(SUI\MISSION 1'()1'111' PIERS 1996 SYMPOSIUM)

IMPULSE RESPONSE SHAPING FOR ULTRA WIDE BAND SAR IN A CIRCULAR FLIGHT PATH

Michael Y. Jin

Jet Propulsion Laboratory
California institute of Technology
Pasadena, California 91109, USA

PHONE: (818) 354-3778

TELEFAX: (818) 393-6943

E-MAIL: michael_jin@radar-email.jpl.nasa.gov

The research described in this paper was carried out by the Jet Propulsion Laboratory, California institute of Technology, under a Contract with the National Aeronautics and Space Administration.

Abstract

An ultra wide band SAR has potential applications on imaging underground objects. Flying this SAR in a circular flight path is an efficient way to acquire high resolution image from a localized area. This paper characterizes the impulse response of such a system. The results indicate that to achieve an image with a more uniformed resolution over the entire imaged area, proper weighting coefficients should be applied to both the principle aperture and the complementary aperture.

1. Introduction

To attain useful resolution for a low frequency SAR (Synthetic Aperture Radar) for image mapping, the radar must be designed with a frequency bandwidth close to or greater than its center frequency. This kind SAR is referred to as the ultra wide band SAR. An ultra wide band SAR is capable of an ultimate resolution, which is a fraction of the wavelength of its center frequency. However, it is relatively difficult to achieve the ultimate resolution from such an airborne SAR flying in a straight line path because of the difficulty in handling its extremely long aperture. On the contrary, the synthetic aperture length for this SAR in a circular flight path has a more reasonable aperture length and allowing a full 360 degree of viewing angle. Due to its potential application in under ground mapping, it is of great interest in understanding the characteristics of this system.

The principle of imaging for an ultra wide band SAR in a circular flight path is similar to the Inverse SAR (ISAR) [1]. in both of these systems, a target is observed by the radar through a complete 360 degree. However, in ISAR, the distance between the radar and targets is usually much greater than the dimension of the targets. Thus, the problem of ISAR is related to tomography dealing with line integral of parallel lines. Due to the constraint of antenna dimension, an ultra wide band SAR has a much broader radar beam angle. Therefore, the dimension of the mapping area is comparable to the distance between the radar and targets. Hence, the problem of an ultra wide band SAR is related to tomography dealing with line integral of concentric circles [2]. Due to the complexity of the mathematics, a comprehensive analysis on this system is yet not available. This paper intends to characterize the point-target impulse response and presents method to control the shape of the impulse response.

2. **Analysis**

The mapping geometry of an ultra wide band SAR in a circular flight path is shown in Figure 1. To simplify the following analysis, we assume that the radar footprint covers the same area as the ground projection of the flight circle. The first part of the analysis shall be focused on a special case where the altitude of the radar is much smaller than the radius of the flight circle. When the radar altitude is equal to zero, the problem of this system is identical to that of an acoustic reflective tomography [2].

Low Altitude Geometry

An exact solution to this problem was given in [3]. However, this solution is yet not practical due to the lack of accuracy and efficiency in implementing the required Hankle transforms. The algorithm of filtered backprojection were also suggested [1] to yield a satisfactory impulse response. However, the analysis was given based on an approximation that is only applicable to targets near the center of the circle. The following analysis will show that the same waveform also holds with very little error for targets near the inner rim of the circle.

