A Methodology for Finite Element Model Updating Using Modal Data

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Abstract

A new methodology for updating the finite element model of a structure using an incomplete set of modal test frequencies and modeshapes is presented. The proposed model updating methodology involves a least squares minimization of the modal dynamic force balance residuals subject to quadratic inequality constraints introduced to properly account for the expected measurement and modeling errors. in particular, applications to structural damage detect ion and structural health monitoring are addressed. It is demonstrated that the predicted accuracy of a model updating methodology can be significantly improved by joi ıtly analyzing multiple sets of test data obtained from repeated laboratory or field experiments. Simulated modal data obtained on a truss structure are used to assess t he st rengths, limitations, and overall performance of the proposed model updat ing methodology in relation to the number and location of sensors, the location and magnitude of errors in the finite element model of the structure, as well as tile number of measured modes.

1. Introduction

The need for model updating arises in the p ro cess of constructing a theoretical model of a structure. In most (“iIS['S, in order to improve the accuracy in the model response predictions, the pre-test finite ele ment 1110(1(’1 of the structure [1(’(’(1 to be updat ed to match available test data. Anoth er important application of a model updat ing met hodology is in t he prediction of structural damage (see, for example, Natke and Yao, 1988; Stubbs, Broome and O segueda 1990; Fahrat and Hemez, 1993; Papadimitriou *et* (11, 1997; Vanik and I](’(”k, 1997; Katafygiotis and Lam, 1997). The loc ation and size of damage can be inferred by monitoring the reduction ni stiffness and ma ıss properties o f the elemer its comprising the finite element model of tile structure.

The general problem of model updati ng involves t he selection of the best model from a parameterized class of mo dels that best fits the [’(’S{)onse or modal test da ıta as judged by an appropriately selected measure of fit. The difficulties associated with this inverse problem are mainly (111(’ to the measurement er ror in the test data, the modeling error, and the incompleteness of the ava ilable data relati ve to the model complexity need ed to produce physically meaninful n ode ls. As a result, the inverse prol)lem leads to non-unique solutions and ill-conditioning (Berman, 1989; Beck, 1989; Mottershead and Friswell, 1993; Beck and Katafygiotis, 1997).

In past years, several studies have been devoted to reconciling finite element models with measured modal data. A literature review of existing finite element model updating and damage detection methods can be found in the survey by Mottershead and Friswell (1993). Each method has its own advantages and shortcomings and there is no acceptable methodology for successfully treating the model updating and damage detection problem. The preferable methods of updating are usually the ones which preserve structural connectivity. Most methods address the problem by choosing some mathematical criteria which hopefully creates a unique optimal model while neglecting other models that can give an equally good fit to the measured data. The present study is based on this class of methods. However, new methods (Beck, 1989; Beck and Katafygiotis, 1997) based on Bayesian statistical inference have been developed recently for properly addressing the non-uniqueness by computing and considering in the predictions all (finite or infinite) models that give acceptable fit to the data (Katafygiotis and Beck, 1997; Vanik and Beck, 1997; Beck and Vanik, 1996). The latter methods are powerful and have shown great promise in properly incorporating modeling and measurement errors, as well as properly addressing many of the difficulties encountered in the model updating problem, especially those associated with model and response prediction accuracy.

This study presents a methodology for updating the finite element model of a structure using an incomplete set of experimentally obtained modal frequencies and modeshapes. It combines the modeshape expansion technique proposed by Levine-West, Milman and Kissil (1996, 1997) with updating capabilities for predicting both the location and size of errors in the pre-test finite element model of a structure. Applications of the proposed methodology to structural damage detection and structural health monitoring are addressed. Specifically, a least squares minimization of the modal dynamic force balance residuals subject to quadratic inequality constrains involving the difference between expanded and model-based modeshapes is considered. The proposed measure of fit and the constraint equations account for the expected measurement error in both modal frequencies and mode shape components, as well as the expected modeling errors. The unknown quantities involved in the proposed error measure include the location and size of errors in the properties of the finite element model of the structure, related to stiffness, mass and geometrical properties, as well as the modeshape values of the contributing modes at all degrees of freedom. Including the complete modeshape as unknowns in the model updating methodology has the advantage of avoiding the problem of identifying the correspondence between model and measured modes. Another advantage of the expanded modeshapes is in their use for predicting potential damage locations or locations of errors in the properties of the finite element model. Other model updating methodologies based on modeshape expansion techniques include the work by Farhat and Hemez (1993), Alvin (1997), and Vanik and Beck (1997).

