POWER LOSS FOR MULTIMODE WAVEGUIDES AND ITS APPLICATION TO BEAM-WAVEGUIDE SYSTEMS*

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ABSTRACT

The conventional way of expressing power loss in dB/meter for a multimode waveguiding system with finite wall conductivity (such as a beam-waveguide system with protective shroud) can be incorrect and misleading. The power loss (in dB) for a multimode waveguiding system is, in general, not linearly proportional to the length of the waveguide. New power loss formulas for multimode system are derived in this paper for arbitrarily shaped conducting waveguide tubes. In these formulas, there are factors such as $(\exp(jx)-1)/(jx)$, where $x = (\beta_a - \beta_b)\ell$, with β_a and β_b being the propagation constants of the different propagating modes and ℓ being the distance from the source plane to the plane of interest along the guide. For a large beam-waveguide supporting many propagating modes, β_a's are quite close to β_b's, thus the mode coupling terms remain important for a very long distance from the source plane.

The multimode power loss formula for a large circular conducting tube has been verified by experiments. This formula was also used to calculate the additional noise temperature contribution due to the presence of a protective shroud surrounding a millimeter-wave beam-waveguide system.

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I. INTRODUCTION AND THE CONSIDERATION OF A FUNDAMENTAL CONCEPT

In textbooks on electromagnetic and guided waves, the usual perturbation technique is used to calculate the attenuation factor of a given propagating mode in a slightly lossy, highly conducting hollow metallic waveguide. Based on this technique, the attenuation constant for the mth mode, $\alpha^{(m)}$, due to conductor loss in a general cylindrical hollow metallic waveguide is found to be:[1-6]

$$\alpha^{(m)} \simeq \frac{R \oint_C \left(\underline{H}^{(m)} \cdot \underline{H}^{(m)*}\right) d\ell}{2Re \iint_S \left(\underline{E}^{(m)} \times \underline{H}^{(m)*}\right) \cdot \underline{e}_z \, dA} \cdot 8.686 \text{ (dB / m)} \quad (1)$$

where R denotes the intrinsic resistance of the metal walls, $\underline{E}^{(m)}$ and $\underline{H}^{(m)}$ are the unperturbed electric and magnetic fields for the mth propagating mode in this waveguide with perfectly conducting walls, Re denotes the real part of the integral, $\underline{e}_z$ is the unit vector in the z-propagating direction, the * denotes the complex conjugate of the integral, C is the contour around the cross-section of the waveguide, and S is the cross-sectional area of the waveguide. Here, $a^{(m)}$ expressed in dB/meter is the attenuation constant for the mth propagating mode per unit length of the waveguide.

A more accurate determination of the attenuation constant, $a^{(m)}$, can be obtained through the boundary-value-problem approach. Here, the fields in different regions (i.e., the metal region characterized by (ϵ, μ, σ) and the vacuum region characterized by (ϵ_0, μ_0)) of the waveguide are matched at the boundary, yielding a dispersion relation from which the complex propagation constant for each mode may be determined. For this approach, in general, all field components must be assumed to be present. In other words, for a hollow circular metal pipe, the field components, (E_z, E_r, E_q, H_z, H_r, H_q) , will all be present, when circular symmetry of the mode is not present. Here, the circular cylindrical coordinates (r, q, z) are assumed. This was the approach (called hybrid-mode approach) used by Chou and Lee to calculate modal attenuation in multilayered coated waveguides.[1]

In all of the above considerations, the power loss has always been expressed by $a^{(m)}$ for each mth mode in dB/meter.

It is the limitation of this way of expressing power loss that we wish to address in the following.

When a single mode, say the mth mode, is propagating in this hollow waveguide, the following expression is normally used to represent the power carried by this mode along this waveguide structure:

$$P^{(m)}(z) = P_0^{(m)} e^{-\alpha^{(m)} z} \approx P_0^{(m)} \left(1 - 2\alpha^{(m)} z\right) \qquad (2)$$

where $P_0^{(m)}$ is the initial input power of the mth mode and z is the distance along the guide. That this expression is valid if and only if a single mode is propagating alone in this waveguide, is usually glossed over in the textbooks. Furthermore, Eqs. (1) and (2) offer the impression that the power loss in a given waveguide may be expressed by the attenuation constant $\alpha^{(m)}$ in nepers/meter. From Eq. (2), for small attenuation, the power loss $P_L^{(m)}$ is

$$P_L^{(m)} = P_0^{(m)}(\text{Power Input}) - P^{(m)}(z)\ (\text{Power Output}) \approx P_0^{(m)} 2\alpha^{(m)} z.$$

Consequently, one may obtain the mistaken impression that the total loss is additive when more than one mode is present simultaneously in the waveguide; after all, we know that the total power is additive. For the multimode propagation case, the total power loss should not be expressed through an attenuation constant as certain nepers/meter (or dB/m). Indeed, due to the contributions of the cross-product terms in $\underline{J} \cdot \underline{J}$, where $\underline{J}$ is the total surface current, the total power loss in the multimode case is no longer a linear function of the length of the guide as in the single-mode case.

Assume that a given source in an infinitely long hollow conducting waveguide excites two equal amplitude lowest order propagating modes. Further assume that the waveguide can only support these two lowest order propagating modes. The walls of the waveguide are made with highly conducting (but not perfectly conducting) metal. Let us find the total power loss at a distance d from the source plane.

Incorrect **Solution**

According to the textbook formula (Eq. (1) of our paper), the attenuation constant for each mode can be calculated using this formula. Say the answer for mode 1 is $a_1 = 0.001$ (nepers/meter) and for mode 2 is $a_2 = 0.002$. (Even if we use the more exact way of calculating the attenuation constant by the boundary-value-problem approach (or the hybrid-mode approach) described in Ref. 1, due to the highly conducting nature of the walls, the attenuation constants for these two modes would not deviate much from the given values.) Let P_0 be the input power for mode 1 as well as for mode 2. So, the power of mode 1 after propagating for a distance z in the waveguide is $P_0 \exp(-2a^{(1)}z)$ and for mode 2 is $P_0 \exp(-2a^{(2)}z)$. Since the power is additive, the total power loss is

$$
\begin{aligned}
P_{\text{Total Loss}} &= P_{\text{Input}} - P_{\text{Output}} \\
&= 2P_0 - P_0(\exp(-2a^{(1)}z) + \exp(-2a^{(2)}z)) \qquad (3) \\
&= 2P_0(a^{(1)} + a^{(2)})z.
\end{aligned}
$$

So, according to the above formula, $P_{\text{Total Loss}}$ is proportional to z. No matter how small z or a is. This is wrong!

