

# New Bandwidth Efficient Parallel Concatenated Coding Schemes<sup>†</sup>

S. Benedetto\*, D. Divsalar<sup>#</sup>, G. Montorsi', F. Pollara<sup>#</sup>

\* Dipartimento di Elettronica, Politecnico di Torino

C. Duca degli Abruzzi 24-10129 Torino, Italy

<sup>#</sup> Jet Propulsion Laboratory

4800 Oak Grove Drive - Pasadena, CA 91109

## Abstract

We propose a new solution to parallel concatenation of trellis codes with multilevel amplitude/phase modulations and a suitable iterative decoding structure. Examples are given for throughputs 2 bits/see/Hz with 8PSK and 16QAM signal constellations. For parallel concate-

<sup>†</sup> Part of the research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration

nated trellis codes in the examples, rate 2/3 and 4/5, 16-state binary convolutional codes with Gray code mapping are used. The performance of these codes are within 1 dB from the Shannon limit at a bit error probability of  $10^{-6}$  for a given throughput. This outperforms the performance of all codes reported in the past for the same throughput.

## 1 Introduction

Trellis coded modulation (TCM) proposed by Ungerboeck in 1982 [1] is now a well-established technique in digital communications. Since its first appearance, it has experienced a highly growing interest, concerning its theoretical foundation as well as its numerous applications, spanning high-rate digital transmission over voice circuits, digital microwave radio relay links, and satellite communications. In essence, it is a technique to obtain significant coding gains (3-6 dB) sacrificing neither data rate nor bandwidth.

Turbo codes represent a more recent proposal in the coding research field [2], which has risen a large interest in the coding community. They are *parallel concatenated convolutional codes* (PCCC) whose encoder is formed by two (or more) *constituent* systematic encoders joined through an interleave.

The input information bits feed the first encoder and, after having been interleaved by the interleaver, enter the second encoder. The codeword of the parallel concatenated code consists of the input bits to the first encoder followed by the parity check bits of both encoders.

The suboptimal iterative decoder is modular, and consists of a number of equal component blocks formed by concatenating the decoders of the constituent codes (CC) separated by the interleavers used at the encoder side. Each decoder outputs weighted soft decoding of the input sequence. By increasing the number of decoding modules, and thus the number of decoding iterations, bit error probabilities as low as  $10^{-5}$  at  $E_b/N_0 = 0$  dB have been shown by simulation [4]. Parallel concatenated convolutional codes yield very large coding gains (10-11 dB) at the expense of a data rate reduction, or bandwidth increase. Typically, they have been proposed for rates not higher than  $1/2$ .

It seems thus worthwhile to merge TCM and PCCC in order to obtain large coding gains and high bandwidth efficiency. A first attempt employing the so-called "pragmatic" approach to TCM was described in [3].

In this paper, we propose a new solution to parallel concatenation of trellis codes (PCTCM) with multilevel amplitude/phase modulations and a

suitable bit-by-bit iterative decoding structure. The performance of the new codes are within 1 dB from the Shannon limit at bit error probabilities of  $10^{-6}$ , and outperform all codes reported previously for the same throughput.

## 2 Parallel Concatenated Trellis Coded Modulation

A pragmatic approach for turbo codes with multilevel modulation was proposed in [3]. Here we propose a different approach that outperforms the results in [3] when MPSK or M-QAM modulation is used. A straightforward method to use parallel concatenated codes with multilevel modulation is first to select a rate  $\frac{b}{b+1}$  constituent code where the outputs are mapped to a  $2^{b+1}$ -level modulation based on Ungerboeck's set partitioning method (i.e., we can use Ungerboeck's codes with feedback). If MPSK modulation is used, for every  $b$  bits at the input of the parallel concatenated encoder we transmit two consecutive  $2^{b+1}$  PSK signals, one per each encoder output. This results in a throughput of  $b/2$  bits/sec/Hz. If M-QAM modulation is used, we map the  $b + 1$  outputs of the first component code to the  $2^{b+1}$  in-phase levels

Should adjust the "bit" to be the "s"?

(I-channel) of a  $2^{2b+2}$ -QAM signal set, and the  $b + 1$  outputs of the second component code to the  $2^{b+1}$  quadrature levels (Q-channel). The throughput of this system is  $b$  bits/sec/Hz.

This approach requires more levels of modulation than conventional TCM, which is not desirable in practice. Moreover, the input information sequences are used twice in the output modulation symbols, which is also not desirable.

