

ABSTRACT: Low Energy Interplanetary Transfers Using the Invariant Manifolds of L1, L2, and Halo Orbits

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Despite the apparent regularity of planetary motions, the dynamics of the solar system is in reality extremely nonlinear and replete with chaos. The difficulty in detecting this chaos is the long time scale of this dynamics as compared with human observations. This chaotic dynamics can be used to great advantage in mission design as exhibited by the many halo orbit missions starting with ISEE3 in 1978 (Ref. 1). More recently, new advances have been made in this area in the Hiten Mission (Ref. 2) and the Genesis Discovery Proposal (Ref. 3). The Hiten Mission sent the spacecraft near the Earth-Sun L2 point first to enable an eventual ballistic capture into lunar orbit. The Genesis Mission uses the same dynamics by sending the spacecraft from a halo orbit at L1 to an orbit near L2 to effect a return to earth on the day-side. In this paper, a new transfer between planetary bodies which further exploits the nonlinear dynamics of the three body system is described.

In all these examples, the role of the libration points is key. Understanding of the underlying nonlinear dynamics of these orbits is provided by the theory of invariant manifolds. The ideas of the mathematics behind this theory is very simple. We define some terms first. A manifold is a smooth and regular high dimensional surface sitting inside our phase space. An invariant manifold, M , has the property that given any point P in M , the orbit with initial state P will remain on M . We also refer to orbits as the flow on M . Thus, M is comprised of orbits in the phase space. Simple examples are: a fixed point, a periodic orbit, a torus on which a quasi-periodic orbit lives, an energy "surface" in a Hamiltonian system. Associated with M are other invariant manifolds known as the stable and unstable manifold. The stable manifold of M , M_s , is the set of orbits which approach M the fastest. The unstable manifold of M , M_u , is the set of orbits which depart M the fastest. The fact that these sets form manifolds is given by the Stable Manifold Theorem. These invariant manifolds provide the necessary structures which enable us to understand and compute the nonlinear dynamics.

For the planetary transfer problem, we model the solar system as a series of Circular Restricted Three Body Problems (CRTBP). For a planetary system, the Sun is the primary body, the planet is the secondary body; for a lunar system, the planet is the primary body, the moon is the secondary body. Within the three body problem are the three collinear libration points, L1, L2, and L3. We focus our attention on L1 and L2 which are closest to the secondary body. These unstable fixed points have a hyperbolic structure: they each have a one-dimensional stable and unstable manifold. This is seen by linearizing the equations of motion at the fixed point and computing the eigenvalues of the linear equations. There is one eigenvalue, e_s , with negative real part (stable); one eigenvalue, e_u , with positive real part

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(unstable); and four with imaginary parts (center). The corresponding eigenvectors, E_s and E_u are linear approximations of the stable and unstable manifolds at the fixed point. By integration, we can globalize the manifolds to obtain M_s and M_u .

We notice that eventually, both M_s and M_u will escape the confines of the L_1 , L_2 region and become orbits about the primary. For example, the heliocentric orbits selected by the **SIRTF** Mission is such an orbit. While it is not on the invariant manifolds of earth, it is very close to the manifolds and is strongly influenced by them. For the inner planets, these manifolds are closely wound around the planet's orbit around the sun, They are separated and distinct for the time interval of our integration (10,000 years). The outer planets present an entirely different story. One finds that the invariant manifolds of the outer planets are extended and tangled. For example, the L_2 unstable manifold of Jupiter is linked with the L_1 stable manifold of Saturn. This means a transfer from Jupiter to Saturn is possible at the point of intersection of the manifolds. However, this takes some 40 years to achieve which makes it unattractive for mission design purposes. If one considers the satellites of Jupiter where the periods are very small, such a transfer becomes once again practical. Indeed, between Ganymede and Europa, such a transfer exists which we examine closer in this paper.

We note in closing that the manifolds of the Moon extend close to 1.5 million km from the Earth which is distance to the L_1 and L_2 libration points of the Earth-Sun system. It is this tangle of the invariant manifolds of the two systems which enables the capture orbit of the Hiten mission.

References

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