The fundamental reason that this result is wrong is that even though the total power for two modes is additive, but the total power loss is not additive. In other words, total power loss is not linearly dependent on z. The correct way of calculating the total power loss is given below.

Correct Solution

For linear EM waves, the fields and currents are additive. So the total induced current flowing on the walls is the sum of the induced current for mode 1 and that of mode 2. It is the currents flowing on the surface of an imperfect conductor that give rise to ohmic loss or the power loss. So, the power loss must be calculated according to Ohm's law. Accepting this fundamental concept will lead to the correct answer. In this case the power is not linearly dependent on z as shown by Eq. (16) below.

This example shows that Eq. (2), representing the power flow for a given mode along a hollow conducting waveguide, is only valid when there is one and only one mode propagating in this waveguide .

Therefore, the purpose of this paper is to address the power loss problem when more than one mode is present simultaneously in the waveguide. This effort is motivated by our desire to verify the measured data for a millimeter wave beam-waveguide with protective shroud consisting of sections of round conducting tube as shown in Fig. 1. Solution of this problem is of great importance in optimizing the design to yield minimum noise temperature for the NASA/Deep Space Network's low-noise microwave receiving system.[8]

II. FORMULATION OF THE PROBLEM AND FORMAL SOLUTION

Shown in Fig. 2 is the geometry of the canonical problem. A uniform conducting waveguide of arbitrary cross-section with its axis aligned in the z direction has a length ℓ. In the z = 0 plane, the transverse electric field $\underline{E}_t(x,y)$ is assumed to be given. Thus the amplitudes of all the modes (propagating and evanescent modes) can be calculated and are assumed to be known. We wish to calculate the power loss of the fields due to the imperfect conductivity of the wall with intrinsic wave resistance (surface resistance) R.

From Ohm's law and Poynting's vector theorem, the power loss is given by[1-6]

$$P_L = \frac{1}{2} R \iint_A \left(\underline{J}_s \cdot \underline{J}_s^* \right) dA , \tag{4}$$

where

$\underline{J}_s = \underline{n} \times \underline{H}$ = surface current density on the wall

$\underline{n}$ = unit vector normal to the wall surface

$\underline{H}$ = total magnetic field in the waveguide

$A =$ surface area of the wall

or

$$P_L = \frac{1}{2} R \iint_A \left(\underline{H}_\tau \cdot \underline{H}_\tau^* \right) dA \,, \tag{5}$$

where $\underline{H}_t$ is the component of the total magnetic field which is tangential to the wall surface. It is known that, in a hollow arbitrarily shaped uniform waveguide with a conducting wall, there can exist two sets of eigenmodes:[1-6] *TE* (Transverse Electric) modes and TM (Transverse Magnetic) modes with a specific propagation constant for each mode. The total fields for *TE* modes are

$$\underline{\underline{E}}^{(TE)}(x,y,z) = \sum_{m=1}^{\infty} A_m^{(TE)} \underline{E}_{mt}^{(TE)}(x,y) e^{j\beta_m^{(TE)} z} \tag{6}$$

$$\underline{H}^{(TE)}(x,y,z) = \sum_{m=1}^{\infty} A_m^{(TE)} \left[\underline{H}_{mt}^{(TE)}(x,y) + H_{mz}^{(TE)}(x,y) \underline{e}_z \right] e^{j\beta_m^{(TE)} z} \tag{7}$$

where $\underline{E}_{mt}^{(TE)}$ and $(\underline{H}_{mt}^{(TE)} + H_{mz}^{(TE)} \underline{e}_z)$ are connected through the Maxwell's equations and $\beta_m^{(TE)}$ is the propagation constant of the *m*th *TE* eigenmode, and the total fields for *TM* modes are

$$\underline{E}^{(TM)}(x,y,z) = \sum_{m=1}^{\infty} A_m^{(TM)} [\underline{E}_{mt}^{(TM)}(x,y) + E_{mz}^{(TM)}(x,y) \underline{e}_z] e^{j\beta_m^{(TM)} z} \tag{8}$$

$$\underline{H}^{(TM)}(x,y,z) = \sum_{m=1}^{\infty} A_m^{(TM)} \underline{H}_{mt}^{(TM)}(x,y) e^{j\beta_m^{(\cdot)} z} \quad , \tag{9}$$

where $\underline{H}_{mt}^{(TM)}$ and $\underline{E}_{mt}^{(TM)} + E_{mz}^{(TM)} e_z)$ are connected through Maxwell's equations and $\beta_m^{(TM)}$ is the propagation constant of the mth TM eigenmode. $A_m^{(TE)}$ and $A_m^{(TM)}$ **are arbitrary amplitude coefficients for** TE and TM modes. The subscript t indicates the transverse components of the field (transverse to the z direction) . The index m is used to tally the modes—it does not necessarily correspond to mode order. One notes that b_m may take on negative values, indicating modes propagating in the opposite direction.

