

A JOINT VITERBI ALGORITHM TO SEPARATE COCHANNEL FM SIGNALS

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ABSTRACT

This paper presents a method for separating cochannel FM signals. We show that the Viterbi algorithm, traditionally limited to estimation of digital quantities, can jointly track analog FM signals by quantizing the derivative of their instantaneous frequencies. We employ per-survivor processing in the trellis to estimate unknown channel effects. The approach works well when the signal to interference ratio (SIR) is less than or equal to zero, in contrast to conventional interference suppression algorithms that degrade as SIR nears 0 and fail catastrophically when $SIR < 0$. Comparisons of mean squared error (MSE) of the estimates are given for varying SIR, SNR, Doppler offsets, and frequency deviations. The method also can be applied to any other continuous phase modulation scheme, such as CPFSK.

1. INTRODUCTION

A single phase-locked loop (PLL) or phase discriminator can effectively demodulate an FM signal because the signal has a constant envelope and an instantaneous frequency that is proportional to the message signal. However, these conventional techniques can suffer severe degradation when the input consists of the superposition of two cochannel FM signals, because the envelope is no longer constant and the instantaneous frequency is not proportional to either of the cochannel FM signals or their sum. The output in this case contains large, inband spikes and the output is unintelligible [6]. As a result, a number of different receivers have been developed to combat cochannel interference [2, 3, 4, 6, 8]. Such work has made an important contribution, but each is missing one or more of the following desirable attributes

1. The estimation of both dominant and subdominant signal components.

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2. The estimation of unknown channel parameters with Per-Survivor Processing ('S1') in a trellis algorithm, instead of with weaker decision directed estimators.
3. The applicability to both digital (CPFSK) and analog (FM) continuous phase modulation schemes.

In this paper, we present a receiver that contains all three features; as a result, it exhibits improved demodulated voice quality for cochannel FM signals. The receiver uses a method of Cahn [1] (and later, [5]) to transform the phase tracking or FM demodulation problem into a discrete sequence estimation problem, which can be solved with the usual Viterbi algorithm. Unlike Cahn's work, however, we use a joint trellis capable of tracking both dominant and subdominant parts of a cochannel signal, and we estimate unknown channel effects with PSP.

2. SIGNAL MODEL

For simplicity, we assume that there are exactly two cochannel signals. Extension to additional signals is straightforward. The complex baseband representation of the sampled received signal is

$$r[k] = A_1[k]e^{j\theta_1[k]} + A_2[k]e^{j\theta_2[k]} + N[k] \quad (1)$$

where $A_i[k]$ and $\theta_i[k]$ is the amplitude and phase, respectively, of the i th signal at time kT_s , where T_s is the sampling period, and where $N[k]$ is a complex noise process. The amplitude is assumed to vary much slower than the phase, which is further decomposed as

$$f_i[k] = \omega_i kT_s + \phi_i + k_i \int_0^{kT_s} m_i(s) ds, \quad (2)$$

where for the i th signal, ω_i is an offset carrier frequency (in rad./sec.), ϕ_i is an initial phase offset, k_i is the frequency deviation (in rad./sec.), and $m_i(s)$ is the message waveform. This is illustrated in Figure 1. This paper assumes that

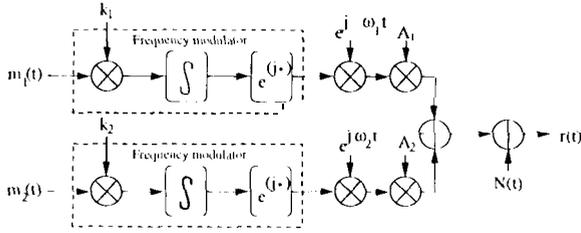


Figure 1: The signal model for two co-channel signals,

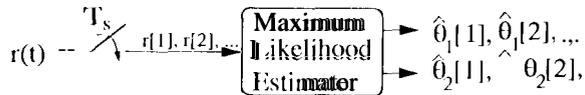


Figure 2: A maximum likelihood joint phase estimator.

