced to the Venus body-fixed reference frame, as defined by Davies et al. (1992).

III. HARMONIC ANALYSIS OF THE TOPOGRAPHY

The planetary radius at latitude $\varphi$ and longitude $\lambda$ with respect to the body-fixed reference frame defined by the Venus rotation axis anti prime meridian is written as
\[ R = \sum_{\ell=0}^{\infty} \sum_{m=0}^{\ell} P_{\ell m}(\sin \varphi) \]

\[ (C'_{\ell m} \cos m \lambda + S'_{\ell m} \sin m \lambda), \]

where \( R \) is the equatorial radius, \( C'_{\ell m} \) and \( S'_{\ell m} \) are the normalized harmonic coefficients of the topography and are the Legendre functions.

### Method

There are two methods of computing the harmonic coefficients of the topography. The first method consists of cast squares fit of Eq. (1) to the data. In practice, this method requires too much computing time if the degree is high. A much less computationally expensive method consists of computing the harmonic coefficients independently, as integrals. The coefficients are given by

\[ d\lambda(R(\varphi, \lambda) - R_0) P_{\ell m} \cos m \lambda \]

\[ d\lambda(R(\varphi, \lambda) - R_0) P_{\ell m} \sin m \lambda, \]

where \( R \) is the a priori reference radius. These coefficients are resealed by introducing a new reference radius \( R_0 \) such that \( C_{00} = 1 \).

### Results

The harmonic coefficients of the topography were computed using individual integrals of degree and order 360. The radius of Venus is found to be \( R_0 = 6051.848 \) km. Determinations led to \( R_0 = 6051.448 \) km (Bills & al. 1985), \( R_0 = 6051.839 \) km (McNamee et al. 1993), and \( R_0 = 6051.839 \) km (Konopliva et al. 1993). The \( \Delta \) between Bills and Kobrick determination and ruminations by McNamee et al., and Konopliva et al. to the fact that Bills and Kobrick used preliminary data that were not completely reduced is due to the fact that the altimetry data that we used in this paper were substantially improved with respect to the data used in the two previous papers. This improvement is a consequence of a reduction in the orbital errors.

The harmonic coefficients up to degree 2 are given in Table 1. In order to obtain an order of magnitude of the uncertainties in the coefficients, we performed three additional calculations in which the data were modified by adding \( \pm 80 \) m to each data point, where the sign was chosen randomly for each point. The uncertainties listed in Table 1 correspond to the maximum difference between the nominal case and the three cases where the data quality had been artificially degraded. These uncertainties are pessimistic, first because the error of \( \pm 80 \) m is itself pessimistic, and then because they cumulate the errors made in the three cases where the data were degraded. On the other hand, the method that we used to evaluate the uncertainties in the individual coefficients assumes that the errors in the topographic heights are uncorrelated and does not take into account the over-sampling near the pole, and in that sense it may be optimistic.

Figure 1 shows topography contours obtained with our topography model. Figure 2, which shows the topography along the equator, both as given by the data (continuous line) and as computed from the model (black triangles), indicates that there is a good agreement between the model and the data. Figure 3 contains a plot of the difference between the topography along the equator computed from the model and the observed topography. This plot...
shows that the error in the model is due to the high-frequency terms.

Our topography model can be obtained by writing to the authors of the paper and it will be archived in the Planetary Data System Geosciences Node at Washington University (Saint Louis, MO).

IV. IMPLICATIONS

This section is devoted to scientific results obtained from our model.

