Clustering of Emerging Flux

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ABSTRACT

Observations show that newly emerging flux tends to appear on the solar surface at sites where there is flux already. This results in clustering of solar activity. Standard dynamo theories do not predict this effect. Moreover, the mean field predicted by dynamo theories is too weak to emerge al the surface of the Sun. '1'0 solve the problem we suggest to take into account fluctuating fields generated by the dynamo. The magnetic fields emerge at the solar surface when the total field (the mean field plus a fluctuation)) exceeds the threshold for buoyancy. A slowly changing enhancement in mean field, predicted by instability theories, provides a persistent basis ("hump") for emergence of the fluctuating field. The enhancements are slowly destroyed by the solar differential rotation, the time scale of which defines the life-tillle of clusters of activity. A simple 2-dimensional model explaining the appearance of persistent clusters of emerging flux is presented.

Subject headings: Sun: active regions · Sun: emerging magnetic fields

1. Introduction

Magnetic fields emerging on the solar surface initiate the solar activity. The distribution of the fields carries clues to the mechanism of magnetic field generation. One striking feature of this distribution is clustering of active regions into large-scale patterns called "complexes of activity" (Bumba and Howard 1965, Gaizauskas et al. 1983, Harvey & Zwaan 1993). The complex of activity is defined as a cluster or sequence of active regions (bipolar magnetic structures) which are related by proximity and by continuity of their emergence. Complexes of activity are typically formed within a solar rotation and live typically 3 to 6 solar rotations during which they are refreshed by repeated injections of magnetic flux. These flux injections appear and disappear in rapid, pulselike succession (Zirin 1974, Gaizauskas et al. 1983). It is the tendency of magnetic flux to emerge near or within regions of existent flux that results in clustering.

The physical mechanism of the clustering remains unknown. The classical concept of emerging flux (Babcock 1961, Parker 1979) is based on the idea of a magnetic loop rising from the convective zone to the solar surface due to buoyancy. The magnetic loops are assumed to be caused by instabilities of the largescale, mean magnetic field generated by the joint action of the differential rotation and mean helicity of the convection (mean-field dynamo). The mean-field dynamo explains the basics of the sunspot cycle: the Maunder butterfly diagram and Hale's law of the field polarities, and predicts a strong toroidal field in the convection zone (c.f. Moffatt 1978, Parker 1979, Krause & Rädler 1981, Zeldovich et al. 1983). Instabilities that could lead to the formation of emerging magnetic loops are studied and incorporated into self-consistent models of storage, instability and dynamical eruption of magnetic flux tubes in the convection zone (see Schüssler et al. 1994, Brummell, Cattaneo & Toomre 1965, Caligari et al. 1995 and references in these papers). According to the instability models, when the field strength of a toroidal flux tube stool at the

core/convective zone overshoot layer exceeds a critical value, about 10^5 G, the flux tube becomes unstable and then rapidly erupts to the surface of the Sun. The models are in a good agreement with the basic observational facts related to sunspot magnetic fields in that the flux tubes emerge at low heliolatitudes and have the correct inclination and asymmetry with respect to the east-west direction. One problem open to resolve is the magnitude of the generated magnetic field. The mean-field dynamo theories do not predict such a strong mean magnetic field (Moffatt 1978, Parker 1979, Krause & Rädler 1981). The predicted field does not exceed the equipartition field which is only about 10^4 G. Modeling the eruption of these weak fields does not go along with observations: they erupt in latitudes which are too high, they show incorrect tilt angle, and their instability time is so long that they cannot survive a destruction by motions in the convection zone (Schüssler et al. 1994). Another problem is to explain why the new flux tends to emerge near the old one.

This paper suggests that solutions to both problems, of insufficient field strength to emerge and of the clustering, can be found by taking into account the random, fluctuating magnetic fields in addition to the regular, mean field. Although it is not easy to characterize the fluctuating fields, their existence is supported by observational and theoretical studies. Observations evidence, for example, that no single large flux tube emerges when sunspots are formed. Instead, the sunspot magnetic field is assembled over a 1)('lied of hours and (Is)'s through the progressive gathering of many flux tubes (Zwaan 1978). In accord with these observations, it has been pointed out that the sunspot field appears as a dynamical clustering of" many separate flux tubes (Parker 1979). Moreover, the most of magnetic flux emerging on the solar surface is apparently associated with fluctuating field. The magnetic flux emerging in active regions represent the largescale end of a continuous emerging flux spectrum (Rabin et al., 1991, Stenflo 1992). Theoretically, the generation of' fluctuating fields in the solar convection zone and their role in the generation of the mean field has been widely discussed: for early studies see for example Krause & Rädler (1981), Zeldovich et al.