Substituting Eqs. (6)-(9) into (5) yields

$$P_L = \frac{1}{2} R \Bigg\{ \sum_{m=1}^{M} \sum_{n=1}^{M} A_m^{(TE)} A_n^{(TE)*} \oint_c \left[H_{mc}^{(TE)} H_{nc}^{(TE)*} + H_{mz}^{(TE)} H_{nz}^{(TE)*} \right] dc \int_0^{\ell} e^{j\left(\beta_m^{(TE)} - \beta_n^{(TE)}\right) z} dz$$

$$+ \sum_{m'=1}^{M'} \sum_{n=1}^{M} A_{m'}^{(TM)} A_n^{(TE)*} \oint_c \left[H_{m'c}^{(TM)} H_{nc}^{(TE)*} \right] dc \int_0^{\ell} e^{j\left(\beta_{m'}^{(TM)} - \beta_n^{(TE)}\right) z} dz$$

$$+ \sum_{m=1}^{M} \sum_{n'=1}^{M'} A_m^{(TE)} A_{n'}^{(TM)*} \oint_c \left[H_{mc}^{(TE)} H_{n'c}^{(TM)*} \right] dc \int_0^{\ell} e^{j\left(\beta_m^{(TE)} - \beta_{n'}^{(TM)}\right) z} dz$$

$$+ \sum_{m'=1}^{M'} \sum_{n'=1}^{M'} A_{m'}^{(TM)} A_{n'}^{(TM)*} \oint_c \left[H_{m'c}^{(TM)} H_{n'c}^{(TM)*} \right] dc \int_0^{\ell} e^{j\left(\beta_{m'}^{(TM)} - \beta_{n'}^{(TM)}\right) z} dz \ . \tag{10}$$

Here, c is the contour around the inner surface of the waveguide, which is also normal to the z-axis (see Fig. 2) . The subscript c represents the component of the transverse field that is tangential to the contour c, M is the number of TE **propagating** modes, and M' is the number of TM **propagating** modes. Simplifying Eq. (10) gives

$$P_L = [\text{Part 1}] + [\text{Part 2}] \tag{11}$$

with

$$[\text{Part 1}] = \frac{1}{2} R\ell \left[\sum_{m=1}^{M} \left| A_m^{(TE)} \right|^2 I_m^{(TE)} + \sum_{m'=1}^{M'} \left| A_{m'}^{(TM)} \right|^2 I_{m'}^{(TM)} \right] \tag{12}$$

$$\begin{aligned}
[\text{Part 2}] = \frac{1}{2} R\ell \Bigg[& \sum_{m=1}^{M} \sum_{\substack{n=1 \\ n \neq m}}^{M} A_m^{(TE)} A_n^{(TE)*} I_{mn}^{(TE)} \left(\frac{e^{j\left(\beta_m^{(TE)} - \beta_n^{(TE)}\right)\ell} - 1}{j\left(\beta_m^{(TE)} - \beta_n^{(TE)}\right)\ell} \right) \\
& + \sum_{m'=1}^{M'} \sum_{\substack{n'=1 \\ n' \neq m'}}^{M'} A_{m'}^{(TM)} A_{n'}^{(TM)*} I_{m'n'}^{(TM)} \left(\frac{e^{j\left(\beta_{m'}^{(TM)} - \beta_{n'}^{(TM)}\right)\ell} - 1}{j\left(\beta_{m'}^{(TM)} - \beta_{n'}^{(TM)}\right)\ell} \right) \\
& + \sum_{m'=1}^{M'} \sum_{n=1}^{M} A_{m'}^{(TM)} A_n^{(TE)*} I_{m'n}^{(TM)(TE)} \left(\frac{e^{j\left(\beta_{m'}^{(TM)} - \beta_{n}^{(TE)}\right)\ell} - 1}{j\left(\beta_{m'}^{(TM)} - \beta_n^{(TE)}\right)\ell} \right) \\
& + \sum_{m=1}^{M} \sum_{n'=1}^{M'} A_m^{(TE)} A_{n'}^{(TM)*} I_{mn'}^{(TE)(TM)} \left(\frac{e^{j\left(\beta_m^{(TE)} - \beta_{n'}^{(TM)}\right)\ell} - 1}{j\left(\beta_m^{(TE)} - \beta_{n'}^{(TM)}\right)\ell} \right) \Bigg]
\end{aligned} \tag{13}$$

where

$$I_m^{(TE)} = \oint_c \left(\left| H_{mc}^{(TE)} \right|^2 + \left| H_{mz}^{(TE)} \right|^2 \right) dc$$

$$I_{m'}^{(TM)} = \oint_c \left| H_{m'c}^{(TM)} \right|^2 dc$$

$$I_{mn}^{(TE)} = \oint_c \left[H_{mc}^{(TE)} H_{nc}^{(TE)*} + H_{mz}^{(TE)} H_{nz}^{(TE)*} \right] dc$$

$$I_{m'n'}^{(TM)} = \oint_c \left[H_{m'c}^{(TM)} H_{n'c}^{(TM)*} \right] dc$$

$$I_{m'n}^{(TM)(TE)} = \oint_c \left[H_{m'c}^{(TM)} H_{nc}^{(TE)*} \right] dc$$

$$I_{mn'}^{(TE)(TM)} = \oint_c \left[H_{mc}^{(TE)} H_{n'c}^{(TM)*} \right] dc. \qquad (14)$$

It should be noted that P_L is always purely real .

Equation (11) shows that power losses or attenuations of different simultaneously existing modes are not just additive, as given by the first bracketed term [Part 1]. The correct expression must include the second bracketed term [Part 2] , which shows the cross-product terms. Indeed, the use of an attenuation constant to describe power loss in a waveguide should be limited to the single-mode uni-directional propagation case only, because only for this case is the power loss linearly dependent on the length of the guide. For the multimode propagation case, the power loss varies with the length of the guide in a rather

complicated manner as shown in Eq. (11). Equation (11) vividly demonstrates the importance of the modal coupling term. Since the factor

$$f(x) = (q(p) - 1) / (jx) \tag{15}$$

(where x = $(b_1 - b_2)\ell$, b_1 and b_2 are the propagation constants for the coupling modes, and ℓ is the distance from the entrance of the waveguide to the point of interest along the guide) determines the importance of the coupling term, let us now examine this factor closely. The function $|f(x)|$ is largest when $x \to 0$ and begins to diminish and approaches zero when x increases. This means that the cross-product terms in Eq. (11), i.e., [Part **2], are important when the difference between the propagation constants of the propagating** modes that are excited in the waveguide is small and/or when ℓ is small, such that the product $(b_1 - b_2)\ell$ is small. This condition is particularly true when the transverse dimensions of the waveguide are very large, such as the beam-waveguide case that we considered.

