ON THE VALIDITY OF THE ASSUMED PDF METHOD FOR MODELING BINARY MIXINGKEACTION OF EVAPORATED VAPOR IN GAS/LIQUID-DROPLET TURBULENT SHEAR FLOW

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ABSTRACT

An investigation of the statistical description of binary mixing and/or reaction between a carrier gas and an evaporated vapor species in two-phase gas-liquid turbulent flows is performed through both theoretical analysis and comparisons with results from direct numerical simulations (DNS) of a two-phase mixing layer. In particular, the validity and added complications of extending single-point assumed probability density function (PDF) methods to two-phase flows involving evaporating droplets as sources of vapor are addressed. Noting that Favre density-weighted averaging is the most convenient form for moment transport equations for these flows, algebraic relationships are derived for the ratios of the Favre and non-weighted scalar *means* and variances. Comparisons with the DNS results indicates that the mixture fraction Favre moments are nearly identical to the corresponding non-weighted moments even at the mixing layer centerline where the rms density fluctuation is > 12% of the mean density. It is therefore considered appropriate to use Favre moments for PDF closure. A transport equation for the Favre mixture fraction variance is then presented which contains four new terms due to evaporation effects. The DNS results indicate that one of these terms due to scalar-source correlations is predominantly responsible for scalar variance production, whereas the remaining three terms are of negligible magnitude. Finally, the β PDF which is known to represent well the DNS generated mixture fraction statistics for single-phase mixing, is shown to be a poor representation for mixing of vapor resulting from droplet evaporation.

INTRODUCTION

Whereas probability density function (PDF) methods applied to binary mixing and reaction in single-phase turbulent flow have been investigated over the past two decadcs[1][2][3][4], the analogous two-phase problem involving a vapor species generated through liquid droplet evaporation remains essentially unaddressed. The simplest and most commonly used of the single-phase methods is the 'assumed' PDF method which is based on an *a priori* specification of an appropriate density for the mixture fraction. Necessary, but not sufficient, characteristics for a physically plausible assumed PDF *are* that it be bounded on a finite domain (i.e. $0 \leq \phi \leq 1$) and be capable of accurately portraying the 'true' scalar density evolution throughout al I stages of mixing; e.g. from initially segregated double delta function through asymptotically full y mixed Gaussian[5]. From the perspective of implementation, the two-parameter β PDF[4] is particularly attractive duc to the simple algebraic relationships defining its parameters in terms of the scalar mean and variance[4]. The β distribution has furthermore been shown to compare well with results obtained by direct numerical simulations (DNS) of a single-phase, non-homogenous reacting shear layer[6]. A procedure for generating PDFs appropriate for single-phase binary mixing was recently proposed by Miller et. al.[5] who introduced a family of PDFs based on the Johnson-Edgeworth Translation (JET). They evaluated one member of this family, the Logit-Normal PDF, along with the β and the Amplitude Mapping Closure (AMC)[7][8] through a comparison with DNS results of binary mixing in homogeneous isotropic turbulence. Although all of the PDFs captured the general features of the DNS scalar evolution, the Logit-Normal was found to most accurately portray the intermediate mixing stages; however, it is was not possible to obtain algebraic moment closures for this PDF and its practical application is therefore questionable. The present study addresses the question of the possible extension of the assumed PDF method to binary mixing/reaction occurring when one species is introduced through the evaporation of droplets in two-phase gas-liquid turbulent flows. This situation is different from the aforementioned studies because of additional complexities not found in single-phase flows.

Consider the transport equations for the gas phase density, evaporated vapor mass fraction and conserved scalar mixture fraction for this two-phase binary mixing problem:

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x_j}[\rho u_j] = -S_V, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho Y_V) + \frac{\partial}{\partial x_j}\left[\rho Y_V u_j - \rho\Gamma\frac{\partial Y_V}{\partial x_j}\right] = -S_V, \tag{2}$$

$$\frac{\partial}{\partial t}(\rho\phi) + \frac{\partial}{\partial x_j}\left[\rho\phi u_j - \rho\Gamma\frac{\partial \phi}{\partial x_j}\right] = 0, \tag{3}$$

where $0 \leq \phi \equiv (1 + Y_c - Y_V)/2 \leq 1$ is the conserved scalar mixture fraction and S_V is a mass source/sink due to vaporization/condensation of the liquid phase (S_V is negative for evaporation). In this study it is assumed that the dispersed phase volume fraction is small, the species diffusivity Γ is constant and no reactions occur. However, for reacting flows Eq.(3) is unaltered, and the statistics of the reactants and products can be related directly to those of the mixture fraction (see e.g. Ref.[6]), so that the methodology presented here remains valid.

In general, the gas phase flow initially consists of a pure carrier species such that the scalar PDF is everywhere unimodal in the form of a delta function located at $\phi = 1$ in mixture fraction space. As evaporation and mixing progress, the scalar distribution will grow inwards ($\bar{\phi} < 1$; see the Nomenclature for averaging definitions) indicating a more mixed state which will be intuitively characterized by a highly skewed distribution function. The actual form of such a PDF will be determined by the coupled physical evaporation/condensation and turbulent mixing processes occurring within the flow and may vary substantially from that observed in single-phase binary mixing.

The present investigation addresses three issues associated with the statistical description of such two-phase flows by assumed PDF methods: (1) If there exists an appropriate PDF capable of portraying the mixture fraction evolution through all stages of mixing, then the non-weighted mean and variance are two moments necessary to determine the PDF. However, since the governing transport equations for the scalar moments are most conveniently cast in the form of Favre density-weighted averaging, we first analyze the conditions under which it is valid to substitute Favre moments for non-weighted moments for PDF determination. (2) Favre averaged transport equations for the mixture fraction mean and variance are then derived and the relative magnitudes of terms resulting from the evaporative source S_v are calculated for the simulated shear flow. If an appropriate PDF exists, and if the Favre moments can be substituted for the non-weighted moments, then the solutions of these modeled equations would eventually be used to determine the assumed two-parameter PDF in mixing/reaction applications. (3) The assumed β PDF is tested with the gas/liquid-droplet mixing layer in order to characterize its applicability for two-phase evaporating droplet flow. The β PDF is chosen due to its relatively wide spread usage for single-phase mixing/reaction and to its algebraic closed form moment relations. The study is conducted through both theoretical analysis and through comparisons with results obtained previously from DNS.