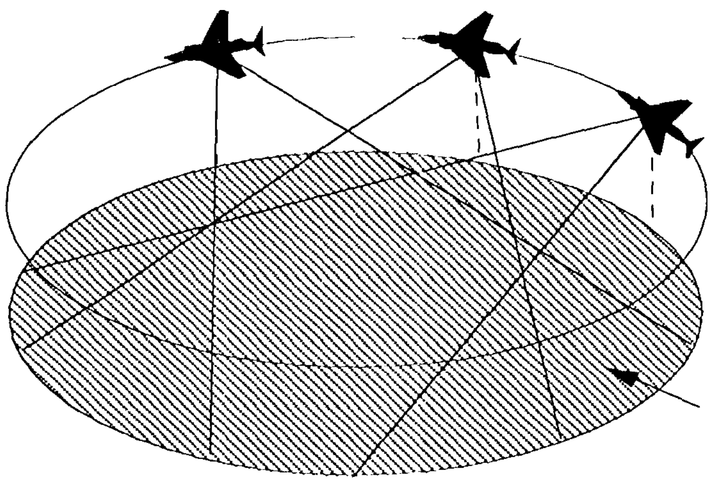


Figure 1 An Ultrawide Band SAR in circular path

Let (r_1, θ_1) be the target position in polar coordinate and $r_{ot}(\theta', r_1, \theta_1)$ be the radar-target distance as a function of the radar polar angle O'. Assuming that the echo from a point target has been compensated for its slant range variation. To simplify the problem furthermore, we assume that the antenna gain over the illuminated area is a constant. The impulse response of the target obtained from the filtered backprojection algorithm may be formulated as a two dimensional integral along the radar angle and radar frequency, i.e.

$$\sigma_{pt}(r,\theta) = \oint \int_{-R_0}^{R_0} e^{-j2\pi\, r_{ot}(\theta',r_1,\theta_1)R} \cdot |R| \cdot e^{j2\pi\, r_{ot}(\theta',r,\theta)R}\, dR\, d\theta'$$

where R is the frequency coordinate of the radar and R_0 is the maximum frequency of the radar. The factor $|R|$ is the filter transfer function which emphasizes the high frequencies. A closed form result for the above integral is difficult to obtain. By denoting $\Delta r(\theta') = r_{ot}(\theta', r_1, \theta_1) - r_{ot}(\theta', r, \theta)$, one may rewrite the above equation as

$$\sigma_{pt}(r,\theta) = \oint \int_{-R_0}^{R_0} |R| \cdot e^{j2\pi\Delta r(\theta')R}\, dR\, d\theta'$$

$$= \oint \int_{0}^{R_0} |R| \cdot cos(2\pi\Delta r(\theta')R)\, dR\, d\theta'$$

let Δr_m represents the maximum value of $\Delta r(\theta')$ and $\rho(\theta')$ be a ratio function of $\rho(\theta') = \Delta r(\theta') / \Delta r_m$, the above equa t ion becomes

$$\sigma_{pt}(r,\theta) = \oint \int_{0}^{R_0} |R| \cdot cos(2\pi\Delta r_m \rho(\theta')R)\, dR\, d\theta'$$

It is apparent that the range of $\rho(\theta')$ is within the interval of [–1, 1]. Since $\cos(-\phi) = \cos\phi$, we can further write the above equation as

$$\sigma_{pt}(r,\theta) = \oint \int_{0}^{R_0} |R| \cdot cos(2\pi\Delta r_m |\rho(\theta')| R)\, dR\, d\theta'$$

Now, one may use a probability density function $p(\rho)$ to express the above equation as

$$\sigma_{pt}(r,\theta) = \int_0^1 \int_{0}^{R_0} p(\rho)\, |R| \cdot cos(2\pi\Delta r_m \rho R)\, dR\, d\rho$$

The above equation can be used to test the closeness of the impulse response of two point targets by comparing their corresponding probability density function p(p). For a point located at the center of the circle, its p(p) is plotted in Figure 2. This is actually the probability density function of the magnitude of a cosine function.