When $|x| \gg 1$, then $f(x) \to 0$, and the coupling terms in Eq. (11) [Part 2] approach zero. This means that, when $\ell \to \infty$, [Part 2] Æ 0, the usual decoupled result given by [Part 1] in Eq. (11) becomes valid. So, as $\ell \to \infty$, the power loss for each mode in the multimode waveguide is additive.

III. THE SPECIAL TWO-MODE CASE

When two propagating modes exist simultaneously in a multimode cylindrical waveguide, the power loss due to imperfection of the waveguide walls can be expressed as follows (from Eq. (11)) .

$$P_L = \frac{1}{2} R\ell\left(|A_1|^2 I_1 + |A_2|^2 I_2\right) + \frac{1}{2} R\ell\left[A_1 A_2^* I_{12} f(x) + A_1^* A_2 I_{21} f(-x)\right] \, , \tag{16}$$

where $f(x)$ is given by Eq. (15) .

Here A_1 and A_2 represent the amplitude coefficients for the two propagating modes, and b_1 and b_2 represent the propagation constants for these modes. It is clearly seen that the term containing $f(x)$ with $x = (b_1 - b_2)\ell$ is the coupling term which approaches zero as $x \to \infty$. If b_1 is near b_2, the distance ℓ must be very large in order that x may be large, implying that the coupling term remains important for a very long distance. So expressing the power loss in dB/m, even for a two-mode case, is misleading at best.

IV. LOSS CALCULATION BASED ON NUMERICAL MODAL FIELD DATA

For many practical situations, the complex modal fields, obtained numerically and expressed in complex numbers and vectorial directions, can not be obtained analytically. In other words, the line integrals in Eq. (14) must be evaluated numerically. This is a task that can easily be accomplished through appropriate summations of the numerical data. This approach was used in the beam-waveguide case which we shall discuss later.

The following normalizations were used:[2]

$$\iint_s \left(\underline{E}_{mt}^{(TE)} \cdot \underline{E}_{nt}^{(TE)*}\right) dA = 1 \quad m = n \tag{17}$$
$$= 0 \quad m \neq n$$

$$\iint_s \left(\underline{E}_{mt}^{(TM)} \cdot \underline{E}_{nt}^{(TM)*}\right) dA = l \quad rn = n \tag{18}$$
$$= 0 \quad m \neq n$$

$$\iint_s \left(\underline{H}_{mt}^{(TE)} \cdot \underline{H}_{nt}^{(TE)*}\right) dA = 1 \quad rn = n \tag{19}$$
$$= 0 \quad m \neq n$$

$$\iint_s \left(\underline{H}_{mt}^{(TM)} \cdot \underline{H}_{nt}^{(TM)*}\right) dA = 1 \quad m = n \tag{20}$$
$$= 0 \quad m \neq n$$

$$\iint_s \left(\underline{E}_{mt}^{(TE)} \cdot \underline{E}_{nt}^{(TM)*}\right) dA = 0 \tag{21}$$

$$\iint_s \left(\underline{H}_{mt}^{(TE)} \cdot \underline{H}_{nt}^{(TM)*}\right) dA = 0 \tag{22}$$

The integration, which may be done numerically, is carried over the cross-sectional area s. Here, $\left(\underline{E}_{mt}^{(TE),(TM)}, \underline{H}_{nt}^{(TE),(TM)}\right)$ are the transverse fields.

V. APPLICATION TO BEAM-WAVEGUIDE NOISE TEMPERATURE COMPUTATIONS

We shall now apply the above theory to calculate the conductivity loss (power loss) in a large beam waveguide (BWG) tube. The noise temperature contributed by the conductivity loss in a BWG can then be easily computed. Computed results are compared with measured data from an experiment, validating the theory.

Large beam-waveguide-type ground station antennas are generally designed with metallic tubes enclosing the beam-waveguide mirrors. The scattered field from a beam-waveguide mirror is obtained by the use of a physical optics integration procedure with a Green's function appropriate to the circular waveguide geometry.[9] In this manner, the coefficients, $A_m^{(TE),(TM)}$, of the circular waveguide modes that are propagating in the oversized waveguides are determined.

Knowing the coefficients $A_m^{(TE),(TM)}$, one may calculate the tangential magnetic fields for the TE and TM modes from Eqs. (7) and (9). The total tangential magnetic field is the sum of these tangential magnetic fields. Substituting the total tangential magnetic field into Eq. (5) and carrying out the integral in Eq. (5) numerically, one may readily obtain the total power loss P_L. This numerical technique is quite general; it can be applied to a metal tube waveguide of arbitrary shape. Another way may also be used: knowing $A_m^{(TE),(TM)}$ for the modes in a circular metal tube (sleeve) waveguide, one may use the analytic expression given in the Appendix to calculate the total power loss P_L.[10,11]

The above numerical approach was used to calculate the conductivity loss of a short length of beam-waveguide tube. The experiment utilized a 3.92-meter length of 2.5-meter-diameter structure steel tube and a very sensitive noise temperature measuring radiometer (see Fig. 3). Noise temperature comparisons were made between several different horns radiating in free space and radiating into the beam-waveguide tube (see Fig. 4). The experiment also included measurements with the steel tube and the tube lined with aluminum sheets. Utilizing the measured conductivity of the aluminum and steel (Table 1), and the computed modes in the beam-waveguide tube, a conductivity loss was computed and converted into a noise temperature prediction. The following formula was used for the conversion:

$$\text{Noise Temperature, } \mathbf{K} = (1 - 10^{-1''''''0})7'_0 \tag{23}$$

where L_{dB} is the total insertion loss in +dB and T_0 is the ambient temperature in K (for room temperature, $T_0 = 293.1$K). A comparison of the measurement with both the new theory (Eq. (11)) and the textbook theory (Eq. (3) expanded to n modes) is shown in Table 2, The most dramatic difference was with the higher gain (22.5-dB) horn. The explanation can be seen in Figure 5, which plots the attenuation loss as a function of tube size. Because the high-gain horn doesn't "illuminate" the wall until further down the tube from its aperture plane, there is only a very small loss near the aperture. This clearly demonstrates the fact that the power loss is not linearly dependent on z and thus validates the analysis.