SIMULATION SUMMARY

Throughout the following analysis, previous DNS results[9] are used in conjunction with theoretical derivations;

the DNS results were for a shear layer formed by the merging of a pure gas stream with a hydrocarbon droplet laden gas stream. Since it is impossible to fully describe here the DNS results, only a brief summary of the simulation is included and the reader is referred to Ref.[9] for further details. The flow configuration considered is that of a three dimensional temporally developing mixing layer formed by the merging of two parallel flowing gas streams. The streamwise, cross stream and spanwise coordinate directions are denoted as x_1, x_2, x_3, respectively (i.e. $u_1 = \pm U_0$ in the far fields of $\pm x_2$). Periodic boundary conditions are employed in both the streamwise and spanwise directions, whereas adiabatic slip wall conditions are employed at the cross stream boundaries through the solution of appropriate characteristic equations[10]. Forcing of fundamental and first harmonic modes is used to stimulate the growth and development of both spanwise and streamwise disturbances resulting in a single primary vortex pairing. The upper/unladen stream ($x_2 > 0$) is a pure gas phase flow (air), whereas the lower/laden stream is uniformly seeded with randomly placed spherical droplets with a liquid to gas mass loading of 0.225. Every individual droplet is tracked in a Lagrangian manner through the solution of equations for position, velocity, uniform internal temperature (infinite liquid thermal conductivity) and mass. Vaporization is governed by the non-equilibrium Langmuir-Knudsen evaporation law[11]. All properties are assumed to be constants, except for the latent heat of evaporation of the droplet species which is a linear function of temperature as required for calorically perfect species[9]. The gas phase viscosity is assumed to be independent of the mixture fraction, and is chosen to produce a resolvable flow (Re = 200 based on the initial vorticity thickness and mean velocity difference across the layer), the Prandtl number is 0.697 and the Lewis number is assumed equal to unity. All remaining properties are calculated from correlations in Ref. [11] evaluated at $350K$, and correspond to those of air and decane for the carrier gas and liquid/vapor species, respectively. Complete two-way mass, momentum and energy exchange between phases is accounted for. The gas phase is initially at a uniform temperature of $350K$ corresponding to moderate evaporation rates, the initial liquid phase temperature is $325K$, and the free stream Mach number is initially 0.5.

The DNS code utilizes fourth order accurate Runge-Kutta explicit time stepping for both the gas and droplet equations, whereas **all** spatial derivatives are approximated by eight order accurate central finite differences 12]. The computational grid is analytically compressed in the cross stream direction with a maximal compression ratio of 0.5 at the centerline. Eulerian gas phase variables required for the dispersed phase transport equations are interpolated to the droplet positions using a fourth order accurate Lagrange interpolation procedure. Phase coupling terms (e.g. S_V) are calculated by summing individual droplet contributions within local computational volumes using a geometric weighting factor based on the distance between each droplet and its eight nearest neighbor grid points. The solution procedure consists of integrating the coupled gas and droplet equations simultaneously in a time accurate manner following the **rollup** and spanwise vortex pairing of the mixing layer. **The** particular simulation addressed in this paper was performed with a gas phase resolution of 96 x 128 x 64 grid points, following 4.5 x 10^5 individual droplets and required approximately 15 hours of CPU time on a Cray **J90 supercomputer**.

Figure 1 shows the final time mixture fraction PDF extracted from the DNS results as a **function** of the cross stream direction x_2 (not the joint PDF). At this stage, the laden stream has reached a saturated state for which evaporation has essentially ceased (although droplets remain present) with $\overline{\phi} \approx 0.94$ (i.e. $\overline{Y_V} \approx 0.06$). In this case, saturation has occurred due to two primary influences: (1) evaporation and associated latent heat effects have reduced the temperature of the laden stream, and (2) the evaporated mass has resulted in a free stream vapor mass fraction approximately equal to the average surface mass fraction of the droplets. At this time the **saturated-**free-stream droplets have undergone an average **20%** reduction in mass; however, droplets which continue to be entrained into the layer undergo **further evaporation** as they are brought into contact with the higher temperature pure carrier gas stream. Inside the layer, a more markedly mixed state is apparent as characterized by a continuous range of mixture **fraction** values; 0.92< #J $\leq$ 1. Finally, the delta function located at $\phi = 1$ for $X_2 > 0$ locations indicates the pure carrier gas stream. The **successful** application of the assumed PDF method to this flow requires the accurate reproduction of the scalar distributions in Fig. 1 for all cross stream locations by an appropriate *a priori* chosen PDF.

RESULTS

The presence of evaporative source terms in the gas phase density equation [Eq.(1)] implies that appropriate moment transport equations are most conveniently formulated in terms of Favre averaged (density-weighted) variables; even in the case of low speed flow. This is duc to the fact that the Favre averaging removes terms resulting from density fluctuations and therefore substantially reduces the number of terms requiring modeling. In this section we investigate whether it is appropriate to determine an assumed PDF using Favre moments in place of the 'true' non-weighted averaged (hereinafter referred to as 'ensemble' averaged for simplicity) moments. Utilizing the standard definitions of the ensemble and Favre averaging operators defined in the Nomenclature, algebraic relationships between the corresponding means and variances of the scalar (or any flow variable) are derived: ,

$$\frac{\tilde{\phi}}{\bar{\phi}} = 1 + C_2\ (\rho, \phi)\ I(\rho)\ I(\phi), \tag{4}$$

$$\frac{\widetilde{\phi''\phi''}}{\overline{\phi'\phi'}} = 1 - [C_2(\rho, \phi)\ I(\rho)]^2 + C_3(\rho, \phi, \phi)\ I(\rho), \tag{5}$$

where the correlation coefficient of order N:

$$C_N(a_1, a_2, ...a_N) = \frac{\overline{\prod_{k=1}^{N} a'_k}}{\prod_{k=1}^{N} \left(\overline{a_k'^{\,2}}\right)^{1/2}}, \tag{6}$$

is bounded by —1 $\leq C_N \leq$ +1, and the fluctuation intensity is defined as the root mean square (rms) fluctuation relative to the mean:

$$I\,(a_k) = \left(\overline{a_k'^{\,2}}\right)^{1/2}\ /\ \overline{a_k}, \tag{7}$$

for any stochastic vector a_k. In this form, the above derivations clearly illustrate the physical constraints which must be satisfied in order to substitute Favre moments for ensemble moments. In particular, these relations suggest that moments calculated from the two methods may be nearly equal even in the case of large density fluctuations *if* either the scalar intensity is small or, more general] y, if the scalar and density fields are uncorrelated at order two and/or three.

The respective ratios of means and variances from Eqs.(4) and (5) are calculated from the final time' DNS results and presented in Fig. 2 along with the corresponding density values and correlation coefficients (averaging is over homogeneous $x_1 - x_3$ planes). These results show clearly that even for the present flow with relatively large density fluctuations $[I(\rho)_{max} > 0.12$; scc Fig.2b] that the Favre and ensemble means and variances are nearly identical. This is true for the means even in regions where the second order correlation coefficient is $C_2(\rho, \phi) \approx -0.5$ (Fig.2c), because the scalar intensity is in all locations < 3% (Fig.2a). The Favre and ensemble variances are also essentially identical, but for different reasons [note that the scalar intensity does not appear in Eq.(5)]: although all terms on the right hand side of Eq.(5) have maximal magnitudes within the layer, the respective terms due to second and third order correlations have opposite sign and cancel each *other* resulting in $\widetilde{\phi''\phi''} \approx \overline{\phi'\phi'}$. These results suggest that it is reasonable to determine the assumed PDF with Favre moments for this flow configuration, and possibly for other flow types containing relatively large evaporation induced density fluctuations.