$-1 \leq m_i(s) \leq 1$ for all $s > 0$ and $i \in \{1, 2\}$. This paper considers the worst case cochannel scenario of $\omega_1 = \omega_2$, which we set to zero. Note that through suitable restrictions on $m_i(s)$ involving symbol times and levels, digital modulating schemes such as CPFSK are implicitly allowed in Equation (2).

3. JOINT VITERBI ESTIMATION WITH PSP

If a vector N samples $\mathcal{Y} = (r(1), r(2), \dots, r(N-1))$ have been received, then for $k \leq N$, the maximum likelihood estimate of $(\theta_1[k], \theta_2[k])$ is

$$(\hat{\theta}_1[k], \hat{\theta}_2[k])_{ML} = \arg \left\{ \max_{\theta_1[k], \theta_2[k]} p_{\theta_1[k], \theta_2[k]}(\mathcal{Y}) \right\},$$

Where $p_{\theta_1[k], \theta_2[k]}(\mathcal{Y})$ is the joint probability density function of $(\theta_1[k], \theta_2[k])$ evaluated at \mathcal{Y} . This is illustrated as a black box in Figure 2. Since $A_1[\cdot], A_2[\cdot], m_1[\cdot]$, and $m_2[\cdot]$ are in general each unknown, computing the maximum likelihood estimate of $(\theta_1[k], \theta_2[k])$ involves a joint maximization over at least four variables. In practice, the joint density function is not known exactly, and in any case such a maximization would be too complex to implement.

To make the problem tractable, we start by modeling the uncountably infinite possibilities for the trajectory of $\theta_i[k]$ by a countable number. This can be done by quantizing $\theta_i[k]$ to one of a finite number of phases; however, better performance can be had by instead quantizing the second derivative of the phase at time kT_s , denoted by $\theta_i''[k]$. This was the approach taken by Cahn for the case of tracking the phase of a single signal [1]. The true second derivative, determined from Equation (2), is given by $\theta_i''[k] = k_i m_i'[k]$, where $m_i'[k]$ is the derivative of the message waveform evaluated at time kT_s , and can be any real number. The estimator, however, assumes that $k_i m_i'[k]$ can take on only the values C or $-C$, where C is a constant. Of course, this is not the

case, but when integrated twice and for a suitable sample rate, little fidelity is lost in the approximation. Once $\theta_i''[k]$ is estimated as $\hat{\theta}_i''[k] = \pm C$, the estimator uses the truncated Taylor series to estimate the phase:

$$\begin{aligned} \hat{\theta}_i'[k] &= \hat{\theta}_i'[k-1] + \hat{\theta}_i''[k-1]T_s \\ \hat{\theta}_i[k] &= \hat{\theta}_i[k-1] + \hat{\theta}_i'[k-1]T_s + \hat{\theta}_i''[k-1]T_s^2/2. \end{aligned}$$

Thus, the sequence of signs for $\hat{\theta}_i''[\cdot]$ determines the sequence $\theta_i[k]$, which together with amplitude estimates gives rise to an estimated remodulated signal of

$$\hat{r}[k] = \hat{A}_1[k]e^{j\hat{\theta}_1[k]} + \hat{A}_2[k]e^{j\hat{\theta}_2[k]}. \quad (3)$$

Since the noise is assumed to be AWGN, the maximum likelihood sequence estimation of $(\theta_1''[k], \theta_2''[k])$ is that which minimizes the Euclidean distance $\sum_{l=1}^k \|r[l] - \hat{r}[l]\|^2$. A joint Viterbi algorithm can be used to trace the optimum sign choice sequences for $\theta_1''[\cdot]$ and $\theta_2''[\cdot]$.

The amplitude estimates are determined with a gradient descent algorithm. The conventional technique would be to compute a single estimate $(\hat{A}_1[k], \hat{A}_2[k])$ of $(A_1[k], A_2[k])$ from tentative decisions in the trellis and then use this estimate in every state for the computation of Equation (3). We abandon this approach and instead keep a separate amplitude estimate at each state of the trellis. This is called per-survivor processing (PSP) [7], and offers improved performance because, unlike the single amplitude estimator approach, when a particular path through the trellis is chosen, the amplitude estimates used in that path are optimized for that path. In other words, there is no penalty when a tentative path does not turn out to be the path ultimately chosen.