VI. CONCLUSIONS

The concept of expressing power loss along a given uniform waveguide in dB/meter must be used with caution. This concept is only generally true for single-mode uni-directional propagation. When more than one mode exists simultaneously, the power loss, expressed in dB, is no longer linearly proportional to the length of waveguide. Depending on the differences for the propagation constants of the co-existing propagating modes and the length of the waveguide, the total power loss may be more than, equal to, or less than the proportional sum of the power losses for each Inode propagating separately, as shown in Eq. (11).

Accurate formulas for the total power loss, taking mode coupling into account, have been derived for an arbitrarily shaped conducting tube, a circular conducting tube, and a rectangular conducting tube. The factor $f(x)$ (see Eq. (15))—where $x = (b_a - b_b)\ell$, with $(b_a - b_b)$ being the differences between the propagation constants of various modes propagating simultaneously in the conducting tube, and ℓ being the length of the waveguide from the source plane to the plane of interest along the guide--appears to be the governing factor that controls the importance of mode coupling between mode a and mode b in affecting the total power loss calculation. Since the factor sin x/x approaches zero as x approaches infinity, the effect of the term containing this factor approaches zero, indicating the diminishing effect of mode coupling on the total power loss calculation. Since $x = (b_a - b_b)\ell$, in order that x may approach a large value quickly, two possibilities exist:

(1) If b_a is close to b_b, as in the case of a very large guide, then ℓ must be very long in order that x may be large, indicating that the mode coupling effect can affect the total loss calculation for a very long distance from the source plane.

(2) If b_a is not close to b_b, as in the case of a smaller guide, then ℓ can be relatively short for x to be large enough so that the $f(x)$ term may be negligible, indicating that the mode coupling term only affects the total loss calculation for a relatively short distance from the source plane.

When applied to the JPL millimeter-wave beam-waveguide case, one notes that b_a is very close to b_b. Thus, the total power loss calculation is greatly affected by the mode coupling effects, requiring the use of the newly developed loss formula described here. One also notes that the concept of expressing power loss in dB/meter is incorrect and misleading for the length of waveguide

typically used in BWG design. The newly developed loss formula for an oversized circular conducting tube was thus used to calculate the additional noise temperature contribution due to the presence of a protective shroud surrounding a millimeter-wave beam-waveguide.

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Table 1. Electrical Conductivities of Shroud Material [12]

Material	Effective Conductivity mhos/meter
BWGantenna shroud ASTM A36 steel	0.003×10^7
0.064 in. thick 6061 aluminum sheet	2.2×10^7
0.024 in. thick galvanized steel	1.2×10^7
High-conductivity copper	5.66×10^7

Table 2. Experimental Results

	Measured, K	Calculated: New Method Eq. (11), K	Calculated: Textbook Method Eq. (3), K
22.5 dB gain horn with steel tube	0.1 ± 0.1	0.1	2.6
14.7 dB gain horn with steel tube	2.5 ± 0.4	2.3	3.0
14.7 dB gain horn with aluminum tube	0.2 ± 0.1	0.09	0.11