Determination of a two-parameter assumed PDF requires the knowledge of at least two moments of the scalar field. In general, closure is obtained through the solution of appropriate transport equations for the two lowest moments; i.e. the scalar mean and variance[5]. The Favre and ensemble averaging operators are therefore applied to Eq.(3) in order to obtain a transport equation for the Favre mean mixture fraction:

$$\frac{\partial}{\partial t}\left(\overline{\rho}\widetilde{\phi}\right) = -\frac{\partial}{\partial x_j}\left[\overline{\rho}\widetilde{\phi}\widetilde{u}_j\right] + \frac{\partial}{\partial x_j}\left[\overline{\rho}\,\Gamma\frac{\partial\widetilde{\phi}}{\partial x_j}\right] - \frac{\partial}{\partial x_j}\left[\overline{\rho}\,\widetilde{\phi'' u_j''}\right], \tag{8}$$

and also, by invoking the continuity equation, for the Favre scalar variance evolution:

$$\frac{\partial}{\partial t}\left(\overline{\rho}\widetilde{\phi''\phi''}\right) = I + II + III + IV + V + VI + VII + VIII + IX, \tag{9}$$

where the right hand side terms are given by:

$$I = -\frac{\partial}{\partial x_j}\left[\overline{\rho}\left(\widetilde{\phi''\phi''}\right)\widetilde{u}_j\right]: \qquad \textbf{Convection}, \tag{lo}$$

$$II = +\frac{\partial}{\partial x_j}\left[\overline{\rho}\Gamma\frac{\partial}{\partial x_j}\left(\widetilde{\phi''\phi''}\right)\right]: \qquad \text{Diffusion}, \tag{11}$$

$$III = -\frac{\partial}{\partial x_j}\left[\overline{\rho}\left(\widetilde{\phi''\phi'' u_j''}\right)\right]: \qquad \text{Third Order Turbulent Transport}, \tag{12}$$

$$IV = -2\overline{\rho}\left(\widetilde{\phi'' u_j''}\right)\frac{\partial\widetilde{\phi}}{\partial x_j}: \qquad \text{Mean Gradient Production}, \tag{13}$$

$$V = -2\,\overline{\rho}\,\Gamma\,\widetilde{\left(\frac{\partial\phi''}{\partial x_j}\frac{\partial\phi''}{\partial x_j}\right)}: \qquad \text{Dissipation,} \tag{14}$$

$$VI = +\,\overline{\phi''\phi''}\;\;\overline{S_V}: \qquad \text{Mean Source Destruction,} \tag{15}$$

$$VII = +\,2\,\tilde{\phi}\,\overline{\phi''}\;\;\overline{S_V}: \qquad \text{Ensemble Mean of Favre Scalar Fluctuation Effect,} \tag{16}$$

$$VIII = +\,2\,\tilde{\phi}\,\overline{\phi'' S_V'}: \qquad \text{Second Order Scalar-Source Correlation,} \tag{17}$$

$$IX = +\,\overline{\phi''\phi'' S_V'}: \qquad \text{Third Order Scalar-Source Correlation.} \tag{18}$$

Due to conserved nature of ϕ, Eq.(8) is unaltered from its traditional single-phase form by the presence of droplets, and therefore will not be addressed in this paper. In contrast, the scalar variance equation contains, in addition to the traditional single-phase sources, four separate terms $(VI - IX)$ which result from droplet evaporation. Note that these terms combine ensemble and Favre averaging of the scalar field due to the source term in Eq.(1) not containing the gas phase density. Terms VI and VII both *involve the* ensemble mean source term: the first of these is a destruction term for a vaporizing liquid phase as S_V *takes* negative values for evaporation, whereas the second term (VII) is proportional to the ensemble mean of the Favre scalar fluctuation (it is straightfonvard to show that $\overline{\phi''} = \overline{\phi} - \tilde{\phi}$) which can be expected to become small under the conditions of Eq.(4). Finally, terms $VIII$ and IX involve second and third order scalar-source correlations, respectively. It is not readily apparent whether these last two terms will act as sources or sinks in general turbulent flows; however, physical intuition dictates that evaporating droplets should enhance the scalar variance. Since term VI is negative definite for evaporation and term VII should be small due to $\overline{\phi''}$, it may be conjectured that either the second and/or the third order scalar-source correlation must be a primary production term.

Figure 3 depicts the normalized magnitudes of the terms on the right hand side of the scalar variance transport equation calculated from the DNS results for three different times. In all cases, the scalar variance production near the center of the layer is primarily due to the action of both the mean gradient production (term IV) and the second order scalar-source correlation (term $VIII$). For this particular flow, all remaining terms due to liquid evaporation are of negligible magnitude during all stages of the mixing layer growth. It is interesting to note that knowledge of the mean evaporation rate $\overline{S_V}$ is of *no* direct use in solving the scalar variance equation since the two terms involving $\overline{S_V}$ arc both of negligible magnitude. Efforts directed at modeling the influence of evaporation on the scalar variance transport will therefore primarily require models for the correlation $\overline{\phi'' S_V'}$.

Assuming an accurate prediction of the scalar moments, the issue of whether or not the assumed PDFs generally applied to single-phase flow can be extended to the two-phase mixing layer is now addressed. Due to its available algebraic moment expressions, we test the utilization of the β PDF as specified by the scalar **Favre** mean and variance (calculated here from the DNS results) by comparing it with the DNS results. The β PDF will be deemed usable if it captures the various forms of the 'true' scalar PDF shown in Fig. 1 for all cross stream **locations**. Although in general the two parameters specifying the β PDF will be calculated from the solutions of the modeled **Eqs.(8)** and (9), in this case the two parameters are calculated from the 'exact' **Favre** mean and variance extraeted from the DNS results. Figure 4 presents a comparison of the normalized third (skewness, μ_3) and fourth (flatness, μ_4) moments extraeted from the DNS results as compared to the corresponding assumed PDF predictions. In the droplet laden stream ($z_2 < O$) the β density predicts an approximate Gaussian PDF (i.e. μ_3 = O and $\mu_4 = 3$), whereas the DNS results show a much narrower distribution about the mean with $\mu_4 \approx 2$. Inside the layer, the assumed PDF is incapable of capturing the hump observed for both moments **located** near $x_2/\delta_\omega \approx -0.8$ which results from the action of the vortex pairing engulfing fluid from the opposite pure gas stream into the layer. Within the $x_2 > 0$ portion of the layer, both the skewness and flatness are over-predicted in magnitude, although the general profiles of the DNS moment curves are reproduced. Figure 5 more closely illustrates the problems with the assumed density by depicting both the DNS and β PDF predictions at the centerline, $x_2 =$ O. The β PDF is incapable of capturing the highly skewed and double hump features of the DNS scalar field (note that **the** β PDF is defined continuously over the range $0 \le \phi \le 1$). **Finally, notc that these results** represent a relatively strenuous test of the assumed PDF method, as the DNS PDFs are highly skewed due to the observed flow saturation. Further tests are recommended at higher temperatures for which the scalar mean will be decreased and the resulting PDF less severely skewed.