(19$3), for numerical simulations see for example Meneguzzi et al. (1981), Brandenburg et al. (1990). Recently Ossendrijver et al. (1996) developed a solar dynamo model with a stochastic kinetic helicity. Fluctuations of the kinetic helicity excite overtones of the basic mode of the mean magnetic field. Schmitt et al. (1996) used magnetic fluctuations as a stochastic forcing control of the dynamo leading to grand minima in solar activity. Fluctuating magnetic fields are produced by random convective motions. The generation of fluctuating magnetic field is less demanding then that of the mean field: almost any three-dimensional random motions are efficient (Molchanov et al. 1984).

A model which involves fluctuating magnetic fields in sunspot formation has recently been developed (Ruzmaikin 1997). Central to the model is a concept of "threshold crossing" recently developed in the study of dynamical systems (see for example Wiesenfeld & Moss 1995). Our paper was restricted to the simplest one-dimensional model of the noise-periodic signal interaction, where the periodic signal simulated the mean solar magnetic field oscillating with 11-year period and the noise simulated fluctuating magnetic fields. The basic objective was to resolve the problem of the weak magnitude of the mean magnetic field. Here we expand the model to two spatial dimensions and suggest an explanation for the persistency of flux emergence, i.e. for magnetic flux clustering. The persistence of solar active regions was previously modeled as a percolation phenomenon (Wentzel &Seiden 1996, Seiden & Wentzel 1996). This approach will be compared with ours in the last section of this paper.

2. Flux emergence as a threshold-crossing phenomenon

In accordance with the current knowledge, we assume that the largescale solar magnetic field is generated near the bottom of the convection zone by joint action of the differential rotation and helical convective motions. This location of the solar dynamo was recently

confirmed by the analysis of the SOHO MDI helioseismic observations (Kosovichev et al 1997). The generated field is mainly azimuthal (toroidal) (Parker 1979, Krause & Rädler 1981, Zeldovich et al. 1983). This field, if sufficiently strong ($B \geq B_{th} \approx 10^5$ Gauss, where subscript "th" stays for "threshold"), is highly unstable: stitches of the toroid rise through the convection zone and eventually erupt at the solar surface to form active regions (Schüssler et al. 1994, Brummell, Cattaneo & Toomre 1965, Caligari et al. 1995 and references in these papers). However the strength of the field generated by the mean-field dynamo is too weak to cross the threshold and emerge.

To resolve this problem, we assume that crossings of the threshold, required for the fields to emerge to the solar surface, are provided by fluctuating magnetic fields (noise) superimposed on the mean field. Note that in the stability analysis by Caligari et al. (1995) this threshold was obtained for the fields with the longitudinal wavenumbers $m = 1, 2$. For the perturbations localized at higher wavenumbers the instability threshold is higher and the demands on the amplitude of fluctuations would be much stronger. However intermittent magnetic fluctuations are not localized in the Fourier space. Typically they have a self-similar power-law spectrum with the power growing towards the largest scale. Thus one can expect that the fluctuations with larger scales (smaller azimuthal m) play the basic role in order to satisfy the requirements of the instability analysis. On the other hand, the preferred emergence of the low-wavelength modes advocated by the instability models seems to be inconsistent with the localized nature of the emerging flux (Zwaan 1996). The problem, perhaps, can be resolved by modeling the emergence process near the solar surface, where such effects as down flows initiated by the emerging fields have to be taken into account. Another way to resolve the problem is to consider small-scale fluctuating fields in the upper part of the convection zone as "piggybacking" on "humps" of the submerged mean field (Ruzmaikin 1990, Zwaan 1996). In this case the approach of the present paper will be even more sound. However, for the sake of consistency we will base

our consideration below on the instability model by Caligari et al. 1995.

An important feature of fluctuations is the absence of an upper limit on their magnitude. Although the variance of fluctuations is limited, say by the equipartion between the kinetic and magnetic energies, the occurrence of large random deviations is restricted only by the form of distribution function of the fluctuating field. Thus, from time to time the fluctuations are strong enough to permit the magnetic flux emergence. Without the mean field, however, the distribution of the emerged fields on the solar surface would be pure random. The mean field, being under-threshold, plays a crucial role in organization of observed solar cycle regularities (Ruzmaikin 1997) and causes the persistency in flux emergence, i.e. the clustering.

