# Heat Flow Induced Anomalies in Superfluid $^4He$ near $T_\lambda$

David Goodstein[1], Talso Chui[2] and Alexa Harter[1]

Condensed Matter Physics [1] and Jet Propulsion Laboratory[2],

California Institute of Technology, Pasadena, CA 91125

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In a recent Letter [ 1], Haussmann and Dohm (HD) presented a renormalization group treatment of the $^4$He lambda transition in a heat current, $Q$. In this Comment, we usc simple arguments that yield the same critical point exponent for the depressed $T_\lambda$, and nearly the same critical velocity, but indicate that HD may not have calculated the proper specific heat anomaly.

Near $T_\lambda$, the heat current is given by $Q = -\rho_s v_s ST$ in standard notations Of the two-fluid model. Of the terms in $Q$, only $\rho_s$ and $v_s$ 1IM% be singular, so for the purpose Of computing exponents, we write $Q_c \sim (\rho_{sc} v_{sc}^2/2)/v_{sc}$. The numerator is a singular term in he free energy density, and every such term goes to zero inversely as the correlation volume i.e. $\rho_{sc} v_{sc}^2 \sim$ -". The denominator is given by [2]

$$v_{sc} = -i(\hbar/m)|\nabla\psi|/\psi \text{ -[V I//[/L//}, \qquad (1)$$

where m is the atomic mass of $^4$He. Thus $v_{sc}$ has the character of an inverse length. Since the correlation length is the only relevant length at a critical point, $v_{sc} \sim \xi^{-1} \sim t^{\nu}$, where $t = (T_\lambda - T)/T_\lambda$. Thus $Q_c \sim \xi^{-d} t^{-\nu}$, or

$$7a(0) - T_\lambda(Q) \sim Q^{1/\nu(d-1)} \qquad (2)$$

which is the same result arrived at by HD.

Equation (1) envisions a wave-function like order parameter which, in uniform flow has the form $\psi = \psi_o e^{i\vec{k}\cdot\vec{r}}$, where $\vec{r}$ is a space vector and $\vec{k}$ is related to vs by $\vec{v}_s = \hbar\vec{k}/m$. The order parameter is governed by a differential equation [3]

$$\xi^2 \nabla^2 \psi = (|\psi|^2 - 1)\psi, \qquad (3)$$

which has a solution $|\psi|^2 = 1 - (k\xi)^2$. Thus $|\psi|^2$ is driven to zero at superfluid velocity.

$$v_{sc} = \hbar / m\xi = 112 t^{\nu} \ \text{[m/see]} \qquad (4)$$

This justifies the argument in eq. (1) that $v_{sc}$ - $\xi^{-1}$. Fluctuations arc taken into account by using the experimental value of v rather than that predicted by man field theory.

Equation (4) maybe compared to the results of HD

$$v_{sc} = \left[1/\sqrt{6} - 0.0112\right] 2^{\nu} \hbar / m\xi = 70.3 t^{\nu} \ [\text{m/sec}] \qquad (5)$$

The difference is due almost entirely to the fact that HD's critical velocity is the consequence of a stability criterion, $\partial Q / \partial v_s \geq 0$, rather than simply the velocity that drives $|\psi|^2$ to zero. The same criterion gives a factor 2" $/\sqrt{6}$ in eq. (4).

We now turn to the heat capacity anomaly. Under superfluid flow the free energy per unit volume is increased by [4] $\Delta F(T, v_s) = \rho_s v_s^2/2$. At constant Q, the proper free energy to use is $\Phi(T, \vec{q}) =$ F' -- $\vec{v}_s \vec{q}$, where $\vec{q} = \rho_s \vec{v}_s$. The molar heat capacity change is:

$$
\begin{aligned}
\text{AC}= & -(TV\ \partial^2\Delta\Phi/\partial T^2)_{\bar{q}} = -\left[TV\,\partial^2\left(-q^2/2\rho_s\right)/\partial T^2\right]_{9} \\
& = \zeta(\zeta+1)\,Q^2 V t^{-(\zeta+2)} / \left(2\rho_o S^2 T_\lambda^3\right) = f(Q/Q_c)t^{-\alpha} \\
@/\text{Q},.) & = 8.65\left(Q/Q_c\right)^2\ [\text{J/ mole K}].
\end{aligned}
\tag{6}
$$

where $\rho_s = \rho_o t^{\zeta}$, $\rho_o$ = 0.37 gm/cm$^3$, $S$ = 1.58 J/gm K, ~==(2--ct)/3= $\nu, \alpha$ is the heat capacity exponent, $V$ = 27.38 cm$^3$/mole is the molar volume and $Q_c = 7580 t^{2\nu}$ [W cm$^{-2}$ ] [1]. The dashed line in Fig. 1 is the scaling function $f(Q/Q_c)$ of HD. The solid line is our result which is based on the two-fluid model neglecting any dependence of $\rho_s$ on $\vec{v}_s$. It is not clear to us why the HD calculation differs so little from these standard arguments in its other principal results, and so much in the predicted heat capacity.