CONCLUSIONS

The results described above have addressed several issues involved in the stochastic modeling of binary mixing and/or reaetion in two-phase turbulent flows by assumed PDF methods. The mixing and/or reaction occurs between a carrier gas and a vapor resulting from liquid droplet evaporation. We first noted that PDF determination is most simply obtained through transport equations formulated in terms of Favre density-weighted moments due to the relatively large density fluctuations which result from evaporation effects. Accordingly, we have made the following three contributions: (1) We have derived algebraic relations which illustrate the conditions under which PDF determination can be accurately performed using Favre moments. We confirmed, using previously obtained DNS results, that this assumption is indeed valid for the two-phase mixing layer under conditions of moderate evaporation. (2) We have derived transport equations for the mixture fraction Favre mean and variance, and used the DNS results to calculate the relative *importance* of the four terms involving evaporation effects. We found that one term due to scalar-source correlations is responsible for variance production, whereas the remaining terms are of negligible magnitude. (3) We used the DNS results to illustrate inadequacies inherent in assuming the β PDF to portray the mixture fraction PDF evolution in two-phase mixing layers involving a dispersed liquid droplet species. Since this result was found for moderate evaporation, it is expected that this will represent a worst case situation since the resulting DNS mixture fraction PDFs are highly skewed. Future research in this area should focus on the derivation of new PDFs capable of capturing the unique characteristics of the two-phase mixture fraction statistics.

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NOMENCLATURE

a_k Generic stochastic vector.

C_N Correlation coefficient of order N.

I Normalized fluctuation intensity.

Re Reynolds number.

S_V Source term due to liquid evaporation/condensation (negative/positive).

t Time.

u_i Velocity vector.

U_0 Mean free stream velocity (A U_o = $2U_0$).

Y_C Carrier gas mass fraction.

Y_V Vapor evaporate mass fraction.

δ_ω Vorticity thickness = $\Delta U_0/ (d\overline{u_1}/dx_2)_{\max}$.

$\delta_{\omega,0}$ Vorticity thickness at $t = O$.

Γ Fickian diffusion coefficient.

ρ Gas phase density.

ϕ Mixture fraction scalar variable (O $\leq \phi \leq$ 1).

μ_N Normalized moment or order N: $\mu_N(a_k) = \overline{(a'_k)^N} / \overline{(a'_k)^2}^{N/2}$

$\overline{a_k}$ Non-weighted ensemble average of a_k.

$\widetilde{a_k}$ Density-weighted Favre average of a_k: $\widetilde{a_k}$ = $\overline{\rho a_k}/\overline{\rho}$.

a'_k Fluctuation of a_k with respect to $\overline{a_k}$.

a''_k Fluctuation of a_k with respect to $\widetilde{a_k}$.

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FIGURE CAPTIONS

Figure 1: Instantaneous mixture fraction PDF extracted from the DNS results as a function of the cross stream direction at time $t\Delta U_0/\delta_{\omega,0}$ = 50.

Figure 2: Pertinent flow statistics across the layer at DNS time $t\Delta U_0/\delta_{\omega,0}$ = 50: (a) Comparison of the ensemble and Favre scalar mean and rms, (b) Ensemble mean and rms density, (c) Second and third order density-scalar correlation coefficients.

Figure 3: Normalized source terms from the Favre variance transport equation as a function of the cross stream direction at times (a) $t\Delta U_0/\delta_{\omega,0}$ = 10, (b) $t\Delta U_0/\delta_{\omega,0}$ = 25, (c) $t\Delta U_0/\delta_{\omega,0}$ = 50.

Figure 4: Comparison of skewness (μ_3) and flatness (μ_4) calculated from DNS results (at time $t\Delta U_0/\delta_{\omega,0}$ = 50) with predictions based on an assumed β PDF.

Figure 5: Comparison of the scalar PDF extracted from the DNS results at $t\Delta U_0/\delta_{\omega,0}$ = 50 and X_2 = *O with the* assumed β PDF prediction.