3. A cause of persistent, flux emergence

The persistency of the flux emergence can be understood as follows. The fluctuating magnetic fields emerge continually whenever and wherever their strength randomly exceeds the threshold. There is always a probability of pattern formation in random distributions, however the observed, at least large-scale, patterns are interpreted as non-random (Harvey & Zwaan 1993). The regular mean-field itself could form these patterns but the magnitude of this field, as considered, is too small to overcome the buoyancy threshold and emerge. What works is a combination of these two fields. The mean field forms an enhancement ("a hump") and whenever a coherent (having the same direction) fluctuating field of a sufficient amplitude appear in the vicinity of the hump the resulting field effectively emerges to the surface of the Sun (Figure 1).

But where do the enhancements come from ? Their origin can be justified as follows. The strongest component produced by the mean-field dynamo is an axisymmetric toroidal

field. This field propagates from high la itudes to the equator as a wave with 11-year period. Thus at a given moment of time he region around its maximum can be considered as a toroidal belt. It is the stability of this belt that has been studied in the instability models (Schüssler et al. 1994, Brummell, Cattaneo & Toomre 1965, Caligari et al. 1995). The unstable modes are found to be non- axisymmetric ($m \neq 0$). This explains why the field emerges in the form of a loop not as a whole belt. The growth times for instabilities are found to be strongly dependent on the magnitude of the field B at the bottom of the convection zone. For $B \geq 10^5$ Gauss the growth time is short enough so that a loop can form and rise up through the convection zone before it could be destroyed by the solar differential rotation. For weaker B, however, the field can "survive" a short phase of instability without erupting because the growth of the perturbation can not keep up with the continuous stretching by differential rotation (Schüssler et al. 1994). The stretching time scale has been identified by these authors as confined between 100 and 500 days. We identify these regions of slow instability with the localized enhancements of the mean magnetic field.

In other words, we identify the life-time of observed clusters of activity with the stretching time scale due to the differential rotation at the bottom of convection zone. Let us estimate the stretching time. Consider a loop-type enhancement of the basic toroidal field produced by dynamo. According to the induction equation the radial component of this loop will be stretched into an azimuthal field as

$$\frac{\partial B_\phi}{\partial t} = r\frac{d\Omega}{dr}B_r$$

We neglect the resisitivity term inessential at the time scale under consideration. Hence the stretching time can be define as inverse proportional to $rd\Omega/dr \approx (r/\Delta r)\Delta\Omega$ where $\Delta\Omega$ the change in the angular velocity over the distance Δr measured in the solar radius. The differential rotation $\Delta\Omega$ in the convection zone has been inferred from the helioseismic

data (for the latest results and references see Kosovichev et al 1997, figure 13). The radial gradient of the angular velocity shows a noticeable latitudinal dependence. It is strong enough at 60 deg so that $(r/\Delta r)\Delta\Omega$ is comparable with Ω and hence the enhancements would not survive more than one rotation. The gradient, however, decreases toward the lower latitudes and even changes the sign at shout 30 deg. Hence in the latitudinal region around 30 deg the enhancements of the mean field can survive for many solar rotations. This gives a clue to the origin of the long-living clusters of activity.

4. A model of flux emergence

Let us incorporate the above ideas into a simple model. Consider two areas of size $La \times Lo$, one represent a bottom of the convection zone and the other represent the solar surface. In the middle of the bottom plane there is a regular pattern of characteristic size $La/4 \times Lo/4$, see Figure 2 where the pattern has the "Gaussian" shape with $La = Lo$. The pattern represents an enhancement of the mean magnetic field $\mathbf{B}$ generated by the dynamo. To further simplify the situation we assume that the regular pattern is time-independent. This assumption is justified by the fact that the life-time of activity clusters (a few solar rotations) is much shorter than the solar cycle (a characteristic time of mean field variations).

At times $t = 1, 2, 3, ...$ each cell of the bottom is given a number produced by a random generator. The numbers are independent at different times and for different cells. This noise distribution represents the fluctuating magnetic fields $\mathbf{b}$. Both the amplitude of mean field and the variance of fluctuating fields are assumed to be subthreshold. When in a cell at some time the total field $c\mathbf{B} + \sqrt{\sigma}\mathbf{b}$ exceeds the threshold $B_{th} = 1$ a crossing mark appears in a corresponding cell on the upper plane. Each crossing is interpreted as a flux emergence event. The emergence in our simplified model is approximated as instant, i.e. we

neglect a time needed to a flux loop to travel from the bottom to the top. The parameters ϵ and σ are introduced to simulate different values of the mean field amplitude and the noise variance. For the sake of simplification, the sign of field is not included so that we model only a magnitude of the emerging flux and do not simulate emerging bipoles. It is clear that the number of crossings depends on the probability distribution of fluctuations, although there will be some crossings for any distribution function that has non-zero values above the threshold.