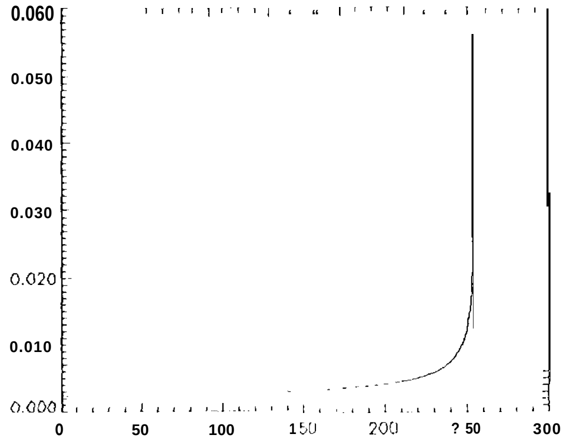


Figure 2. The probability density function $p(\rho)$ for the center target

One may use numerical method to obtain p(p) for all target within the radar footprint. The resultant p(p) of all the targets are very close to that of the center target. As shown in Figure 3. are the ratios of the $p(\rho)$ of each point target to the p(p) of the center target. The high frequency noise in these plots are due to limited samples involved in generating these functions. It is clearly shown that the difference of the probability density is within 0.5%. The 1-D cut of the impulse responses in the horizontal axis of these three targets are plotted in Figure 4. With the presented plotting scale, no difference can be seen. in addition, these impulse responses follow the waveform of a]inc function. These impulse responses may be shaped by applying a weight function to the radar spectrum. The impulse responses with reduced sidelobe energy by the Harming weight function is shown in Figure 5.

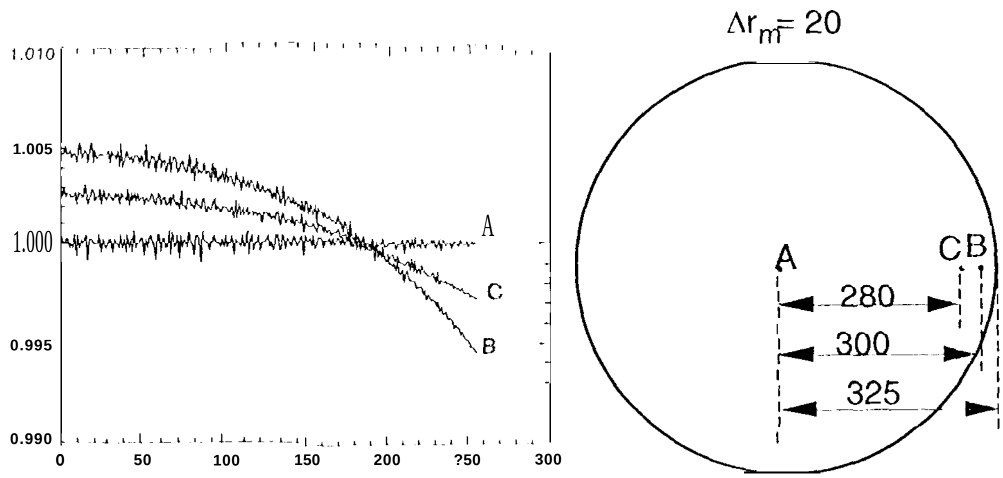


Figure 3. The ratio of probability density function for three points

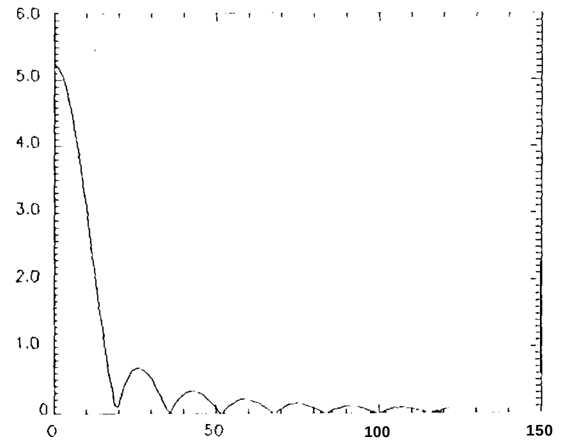


Figure 4. The 1-D cut of the impulse response of three points

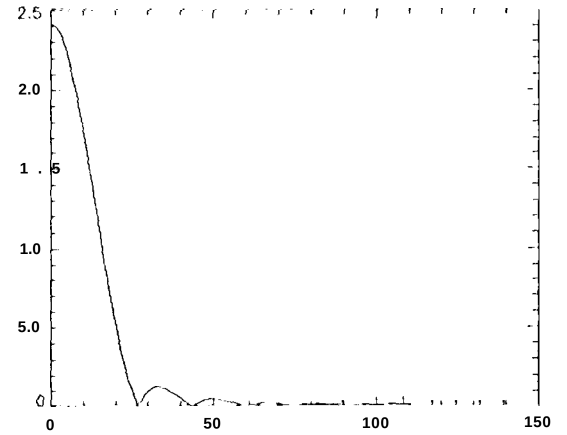


Figure 5. The 1-D cut of the Hanning weighted impulse response

High Altitude Geometry

The analysis for the low altitude case can not be applied here since in general the range of $\rho(\theta')$ is within the interval of [- ζ, 1] where $|\zeta| < 1$, This fact makes the pdf p(p) change significantly from point to point. Consequently, the shape of the point-target impulse response varies from point to point.