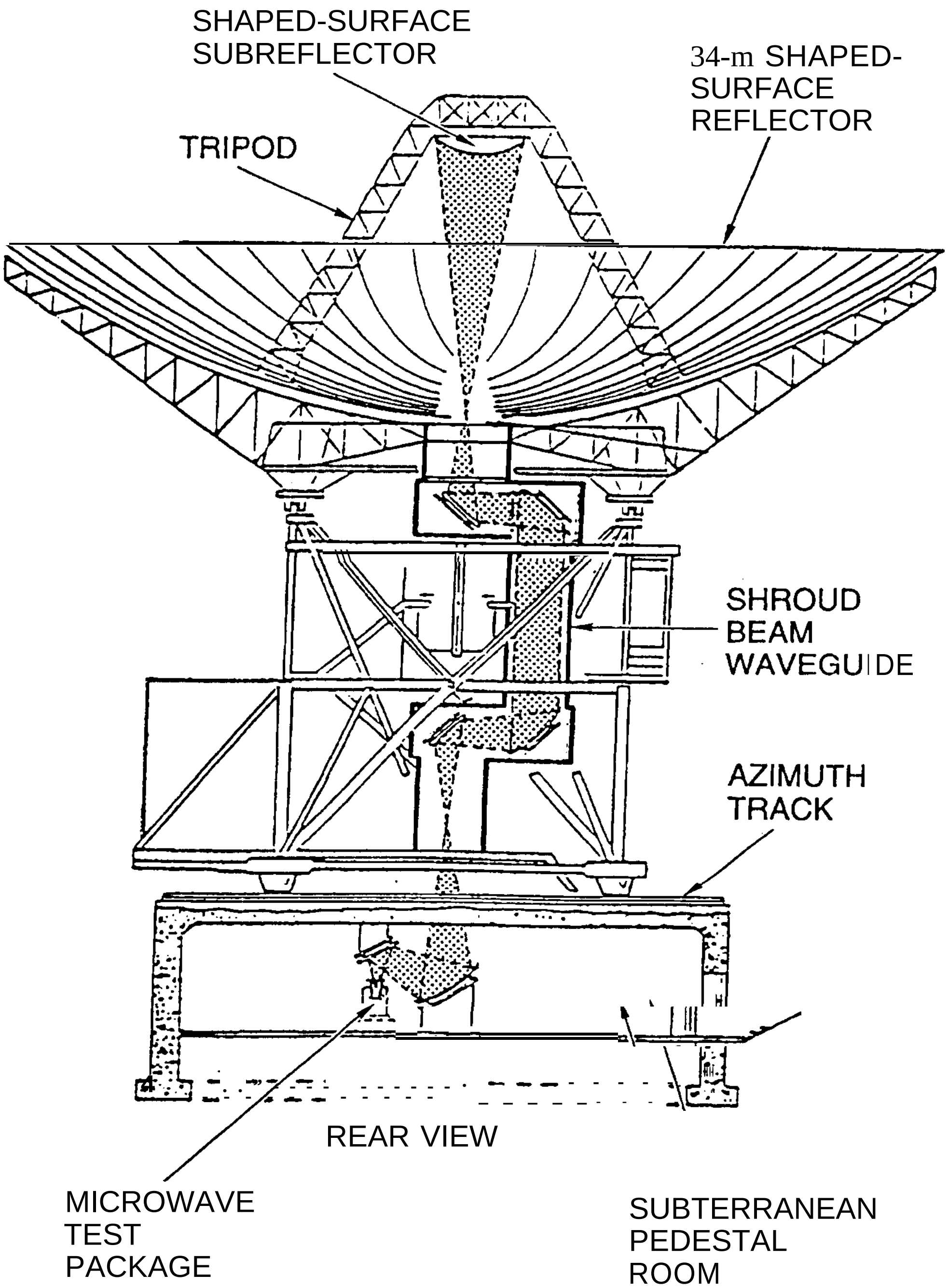

Figure 1. View of JPL's DSS-13 Beam-Waveguide Antenna

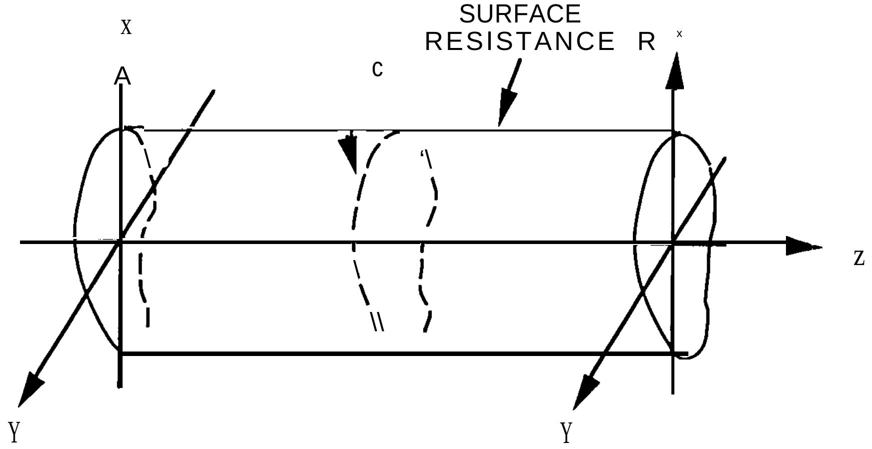

Figure 2. Geometry of the Problem