A simplest choice for the distribution of fluctuations is the normal distribution. We used this distribution in modeling. However there is 110 physical reason to make this choice. Observational studies of magnetograms show that the solar magnetic field is not distributed normally (Lawrence et al. 1993). Theoretical considerations show that the distribution function of dynamo generated magnetic fields is not expected to be normal (Molchanov et al. 1984, Finn & Ott 1988). The main reason is the following. The normal distribution characterizes the additive random variables, such as random experimental errors. This is a consequence of the law of large numbers. The magnetic field generated by random motions is however not an additive but a multiplicative variable (Molchanov et al. 19s-1). The distribution functions for multiplicative variables have higher tails, i.e. higher probabilities of strong fluctuations compared to those produces with the normal distribution. [To better understand this point take for example a multiplicative variable which is a product of normally distributed variables. Then the logarithm of this multiplicative variable will be an additive variable obeying the normal law. Hence the variable itself obeys to the log-normal law.] In the numerical example below we approximate the high intensity tail of the distribution by an exponential.

Figure 3 shows a snapshot of a numerical simulation 011 the square area $La = Lo = 15$. The threshold (B_{th}) is taken as a unit for the field amplitude. The magnitude of the

mean field for the case shown is 0.8 of the threshold. The fluctuating field is exponentially distributed with the variance σ set to 0.2 of the threshold. '1'0 simulate the magnetic flux disappearance (due to reconnections, diffusion and lift into the corona) we assume, quite arbitrary, that every flux distribution, shown on one of the panels in Figure 3, is a sum of what have emerged at the given moment plus the flux left from two previous steps. The number of these "II1C1I101'J'" stepscanbe, of course, changed to better simulate the solar conditions.

In spite of random character of the emerging patterns there a preferred place in the central region of pictures is clearly seen. The flux emerging in this region carries evidence of the hidden, subsurface distribution of the mean field.

Figure 4 shows the emergence rate per unit time and per unit area for different values of c (the magnitude of the hidden mean field) and σ (the amplitude of fluctuations).

5. Discussion

The simple model presented in this paper serves as a qualitative demonstration of how active regions could persistently result from random fluctuations superposed 011 a weaker mean field. The mean field plays a vital role in producing the regular features of the sunspots and the solar cycle itself. The fluctuating field is responsible for allowing the mean field to be observed. The persistence of the emerging flux arises then as the result of interaction between the fluctuating and mean fields: Enhancements of the mean field serve as "humps" increasing the rate of fluctuating field emergence. We identify the life-time of these enhancements, and hence the life-time of the clusters of activity, with the stretching time scale due to the differential rotation at the bottom of the convection zone.

Our approach to explanation of the persistence of emerging flux has to be compared

with the approach basal on percolation ideas (Wentzel & Seiden 1996, Seiden & Wentzel 1996). The percolation model starts computation by randomly designating 1 % of cells on a two-dimensional region as active. An active cell then can induce activity in its neighbor cell - like a tree hops a fire to another tree in the forest fire with a prescribed stimulated probability P_{st}. The flux disappears by diffusing from active cells into empty cells with another probability D. There is also a small spontaneous probability P_{sp} to support new activity. The assumption on hopping the activity to neighbor cells is crucial for the explanation of clustering of the emerging flux. The model is justified observationally by fitting the calculated size distribution of active regions to the histogram built from the data on active regions (Harvey & Zwaan 1993). A key (for the percolation model) interpretation of the observed distribution is a break at about 15 — 20 deg^2 that separate smaller scale power-law and large scale exponential behavior (Seiden & Wentzel 1996). The identification of the exponential distribution from the observational histogram, however, is not quite certain due to a small number of the large-scale clusters. (The author acknowledges tile discussion of this point with K. Harvey.)

In our model no assumption on influence of an active cell (a site for the emerging flux) on its neighbor is made. Active cells are considered as independent, i.e. a new flux does not know anything about the old one. The persistence of the emergence flux and hence its clustering arises due to subsurface enhancements, "humps", made of the mean field. A "hump" effectively lowers the threshold so that more random fields can emerge on top of it. As an extra benefit, our model suggests the explanation of the life-times of observed clusters of activity. The observed cluster size distribution (Harvey & Zwaan 1993), not discussed in the present paper, gives a few important insights that can help to further develop the model. According to our model, the flux in the smaller (than the enhancement size) scale region emerges independently so that its clustering is random and hence has a self-similar, power-law distribution (because there is no preferred scale). The break-point

scale can be identified with a characteristic scale of the mean field enhancements. The size distribution of these enhancements can, in principle, be found from the instability modeling and compared with the observed distribution.