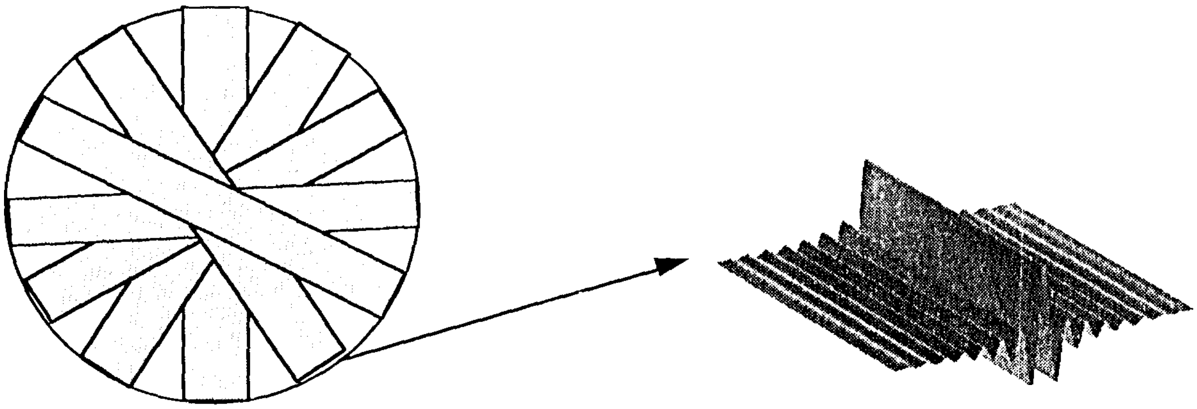
Some analysis for this geometry can be found in [4]. Here, we shall again use the filtered backprojection algorithm for data processing. The shape of the impulse response may be obtained directly from simulation. However, the following analysis provide a significant insight to the waveform of the impulse response. According to the filtered backprojection algorithm, the impulse response in the ground projection coordinated may be viewed as the superposition of ground projected images constructed from each radar echo. Near the location of the point-target, the point-target image constructed from each echo is a line-shaped response, as shown in Figure 6, which is uniform in one dimension and of the following form in another dimension

$$h(x) = 2R\,\mathrm{sinc}(2R_1 x) - \mathrm{sinc}^2(R_1 x)$$

The resolution of this function is determined by the bandwidth R_1, which is determined by the radar bandwidth R_0 as well as the sine of look angle of the radar at the time acquiring this echo. Or,

$$R_1 = R_0 \cdot \sin\theta_L(\theta') = R_0 \cdot \frac{d_{st}(\theta')}{\sqrt{d_{st}^2(\theta') + h^2}}$$

where $d_{st}(\theta')$ is the distance between the ground projection of the radar to the target and h is the aircraft altitude. The spectrum of this line-shaped image is also a single line perpendicular to the image line. Since the line-shape image is a real function, this spectrum is consisted of two segments, each being the complex conjugate of the other. By superposing only one segment of this line spectrum for all echoes covering 360 degree, a single sideband spectrum can be formed. An example of this spectrum is given in Figure 7. It is shown that the frequency band coverage varies as a function of polar angle. By combining both segments of the line spectrum, one may form the full (double sideband) spectrum. Two examples of this spectrum are given in Figure 8 representing those of two point targets. From this spectrum model for a point-target, it is obvious that the look angle variation has great effect on the shape of the final impulse response.