An obvious remedy would be to puncture the output symbols of each trellis code and select the puncturing pattern such that the output symbols of the parallel concatenated code contain the input information only once. If the output symbols of the first encoder are punctured uniformly, the puncturing pattern of the second trellis code is non-uniform and depends on the particular choice of interleave. In this way, for example, for  $2^{b+1}$ -PSK a throughput  $b$  can be achieved. This solution has two drawbacks: first, it complicates the encoder and decoder, and second, the reliability of punctured symbols may not be fully reproducible at the decoder.

An alternative solution, for  $\frac{b}{b+1}$  ( $b$  even), is to select the  $b/2$  systematic outputs and puncture the rest of the systematic outputs, but keep the parity bit of the  $\frac{b}{b+1}$  code (Note that the  $\frac{b}{b+1}$  may have been already obtained by puncturing a rate  $1/2$  code). Then do the same to the second constituent

code but select only those systematic bits which were punctured in the first encoder.

This method requires at least two interleaves: the first interleaver permutes the bits selected by the first encoder and the second interleaver those punctured by the first encoder. For MPSK (or MQAM) we can use  $2^{1+b/2}$  PSK symbols (or  $2^{1+b/2}$  QAM symbols) per encoder and achieve throughput  $b/2$ . For M-QAM we can also use  $2^{1+b/2}$  levels in the I-channel and  $2^{1+b/2}$  levels in the Q-channel, and achieve a throughput of  $b$  bits/sec/Hz. Extension to odd values of  $b$  is straightforward.

This method is equivalent to a multi-dimensional trellis coded modulation scheme (in this case, two multi-level symbols per branch) which uses  $2^{b/2} \times 2^{1+b/2}$  symbols per branch, where the first symbol in the branch (which only depends on uncoded information) is punctured. In this way, the reliability of the punctured symbols is fully reproducible at the decoder.

To optimize the PCTCM code, the constituent codes for a given modulation should be redesigned based on the Euclidean distance. This is left to further research. In this letter, we give two examples of application, using 8PSK and 16QAM constellations and Gray code mapping. Note that this may result in suboptimum constituent codes for multi-level modulation.

The first code has  $b = 2$ , and employs an 8PSK modulation in connection with two 16-state, rate 4/5 codes and four interleavers of size 4096 designed as before. The structure of the PCTCM with 8PSK and two clock cycle trellis termination is shown in Fig. 1.

To obtain the bit error probability performance, we simulated the iterative decoding structure for two codes discussed in [5] where the MAP algorithm is extended from bits to symbols.

The bit error probability performance of this code is shown in Fig. 2 for two values of the number  $m$  of iterations. The required signal-to-noise ratio to obtain a bit error probability of  $10^{-6}$  with 5 iterations is 3.7 dB. This yields a coding gain with respect to uncoded 4-PSK of 7 dB, which outperforms the result of [3] by 1 dB and is only at 0.8 dB from the Shannon limit.

The second code we propose has  $b = 2$ , and employs a 16QAM modulation in connection with two 16-state, rate 2/3 constituent codes and two interleaves of size 16384 bits designed according to the procedure described in [4] with parameters  $S=40$  and  $S=32$ . The structure of the PCTCM with 16QAM and two clock cycle trellis termination is shown in Fig. 3.

The bit error probability performance of this code is shown in Fig. 4 for several values of the number  $m$  of iterations. These results outperform those

obtained with 8-PSK constellations, and are at 0.9 dB from the Shannon limit for a bit error probability of  $10^{-5}$ .

## References

- [1] G. Ungerboeck, "Channel coding with multilevel phase signaling", *IEEE Transactions on Information Theory*, vol. IT-25, pp. 55-67, Jan. 1982.
- [2] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon **Limit** Error-Correcting Coding: Turbo Codes," *Proc. 1993 IEEE International Conference on Communications*, Geneva, Switzerland, pp. 1064-1070, May 1993.
- [3] S. LeGoff, A. Glavieux, and C. Berrou, "Turbo Codes and High Spectral Efficiency Modulation", *Proceedings of IEEE ICC'94*, May 1-5, 1994, New Orleans, LA.
- [4] D. Divsalar and F. Pollara, "Turbo Codes for PCS Applications," *Proceedings of IEEE ICC'95*, Seattle, Washington, June 1995,

- [5] S. Benedetto, D. Divsalar, G. Montorsi and F. Pollara, "Soft-output decoding algorithms in iterative decoding of parallel concatenated convolutional codes", *ICC '96*, submitted.