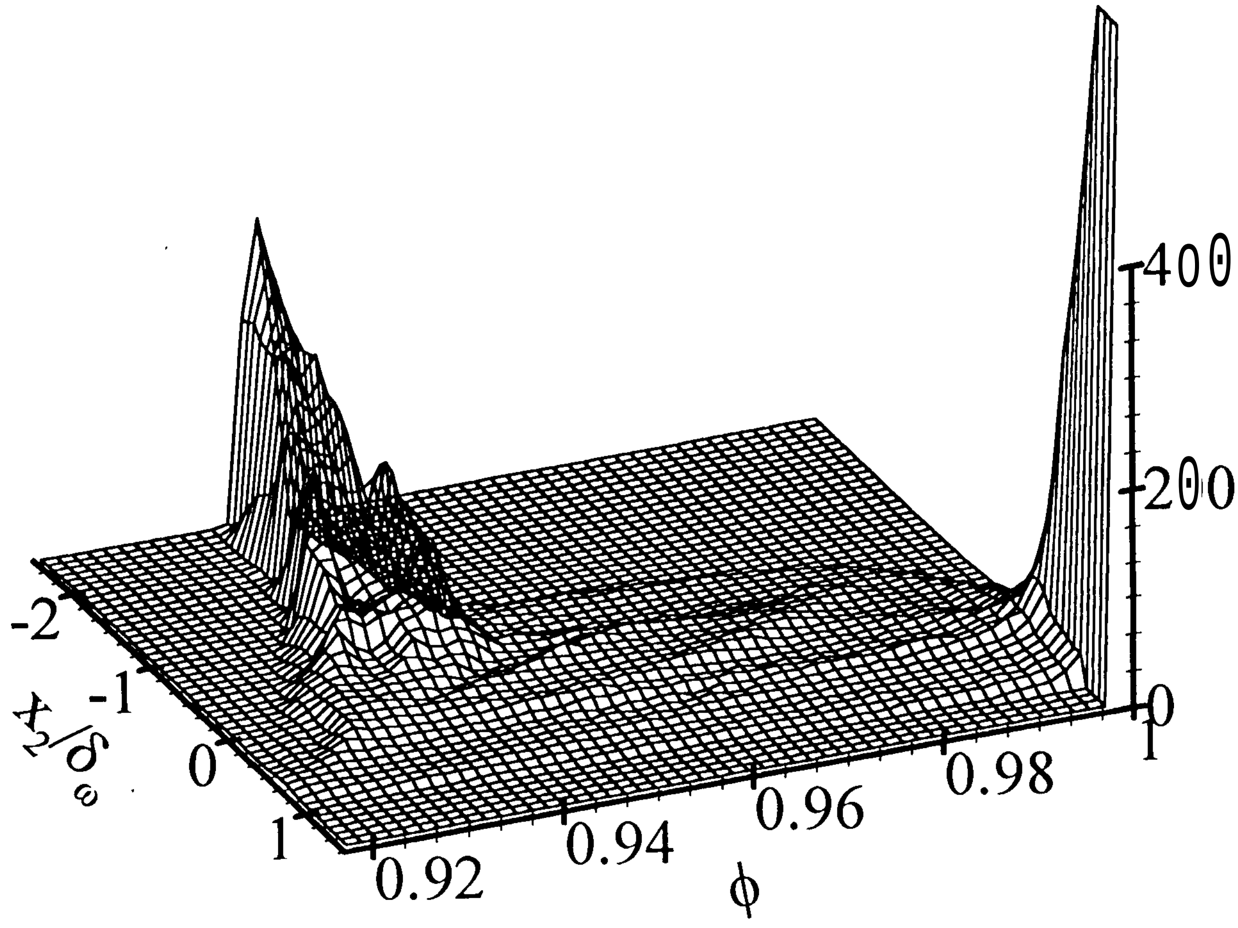
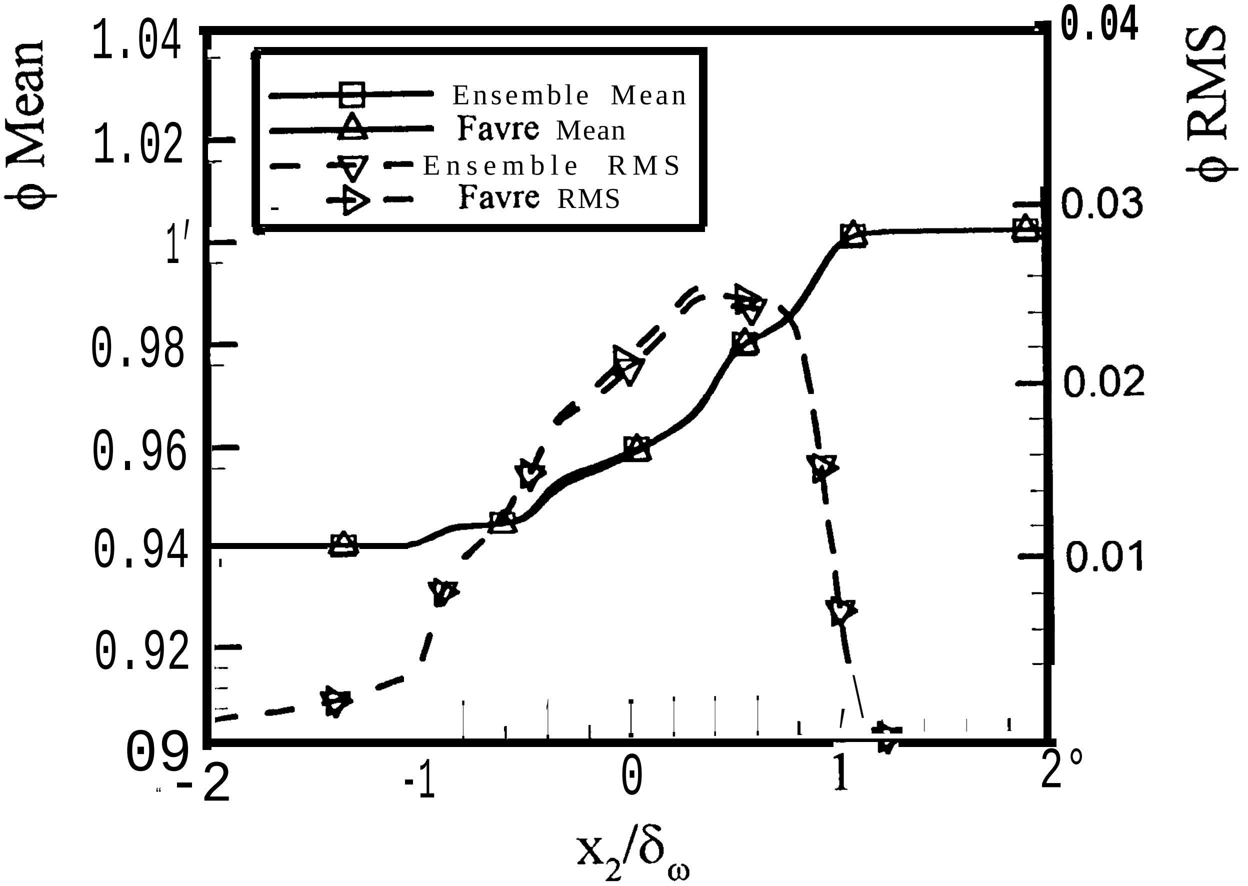


Figure 1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1.02
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0.94
0.92
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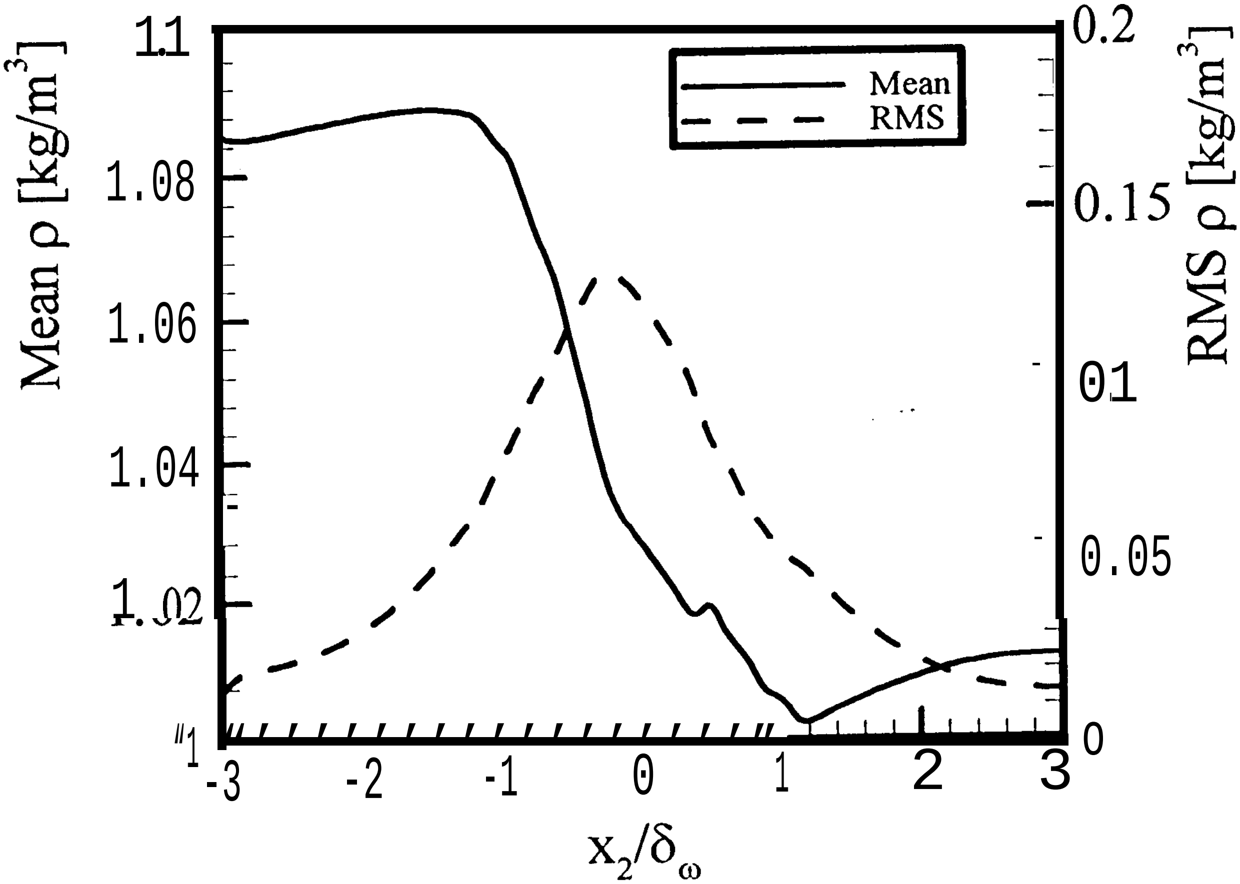
Ensemble Mean
Favre Mean
Ensemble RMS
Favre RMS
x_2/δ_ω