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[1] R. Haussmann and V. Dohm, Phys. Rev. Lett. 72, 3060(1 994), and references therein.

[2] 11.1,. Goodstein, *States of Matter* (Dover, New York, 1985) p. 483.

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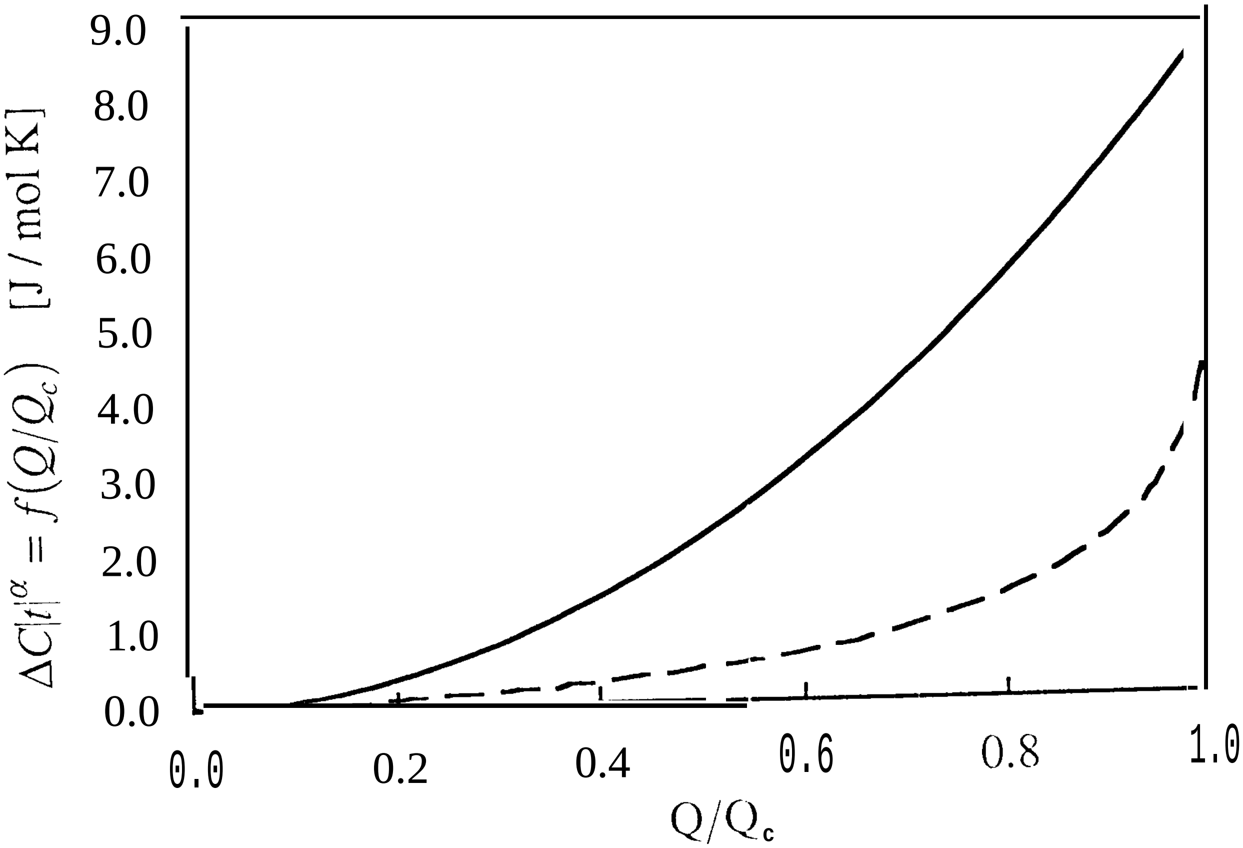
Figure Caption:

Figure 1: The scaling function discussed in the text.



9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
$\Delta C|t|^{\alpha} = f(Q/Q_c)$ [J / mol K]
0.0
0.2
0.4
0.6
0.8
1.0
$Q/Q_c$