There are a number of design issues for which space does not permit a full description. The choice of the constant C affects performance and must be carefully chosen. More than the two levels C and $-C$ may be used to increase performance; for example, an additional level of zero could be added. Also, the appropriate size of the trellis must be determined. The size of the trellis here is determined by the "memory" of $\theta_i''[k]$, just as the trellis size of a maximum likelihood sequence estimator in an ISI environment is determined by the memory of the channel. We have found that devoting three "bits" (sign assignments) of memory to each of $\theta_1''[k]$ and $\theta_2''[k]$ works well. This results in a $2^{2 \cdot 3} = 64$ state trellis. If it is known that digital modulation schemes are being used, this can be easily modeled in the trellis and may result in improvement over the analog quantization presented here. Finally, internal interpolation of the received signal to obtain a higher sampling rate can also improve performance, as can appropriate pre- and post-processing filters to reduce the effects of out-of-band noise.

4. PERFORMANCE

The performance results are stated in terms of the mean squared error between the true sampled message signal $m_i[k]$ and the estimate $\hat{m}_i[k]$, normalized by the signal power:

$$\text{Normalized MSE} = \frac{\sum_{l=1}^M (\hat{m}_i[l] - m_i[l])^2}{\sum_{l=1}^M m_i[l]^2}, \quad (4)$$

This metric is somewhat problematic because it fails to capture some important information. For example, an algorithm which contains a rare spike but otherwise perfectly tracks the phase may have a higher relative error than a generally noisy phase tracker. A better test may be a qualitative assessment of the output audio. As a rule of thumb, we view an algorithm as “working” if the normalized MSE is less than about 1.0. This may seem to be a very liberal rule in view of the fact that the all zero output achieves this MSE. However, we have found that in nearly all cases, when the algorithms achieve a relative error of about 1.0, or even a little higher, they lock on to significant portions of the intended signal and contain adequate voice quality.

The normalized MSE performance for a cochannel signal is shown in Table 1 for varying SIR, SNR, frequency deviations and Doppler offsets. It compares the joint Viterbi algorithm with PSP estimates, a PLL, and a differential phase detector (DPD). In all cases reported, the sampling rate is $T_s = 1/65536$ and the frequency deviation of the first signal is $k_1 = 24000\pi$ (i.e., 12 kHz). The two signals consist of simulated voice waveforms, each with a 3.7kHz bandwidth and 3 seconds in length. This allowed for the processing of about 200,000 samples in each case. The signal to interference ratio, defined by $\text{SIR} = 20 \log_{10}(A_1/A_2)$, was varied between 2dB and 6dB. The SNR, defined as the ratio of the power of the *first* signal to the noise power, was varied between 10 and ∞ . The subdominant frequency deviation was varied between 12kHz and 24kHz. A Doppler offset between the signals was varied between 0 and 1 kHz.

This is among the first work which demonstrates that a subdominant FM signal may be captured when a cochannel FM signal at twice the power is interfering. Thus, by switching one’s perspective of the “first” and “second” signals, the SIR’s of 2dB and 6dB may actually be considered as SIR’s of -2dB and -6dB, and still with excellent results for a variety of transmission parameters. Conventional techniques do not attempt to estimate the subdominant signal and can only hope to *reject* the interference, at best. Thus, such techniques have no hope for operating in situations in which $\text{SIR} \leq 0$. Indeed, in Table I we can only report the MSE error with respect to their lone output: their estimate of the dominant signal.

An example of the estimated voice waveforms is shown in Figure 3. The waveforms were taken from the simulation given in the third row of Table I. As can be seen, both

Table I: Comparison of phase trackers for co-channel FM signals.