IV.1. Geometrical Implications

Since the flattening of Venus is very small, its topography can be approximated by an axis off-set from the origin of the coordinate system, which is Venus' center of mass. According to this definition of the offset between the center of mass and the center of figure, the location of the center of figure is given by

\begin{align*}
X_f &= R_c C_{11} = 118 \text{ m}, \\
Y_f &= R_c S_{11} = 108 \text{ m}, \\
Z_f &= R_c C_{10} = 6 \text{ m}.
\end{align*}

The above definition is used by scientists who are interested in understanding the internal structure of Venus. On the other hand, there exist other areas of planetary sciences where scientists use approximations of planetary
body’s shapes as ellipsoids offset from the center of mass (Davies et al. 1992). An example of a situation in which the ellipsoid model is useful is that of occultation experiments. To first order with respect to the harmonic coefficients, the location of the center of figure of the ellipsoid is given by Eq. (3). A more accurate method consists of fitting an offset ellipsoid by least squares to the topography. This calculation yields

\[ \begin{align*}
X_f &= 205 \text{ m}, \\
Y_f &= 181 \text{ m}, \\
Z_f &= 71 \text{ m},
\end{align*} \]

which corresponds to the latitudinal coordinates

\[ \begin{align*}
\varphi_f &= 282 \text{ m}, \\
\varphi_f &= 14^\circ, \\
\lambda_f &= 139^\circ,
\end{align*} \]

and indicates that the center of figure lies under the northeast part of Thecis Regio.

The least square fit also gives the orientation and the length of the principal axes of figures. These are given in Table 11, together with the orientation of the principal axes of inertia. The axes of figures and the axes of inertia do not coincide. The orientation of the axes of figures is governed by the equatorial and mid-latitude highlands. The longest axis of figure passes through Phoebe Regio and Niobe Planitia, North of Ovda in Western Aphrodite. The second axis of figure passes through Fistlia Regio and South of Atla in Eastern Aphrodite.

IV.2. Statistical Implications

The rms magnitude of the normalized coefficient, defined by

\[ \text{rms}(\ell) = \frac{\sqrt{\sum (C_{\ell m}^2 + S_{\ell m}^2)}}{2\ell + 1}. \] (4)

where the sum goes from \( m = 0 \) to \( m = \ell \) is shown on Fig. 4, in logarithmic scale. As in Section 111.2, we degraded the topography data by adding randomly \( \pm 80 \text{ m} \) to each data point in order to estimate uncertainties in the spectrum. Even though the individual coefficients change, the spectrum remains very stable and the curve corresponding to the degraded topography data practically coincide with the nominal curve.

For \( \ell \geq -100 \), the slope of the spectrum changes and the spectrum becomes flatter. This may not be a real property of the spectrum but rather an artifact associated with our method. As mentioned in Section 111.2, the errors in the topography model are essentially in the high-frequency terms. One possibility is that the oversampling at high latitude had the effect of producing too much power in the high-degree harmonic coefficients.

We compared the variance of the observed topography with the variance of the topography computed from our model. Denoting by \( \tilde{f} \) the topography, the variance was computed using the equation

\[ \sigma_f^2 = \frac{1}{S} \int_{-\theta/2}^{\theta/2} \int_{-\pi/2}^{\pi} \left[ f(\varphi, \lambda) - \tilde{f} \right]^2 \, dS, \] (5)

where \( dS \) represents an element of surface and

<table>
<thead>
<tr>
<th>Axes of figure</th>
<th>Axes of inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>13.8°</td>
</tr>
<tr>
<td>Longitude</td>
<td>98.2°</td>
</tr>
<tr>
<td>Length</td>
<td>6052.3 km</td>
</tr>
<tr>
<td>Longitude</td>
<td>11.2°</td>
</tr>
<tr>
<td>Length</td>
<td>6051.8 km</td>
</tr>
<tr>
<td>Latitude</td>
<td>168.4°</td>
</tr>
<tr>
<td>Length</td>
<td>6051.4 km</td>
</tr>
</tbody>
</table>
With the observed topography, we obtained $\sqrt{\sigma_f} = 0.943$ while our model gives an almost equal value of $\sqrt{\sigma_f} = 0.941$. The agreement between these two values is due to the fact that the variance is essentially due to the low-degree terms and does not allow one to rule out that the model overestimates the power of the high-degree harmonic coefficients.