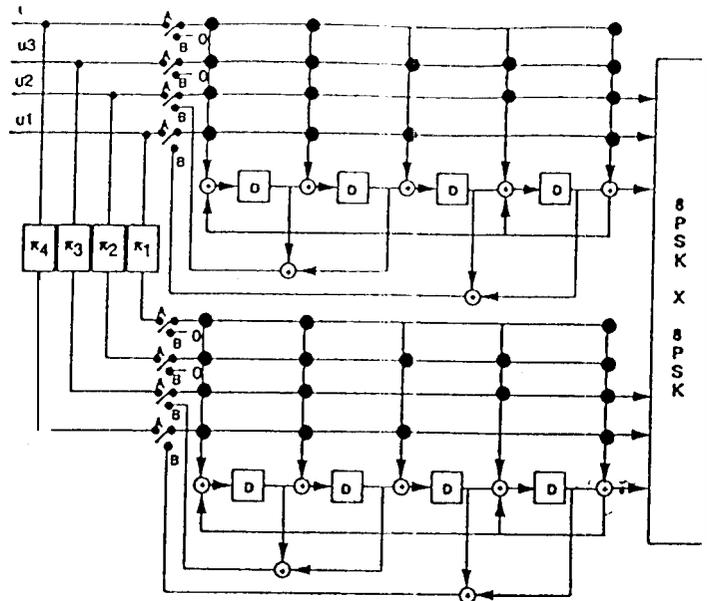


Fig. 1 : Parallel Concatenated Trellis Coded Modulation, 8PSK, 2 bits/sec/Hz.

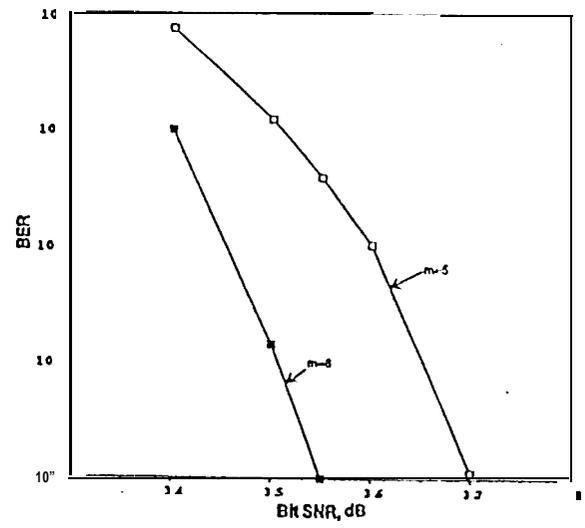


Fig. 2 : BER Performance of Parallel Concatenated Trellis Coded Modulation, 8PSK, 2 bits/sec/Hz.

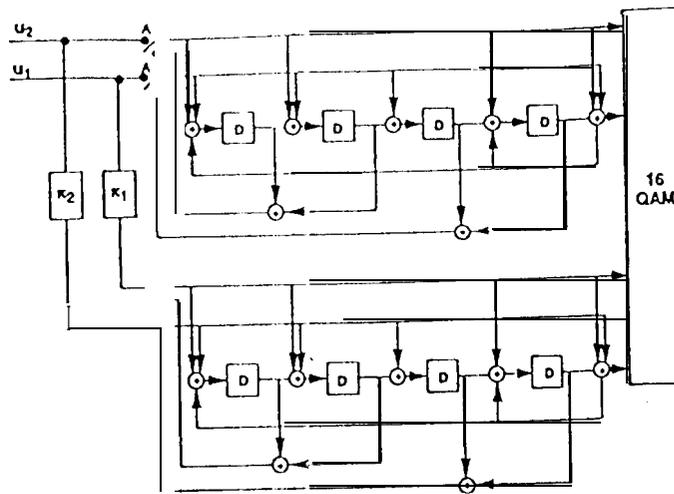


Fig. 3: Parallel Concatenated Trellis Coded Modulation, 16QAM, 2 bits/sec/Hz.

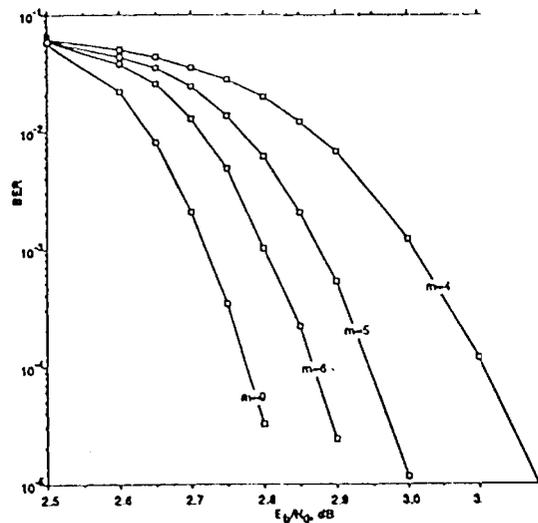


Fig. 4: BER Performance of Turbo Trellis Coded Modulation, 6QAM, 2 bits/sec/Hz.