Impulse Response from many views (echoes)

Line Response from one view (one radar echo)

Figure 6 impulse response Superposed from Line Responses

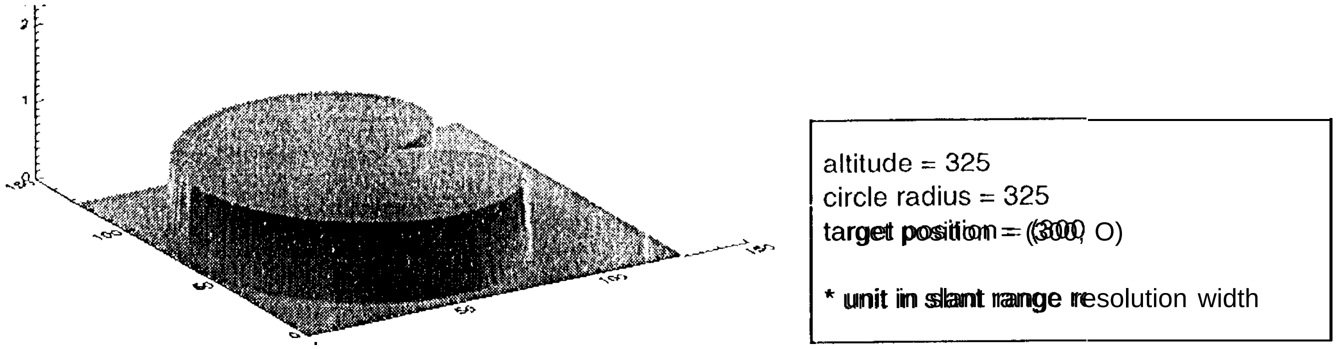


Figure 7. Single Side-Band Spectrum of the PT impulse Response in Ground Projection

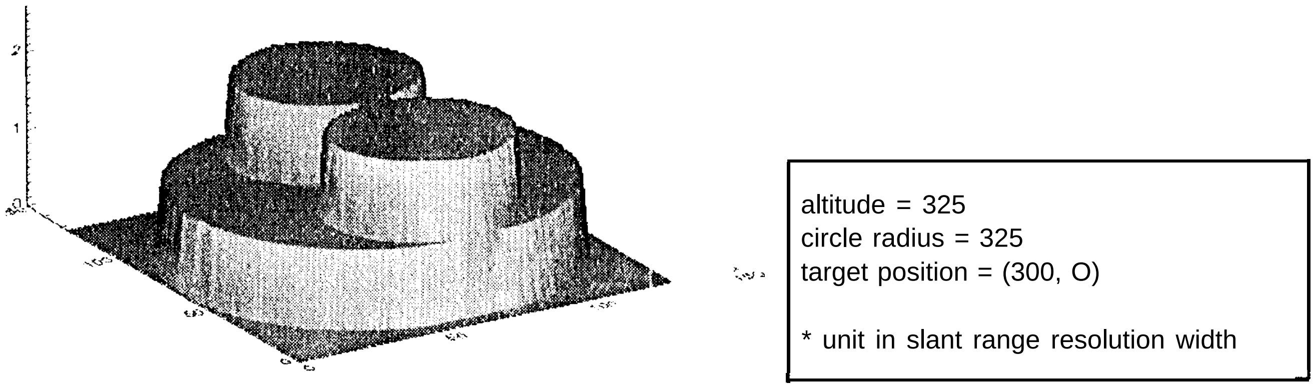


Figure 8a. Full (Double Sideband) Spectrum of the PT impulse Response in Ground Projection