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Figure Captions

Figure 1. A schematic illustration of cooperative emergence of the mean, B, and fluctuating, b, magnetic fields. Shown is a latitudinal cut of the Sun, say at 35 deg, viewed from the north pole. A loop of a random field is rising on the slowly changing enhancement of the mean field (shown by a thick line.

Figure 2. The form of mean magnetic field enhancement used in our calculations. The enhancement has a Gaussian shape with a width equal to a half-size of the plotted region.

Figure 3. A time series of the random flux emerging on the solar surface. A rectangular region of size 45 x45 cells is shown. At every time step (12 steps are shown) the realizations, whose amplitudes exceed the value "threshold-mean-field" are plotted (new emergence) as additions to the flux no more than two steps old. These realizations are obtained from the exponential distribution function with $\sigma = 0.2$. The preferred emergence around the mean field hump which has maximum 0.8 of the threshold value and is hidden in the center of the picture, is clearly seen.

Figure 4. The rate of flux emergence per unit time and per unit area in arbitrary units. The asterisks mark the rate of emergence at $\epsilon = 0$, i.e. in the absence of the mean field enhancement. The rest four curves correspond to different values of ϵ: 0.2 (solid), 0.4 (dashed)), 0.6 (dotted), and 0.8 (dashdot).

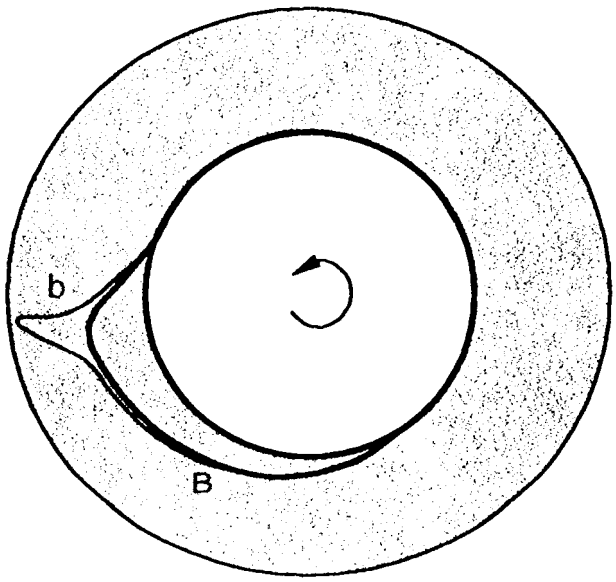


Fig 1

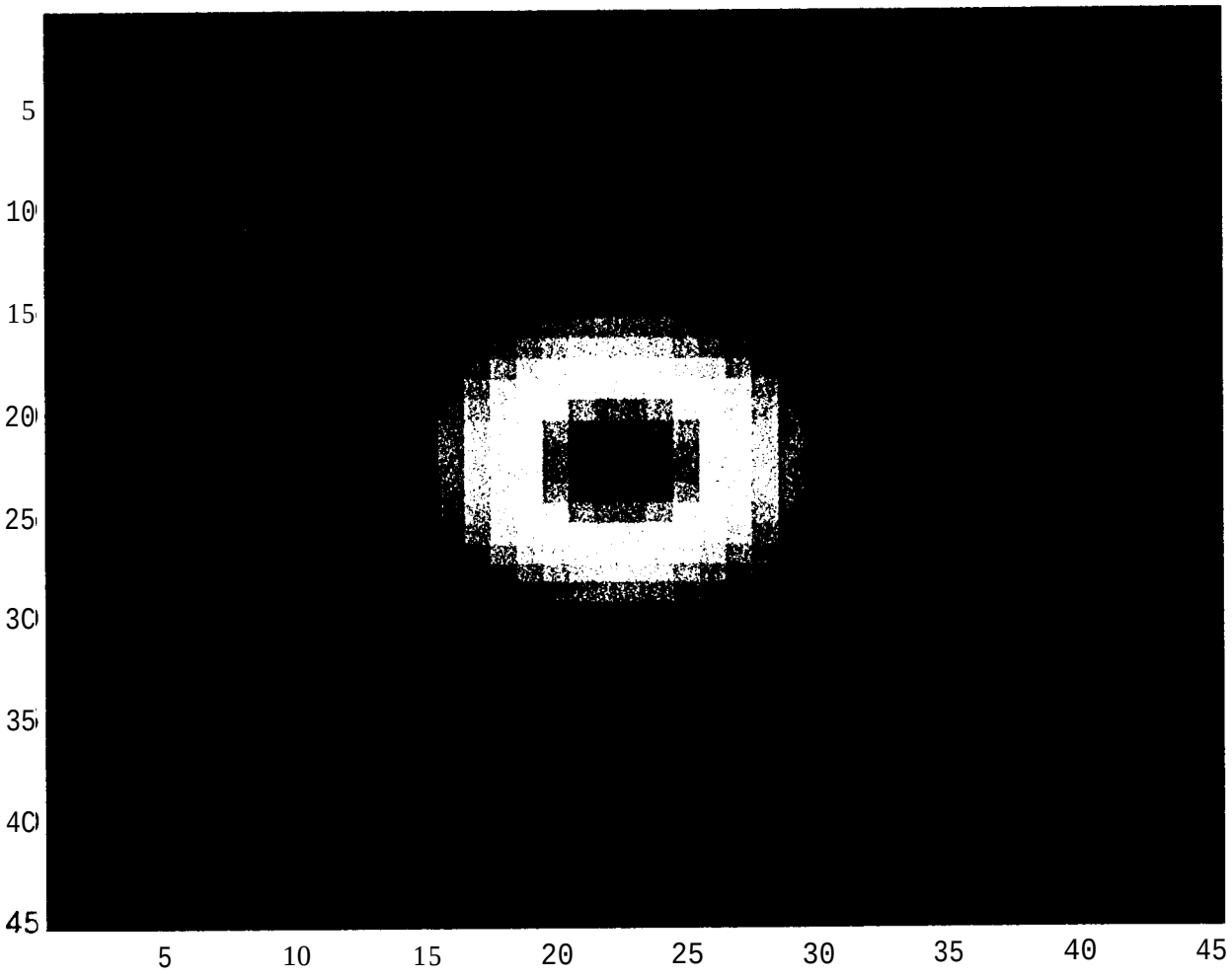


Fig. 2

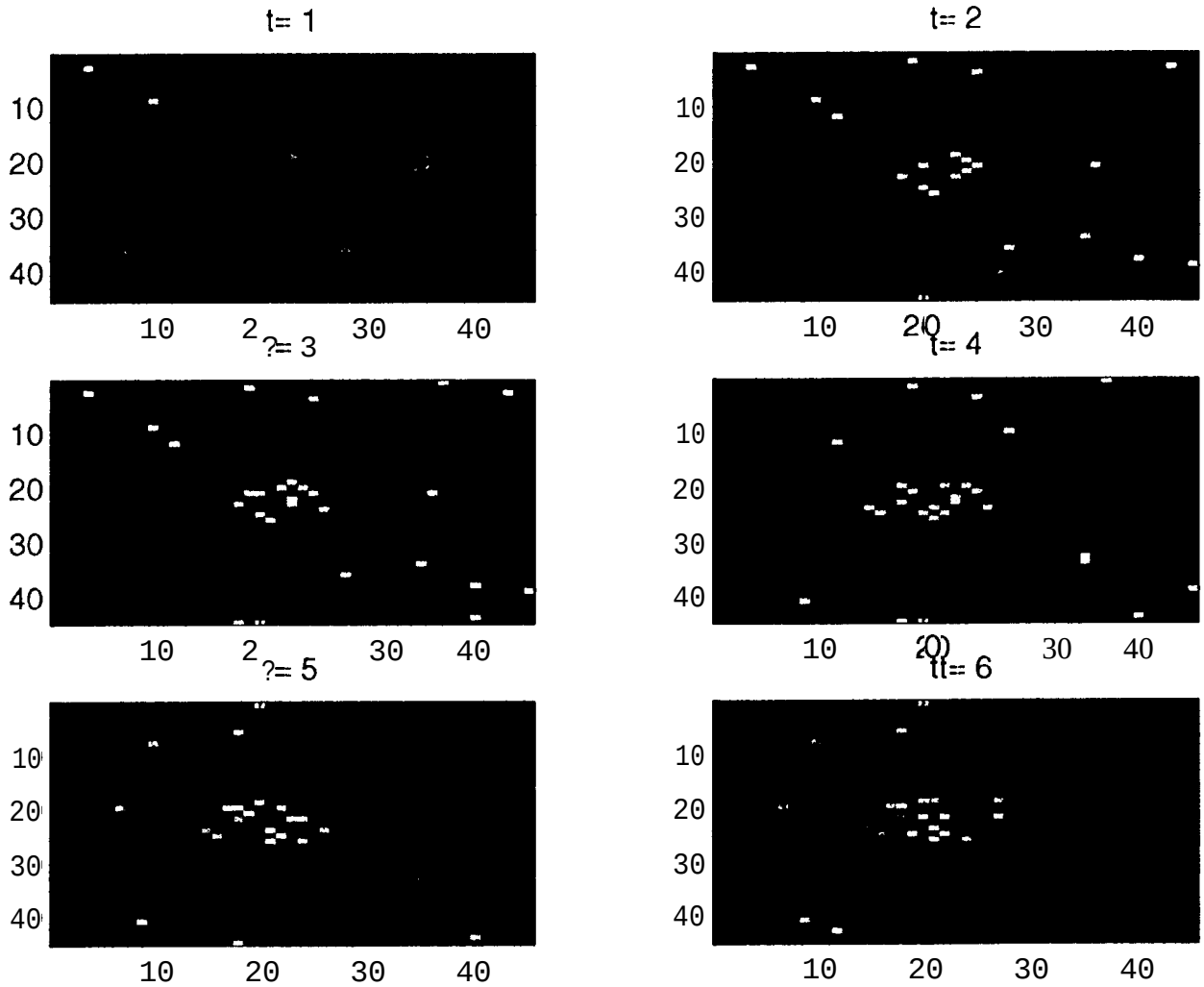
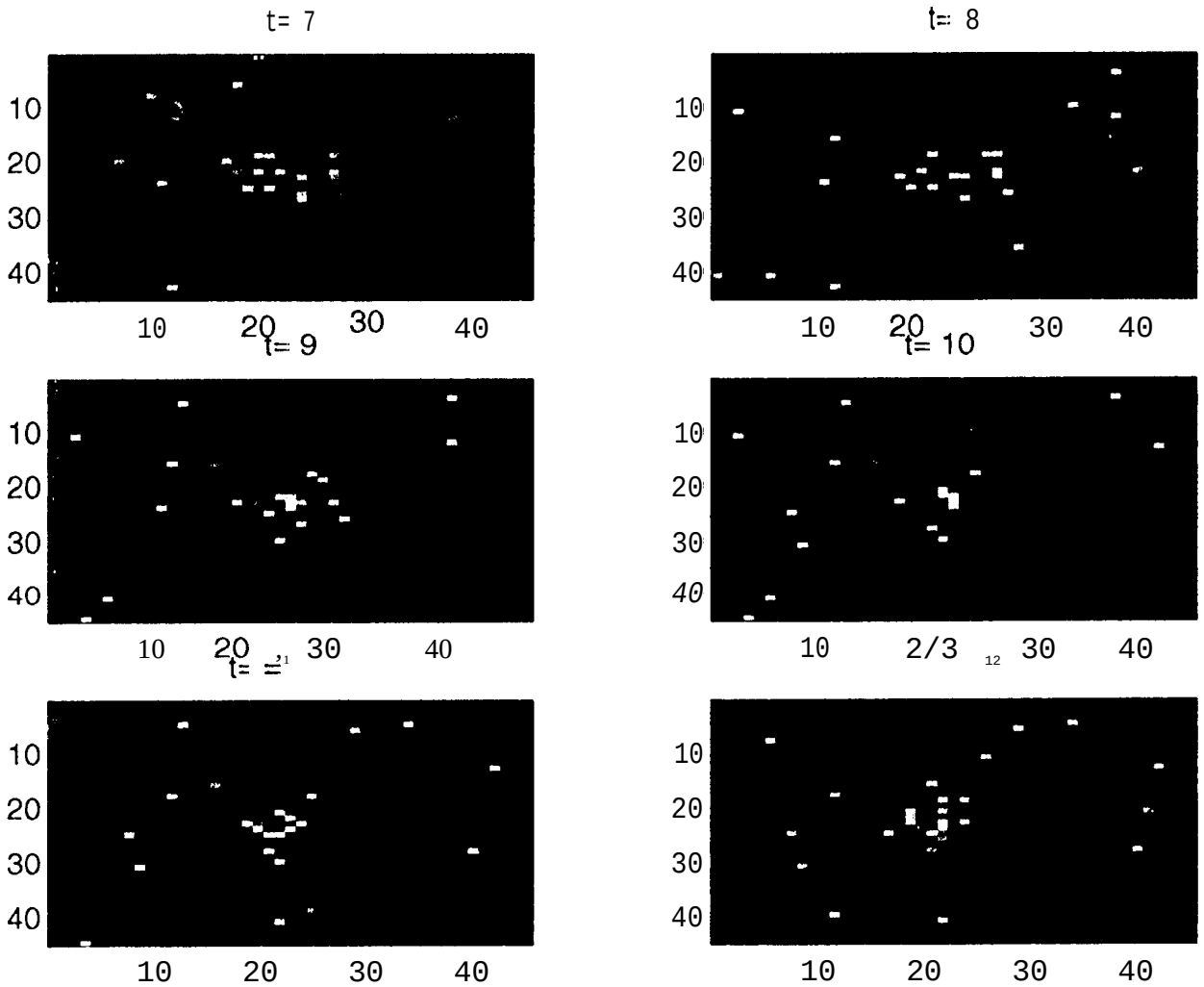