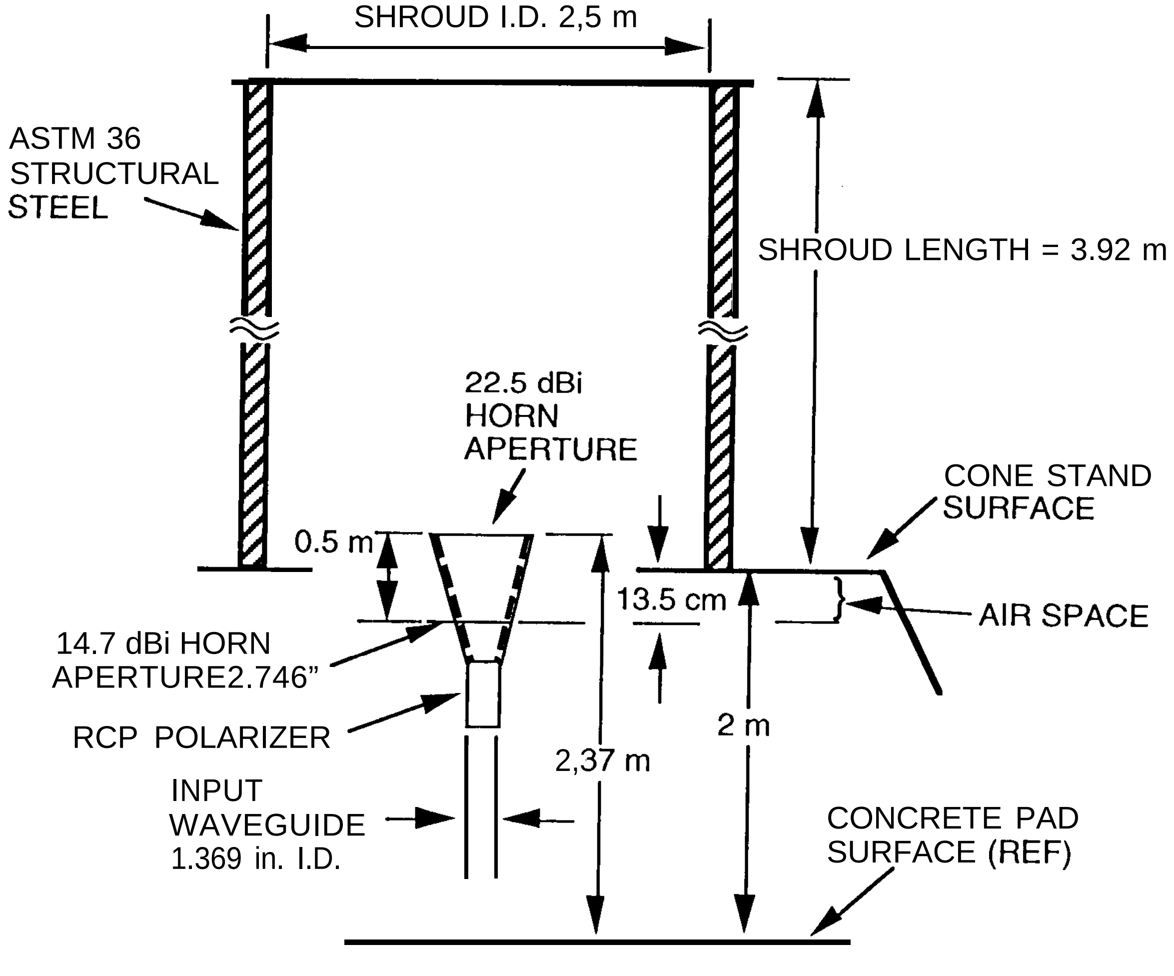

Figure 3. Experimental Setup

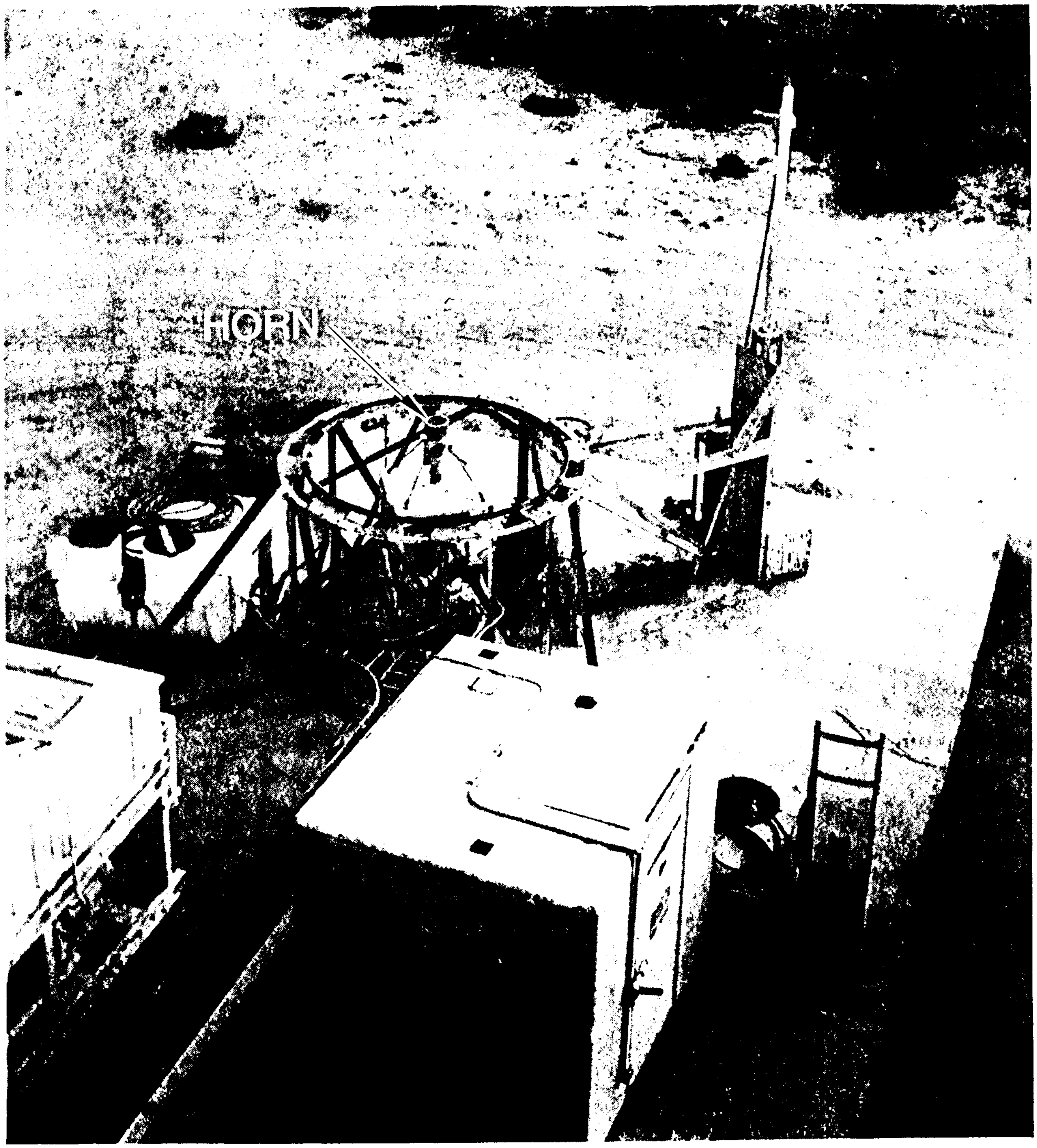

Fig 4a   Horn in Free Space

Figure 4 Measurement Technique