SIR dB	SNR dB	k_2 kHz	ω_2 kHz	Normalized MSE		
				Viterbi*	PLL†	DPD†
6	∞	12	0	0.12/0.68	0.07	0.41
6	∞	12	1	0.15/0.65	0.08	0.41
6	∞	24	0	0.13/0.51	0.09	0.79
6	∞	24	1	0.15/0.47	0.09	0.82
6	10	12	0	0.55/2.09	0.29	2.46
6	10	12	1	0.37/1.68	0.30	2.51
6	10	24	0	0.31/1.37	0.50	2.89
6	10	24	1	0.41/1.05	0.47	2.90
2	∞	12	0	0.42/0.65	0.18	1.58
2	∞	12	1	0.46/0.65	0.19	1.65
2	∞	24	0	0.61/0.34	0.20	2.79
2	∞	24	1	0.56/0.40	0.20	2.85
2	10	12	0	0.86/1.01	–	5.44
2	10	12	1	0.98/1.03	–	5.77
2	10	24	0	1.08/0.54	0.74	6.88
2	10	24	1	1.21/0.54	–	6.84

* Performance for dominant and subdominant signals

† Performance for dominant signal only

the PLL, and the joint Viterbi algorithm effectively capture the dominant signal, and the DPD is somewhat affected by the rechannel interference. The subdominant signal is accurately tracked by the joint Viterbi algorithm.

5. CONCLUSIONS

The joint Viterbi phase tracker enables separation of co-channel signals for a variety of SNR, SIR, frequency deviations, and Doppler offsets. It works for FM, CW, and CPFSK signals. Its estimate of the dominant signal is usually slightly worse than a conventional PLL, but in cases in which the desired signal is not dominant it enables proper phase tracking, whereas the PLL can track only the dominant signal. When the constituent co-channel signals have equivalent power, the PLL also breaks down and the advantages of the joint Viterbi approach may become more apparent.

It is clear how this joint estimator may be extended to more than two interfering signals with the addition of more trellis states. Since the trellis size grows exponentially in the number of interfering signals, there is a practical limit to the model, however.

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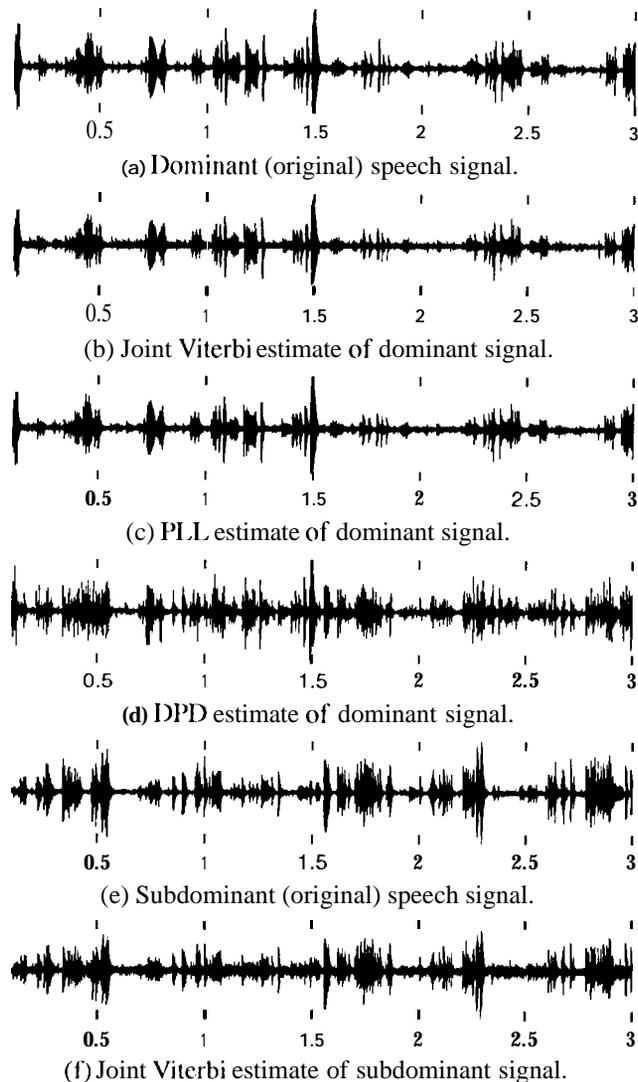


Figure 3: Estimated dominant and subdominant message waveforms .

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