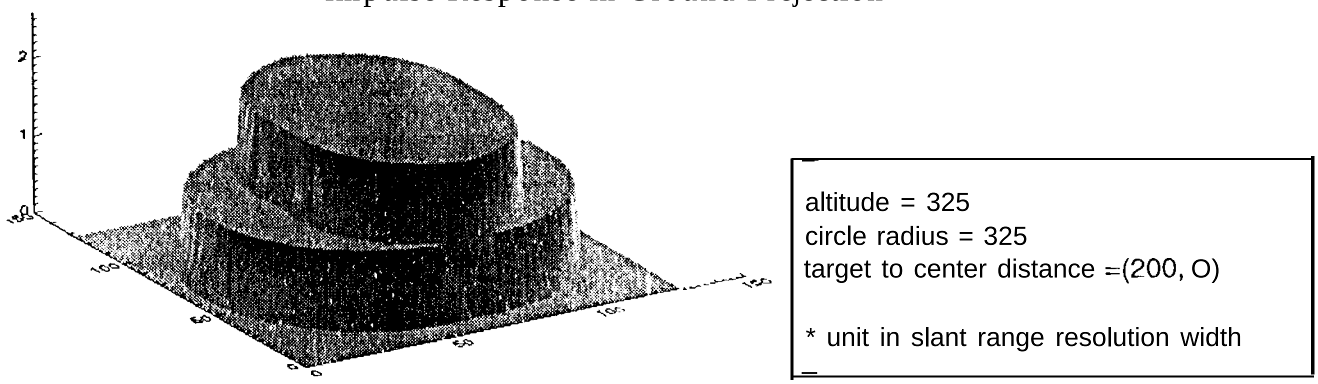


Figure 8b. Full (Double Sideband) Spectrum of the PT Impulse Response in Ground Projection

in Figure 8, the spectrum may also be viewed as the superposition of the amplitudes of two layers. The origin of the top layer is from the echo in the principle aperture [5]. The origin of the bottom layer is from the echo in the complement aperture [5]. Since the frequency coverage of the top layer is less than the bottom layer, the resolution performance shall be worse by processing echoes including the principle aperture.

A simulation has been carried out to characterize the point target impulse response. By including both the principle aperture and complement aperture echoes in data processing, the artifact clue to the principle aperture is evident for target near the edge of the flight circle projection. This is shown in cut c in Figure 9a. This artifact is less obvious for target more close to the center of the circle as shown in Figure 9b. By excluding the echoes in the principle aperture, one may obtain a more sharp impulse response as shown in Figure 10.

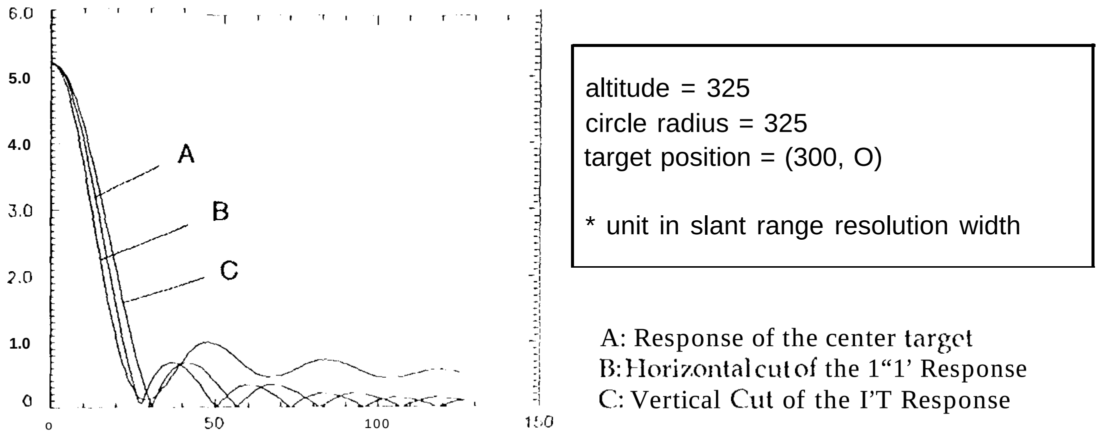


Figure 9a 1-D Cut of Impulse Response Processed by both Principle and Complement Apertures