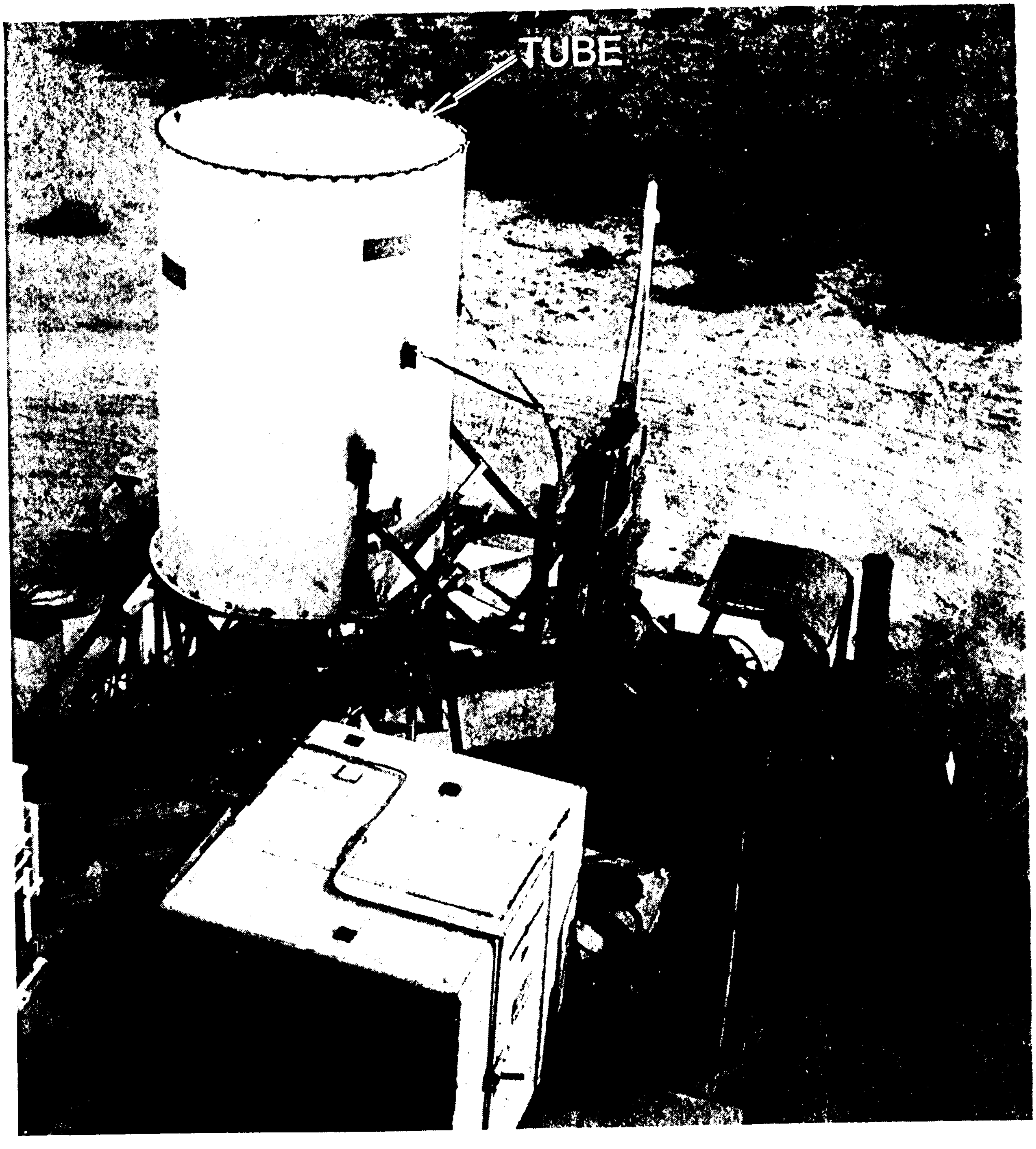

Fig 4b   Horn with BWG Tube

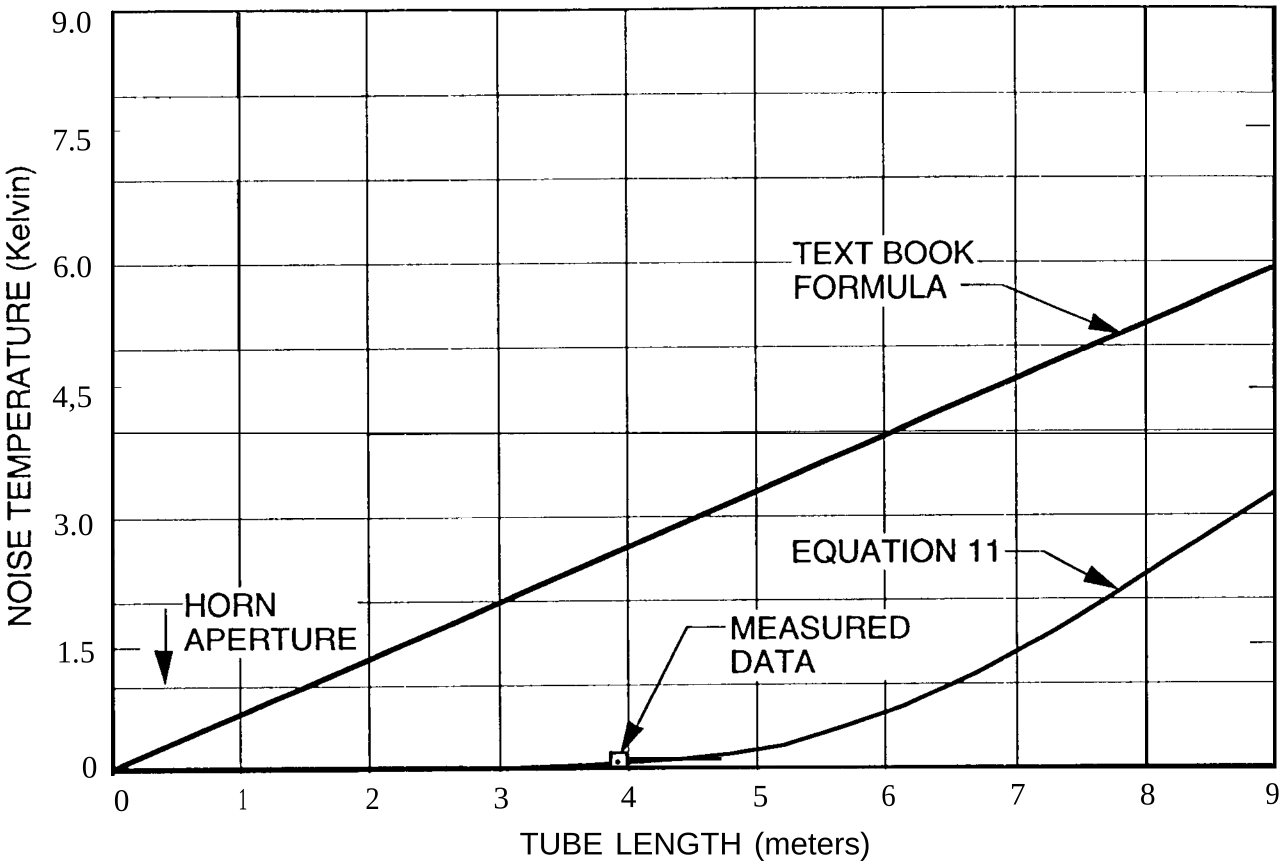

Figure 5. Noise Temperature versus Tube Length for 22.5-dBi Gain Horn