The correlation between Venus' topography and gravity was also investigated. The gravity field at a point $P$ outside the planet is

$$P'(r', y, \lambda) = \frac{GM}{r} \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \left( \frac{R_e}{r} \right)^{\ell} \left( \frac{R_e}{r} \right)^{m} \left( \sin \phi \right) \left( C_{\ell m} \cos m \lambda + S_{\ell m} \sin m \lambda \right),$$

where $G$ is the gravitational constant, $M$ is the mass of the planet, $R_e$ is a reference radius, and the $C_{\ell m}, S_{\ell m}$ are the harmonic coefficients of the gravity field.

The correlation per degree was computed as

$$\gamma(\ell) = \frac{\sum C_{\ell m} C_{\ell m}^* + S_{\ell m} S_{\ell m}^*}{\sqrt{\sum (C_{\ell m})^2} \sqrt{\sum (S_{\ell m})^2}},$$

where the sums go from $m=0$ to $m=\ell$. It is shown in Fig. 5 for three topography models, the one by Konopliv et al. (1993), the one by Balmino (1993), and the model presented in this paper. All calculations used the most recent gravity field model of Konopliv and Sjogren (1994). The uncertainty in the correlation due to the errors in the harmonic coefficients of the topography was computed.
### Table 111

<table>
<thead>
<tr>
<th>Region</th>
<th>( \gamma_{\text{min}} )</th>
<th>( \gamma_{\text{max}} )</th>
<th>( \lambda_{\text{min}} )</th>
<th>( \lambda_{\text{max}} )</th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
<th>( \gamma_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>1° 0'</td>
<td>105°</td>
<td>55°</td>
<td>87.7</td>
<td>0.819</td>
<td>0.934</td>
<td></td>
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<tr>
<td>Phoebe</td>
<td>10°</td>
<td>30°</td>
<td>30°</td>
<td>87.5</td>
<td>0.821</td>
<td>0.845</td>
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<tr>
<td>Maxwell</td>
<td>45°</td>
<td>80°</td>
<td>30°</td>
<td>84.3</td>
<td>0.857</td>
<td>0.864</td>
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<tr>
<td>Gula</td>
<td>0°</td>
<td>30°</td>
<td>15°</td>
<td>89.0</td>
<td>0.897</td>
<td>0.918</td>
<td></td>
</tr>
<tr>
<td>Bell</td>
<td>0°</td>
<td>40°</td>
<td>30°</td>
<td>77.5</td>
<td>0.790</td>
<td>0.812</td>
<td></td>
</tr>
<tr>
<td>Ovda</td>
<td>15°</td>
<td>10°</td>
<td>75°</td>
<td>79.0</td>
<td>0.790</td>
<td>0.805</td>
<td></td>
</tr>
<tr>
<td>Atla</td>
<td>10°</td>
<td>30°</td>
<td>180°</td>
<td>84.3</td>
<td>0.949</td>
<td>0.968</td>
<td></td>
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<tr>
<td>Planet</td>
<td>90°</td>
<td>0°</td>
<td>120°</td>
<td>75.3</td>
<td>0.745</td>
<td>0.772</td>
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</tr>
</tbody>
</table>

**Note:** \( \gamma \) is based on the topology model of Konopliv et al. (1993); \( \gamma_1 \) is based on the topography model of Balmino (1993); \( \gamma_2 \) is based on the topography model of Konopliv and Sjogren (1994).

V. SUMMARY

The first purpose of this paper is to make available to the scientific community the best model developed of the JPL Models of Venus topography in spherical harmonics (1992). The model gives the highest correlation of the principal axes is determined by the equatorial and mid-latitude highlands.

(c) The topography field of Venus is very strongly correlated. Regional correlation is higher in Beta Regio and Atla Regio, which are thought to be dynamically supported, than in Ovda Regio, which is thought to be essentially supported by passive isostasy.

(d) The rms spectrum of the topography follows a power law up to degree 100. The flattening of the rms spectrum for \( n > 100 \) may not be real.

### REFERENCES


gravity model to degree and order 60. *Icarus* **112**, 42-54.