Fig 3 part 1



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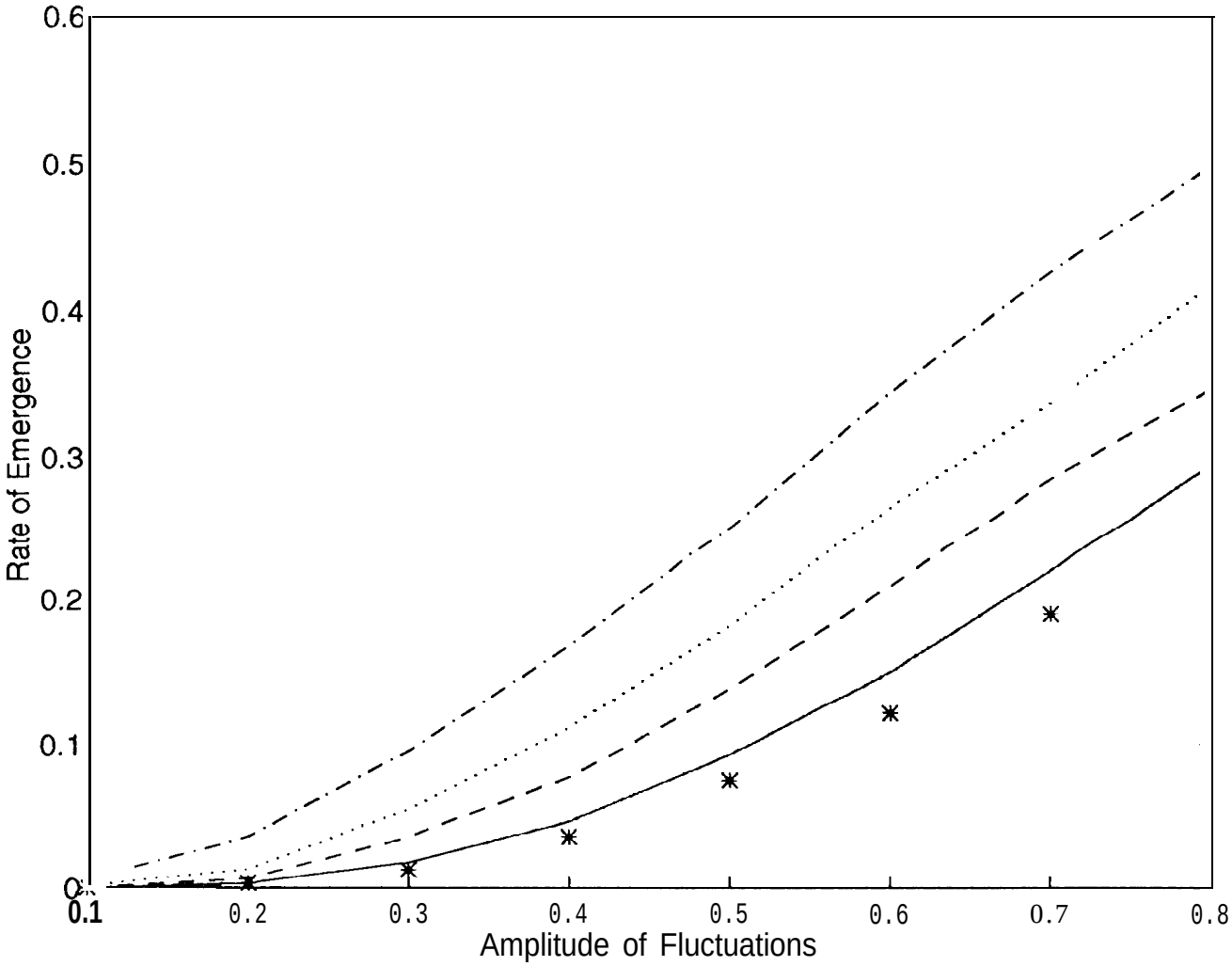
0.6
0.5
0.4
0.3
0.2
0.1
0
Rate of Emergence
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Amplitude of Fluctuations

Fig. 4