Figure 2a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RMS ρ [kg/m³]
Mean
RMS
x₂/δ_ω
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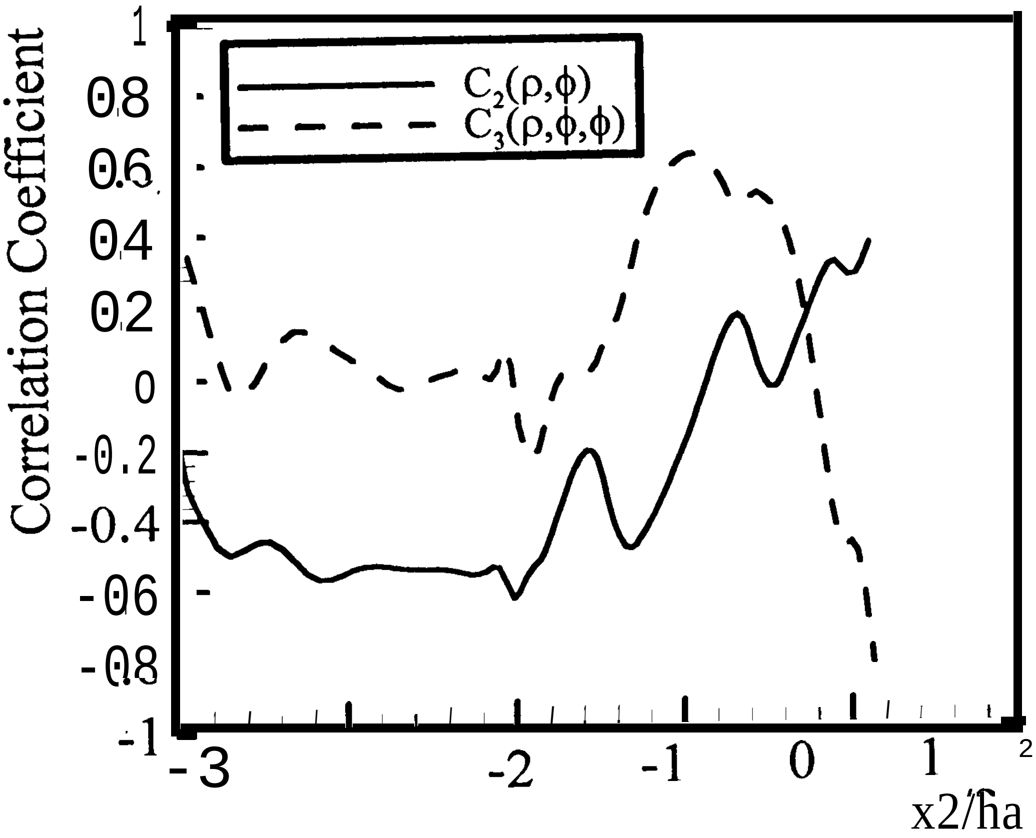
2
3

Figure 2b



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0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
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$C_2(\rho,\phi)$
$C_3(\rho,\phi,\phi)$
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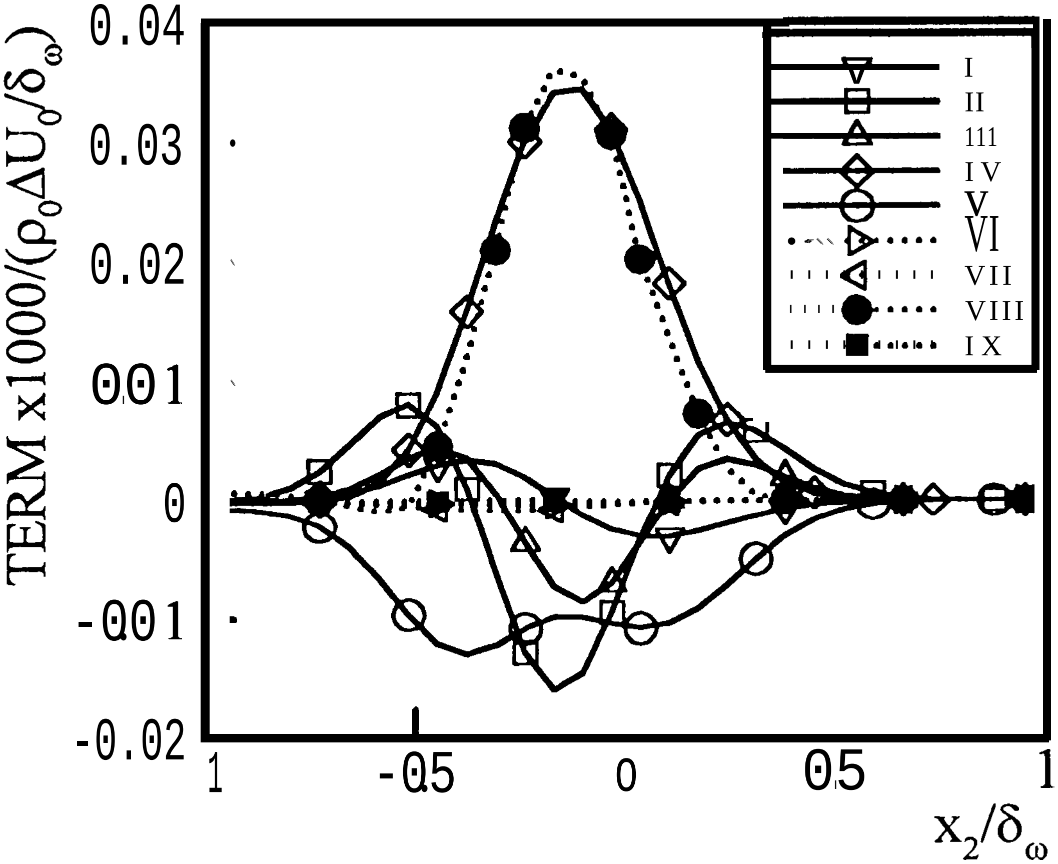
-2
-1
0
1
2
x2/ħa