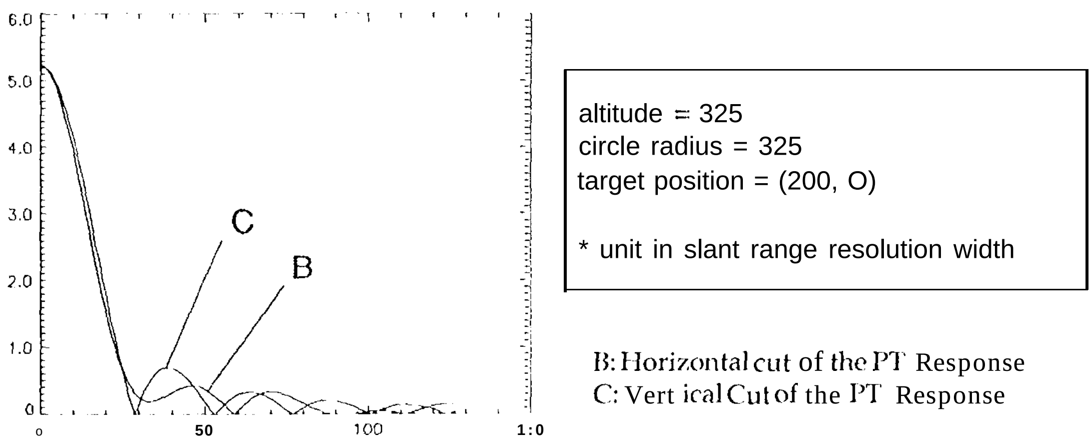


Figure 9b 1-D Cut of Impulse Response Processed by both Principle and Complement Apertures

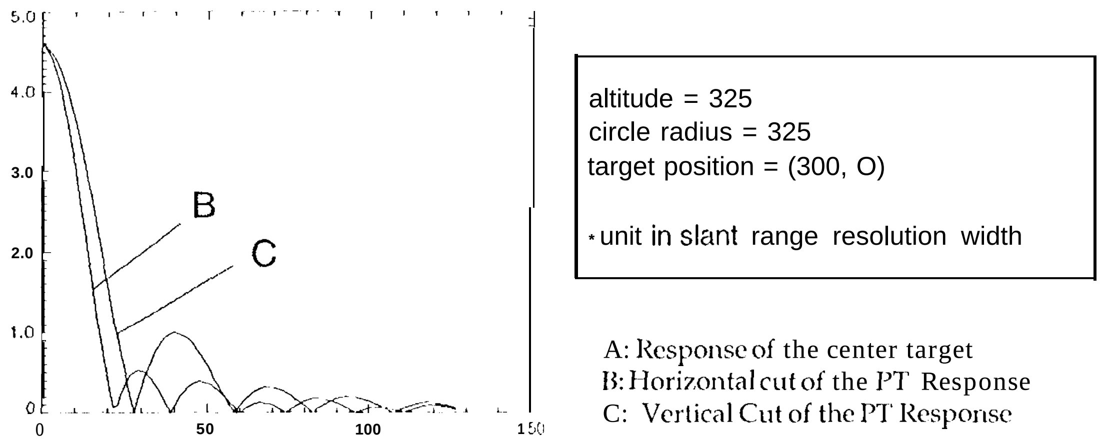


Figure 10 1-D Cut of Impulse Response Processed by the Complement Aperture

Summary

This paper has presented the analysis and simulation results for the impulse response of a an ultra wide band SAR system. For the low altitude case, a probability density function provides the insight of the little difference in the waveforms between targets near the inner rim of the circle and the center target. For the high altitude case, it has shown that the shape and the resolution of the impulse response are relatively uniform for targets in the inner part of the circle. The shape starts to change when the difference of the look angles between the principle aperture and the complement aperture becomes more significant. To achieve a relatively uniform shape and resolution, one may put proper weights on the echo response of both the principle aperture and the complement aperture.

Reference:

1. Mensa, D. L., 1991, High Resolution Radar Cross-Section imaging, Boston: Artech House.

2. A. C. Kak and M. Slaney, Principle of Computerized Tomographic Imaging - Chapter 8: Reflective Tomography, IEEE Press, 1988.

3. S. J. Norton, "Reconstruction of a two-dimensional reflecting medium over a circular domain: exact solution", J. Acoust. Sot. Am., 67(4), 1980.

4. S. J. Norton, "Acoustic I holography with An Annular Aperture", J, Acoust. Soc. Am., 71 (5), May, 1982.

5. M.]in and M. Chen, "Analysis and Simulation for a spotlight-mode Aircraft SAR in a Circular Flight Path", IGARSS Symposium, Tokyo, August, 1993.