Figure 2 c



TERM x1000/(ρ0ΔU0/δω)
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0.03
0.02
001
0
-001
-0.02
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0
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1
x2/δω
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II
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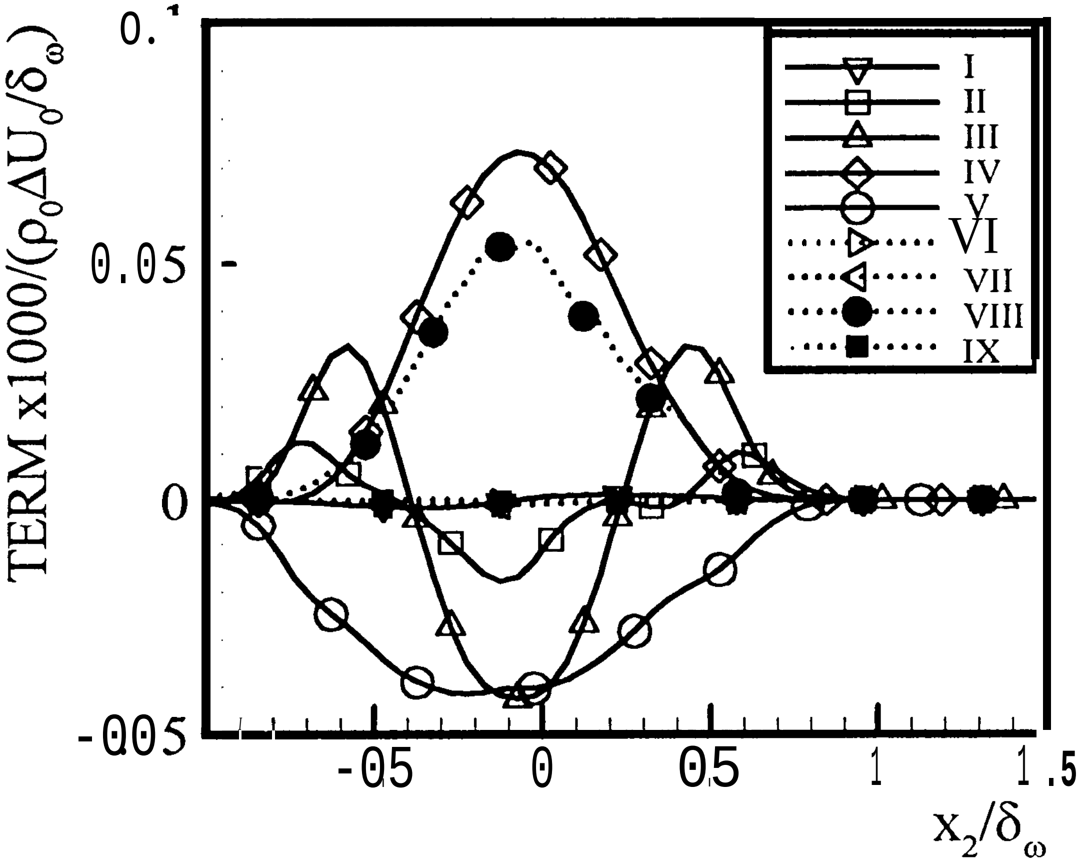
IV
V
VI
VII
VIII
IX

Figure 3a



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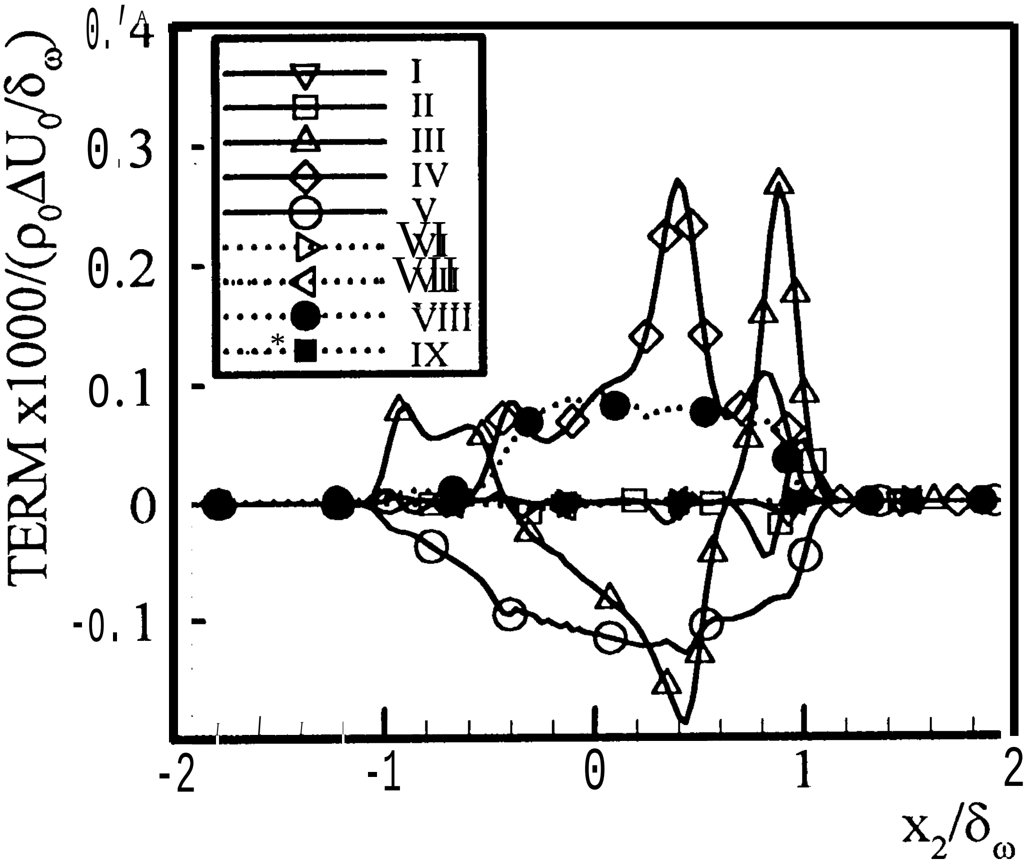
x2/δω
I
II
III
IV
V
VI
VII
VIII
IX

Figure 3b



TERM x1000/($\rho_0 \Delta U_0/\delta_\omega$)
0.4
0.3
0.2
0.1
0
-0.1
I
II
III
IV
V
VI
VII
VIII
IX
-2
-1
0
1
2
x_2/δ_ω

Figure 3c

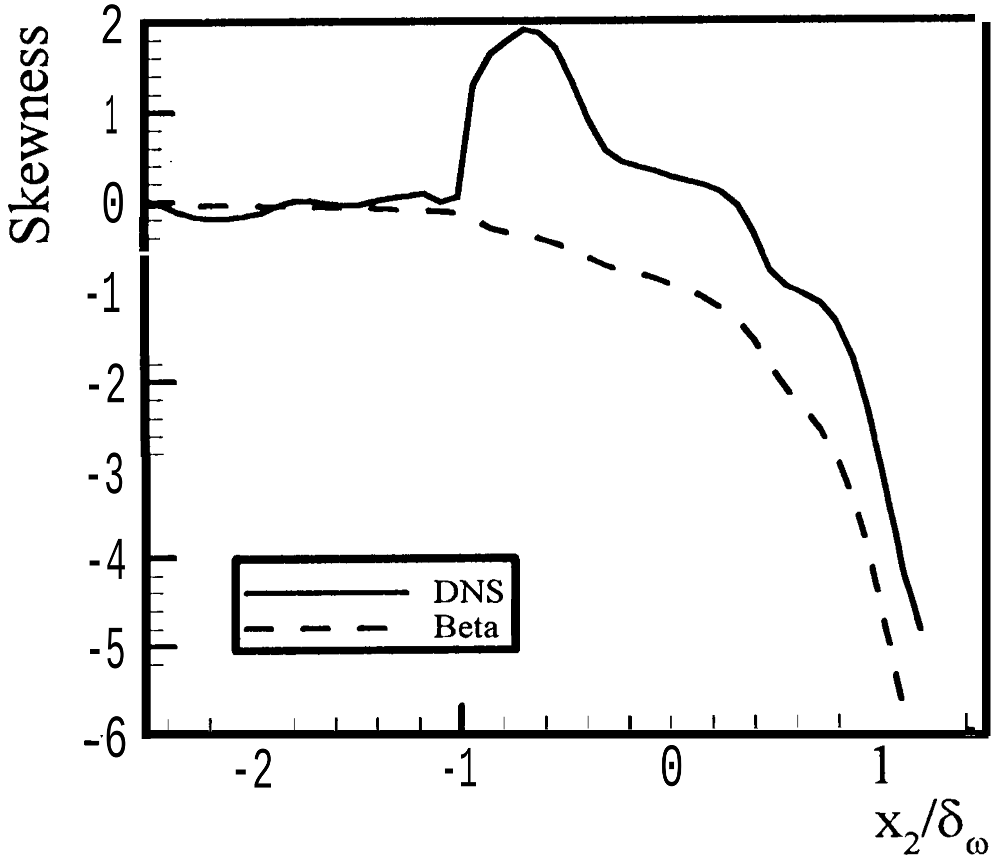


Figure 4a

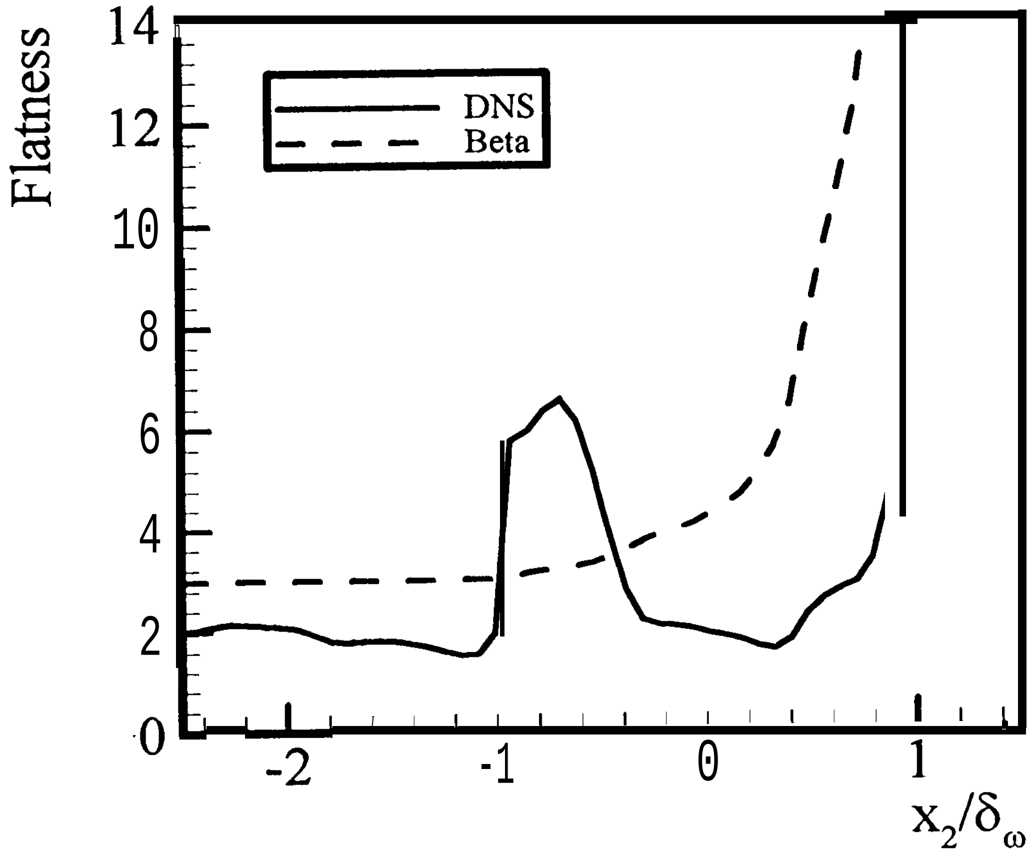


Figure 4b

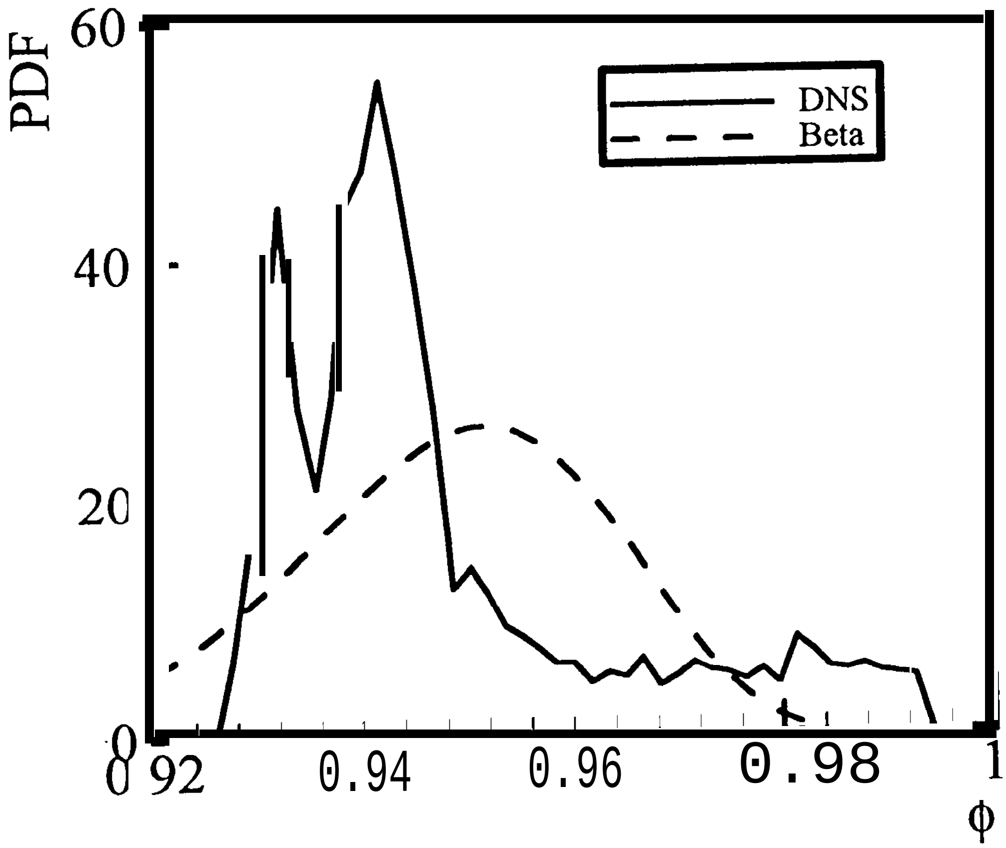


Figure 5