2. Structural Model Updating Methodology

A structural model updating methodology involves choosing the best model from a specified class of parameterized structural models based on evaluations using available test data. For the purposes of the present study, the following class of linear structural models is used:

$$M(\theta)\ddot{x} + K(\theta)x = f(t) \tag{1}$$

where the global mass and stiffness matrices $M(\theta)$ and $K(\theta)$ are assembled, using a finite element analysis, from the element (or substructure) mass and stiffness matrices, respectively. The set θ includes the uncertain parameters of the model to be assigned values during the search for the optimal model. The parameter set θ may represent mass and stiffness properties at the element or substructure level. Examples of finite element properties that can be included in the parameter set θ are: modulus of elasticity, cross-sectional area, thickness, moment of inertia and mass density of the finite elements comprising the model, as well as spring (translational or rotational) stiffnesses used to model fixity conditions at joints or boundaries. For convenience, the parameterization is chosen such that the pre-test finite element model of the structure corresponds to $\theta = 1$.

In particular, the objective in a modal-based model updating methodology is to find the values of the parameter set θ so that the modal data generated by the model of the class of models best matches, in some sense, the experimentally obtained modal data. Let m be the number of measured modes and let $\hat{\omega}_i$ and $\hat{\phi}_{ai}$ be the experimentally obtained i-th modal frequency and modeshape of the structure at the measured 1110(1 ('1 degrees of freedom a. A measure of fit that is explored herein is directly [('lilt('(l to the modal dynamic force balance residuals defined by

$$r(\omega, \phi, \theta) = [K(\theta) - \omega^2 M(\theta)]\phi \tag{2}$$

Note that the modal dynamic force balance residuals satisfy tile equations $r(\omega_i(\theta), \varphi_i(\theta), \theta) = 0$, $i = 1, \cdots, m$, where $\omega_i(\theta)$ and $\varphi_i(\theta)$, $i = 1, \ldots, m$ are respectively the modal frequencies and mass-normalized mode shapes of the first m modes of the model (1).

The proposed method for model updating searches for the optimal model parameters θ which minimize an appropriately selected norm of the modal dynamic force balance residuals $r(\omega_i, \phi_i, \theta)$ subject to conditions that reflect the fact that both the modal frequencies ω_i and modeshapes ϕ_i are sufficiently close, depending on the experimental error expected, to the measured modal frequencies and modeshape components. Mathematically, the model updating problem is stated as a constrained minimization problem:

$$\min_{\theta,\phi_i} \sum_{i=1}^{m} \|r(\tilde{\omega}_i, \phi_i, \theta)\|_{R_i} = \min_{\theta,\phi_i} \sum_{i=1}^{m} \left\|(K(\theta) - \tilde{\omega}_i^2 M(\theta))\phi_i\right\|_{R_i} \tag{3}$$

subject to

$$\left\|P\phi_i - \tilde{\phi}_{ai}\right\|^2 \leq \alpha_i^2 \left\|\tilde{\phi}_{ai}\right\|^2, \quad i = 1, \cdots, m \tag{-1}$$

where $\|x\|^2 = x^T x$ is the usual Eucledian norm, $\|x\|_R = x^T Rx$, R_i is an appropriately selected weighting matrix which scales the contribution of each mode in the measure of fit **(3)**, and P is a constant matrix of zeroes and ones such that $\phi_{ia} = P\phi_i$.

The inequality constraints (4) are introduced to account for the expected measurement error in the mode-shape components, with α_i controlling the expected magnitude of these errors. The value of α_i can be computed from a statistical analysis of measurement data taken from repeated modal test analyses. It is worth pointing out that the methodologies presented by Farhat and Hemez (1993), Hemez and Farhat (1995) and Alvin (1997) are special cases of the measure (3) and condition (4). In particular, both methods correspond to values $\alpha_i = 0$, which fail to directly incorporate the expected measurement error. In contrast, the proposed inequality condition provides more flexibility in improving the fit between model and measured modal data over the space of the parameter set θ.

The weights R_i are selected to make the i-th modal term, designated by $J_i(\phi_i, \theta)$, in the overall measure of fit (3) non-dimensional and proportional to ii scalar weight β_i. Herein, attention is only given to the following two choices (Papadimitriou *et al* (1997):

$$R_i = \beta_i M^{-1}(\theta)/\tilde{\omega}_i^4 \quad \Longrightarrow \quad J_i(\phi_i, \theta) = \sum_{j=1}^{N} \beta_i (\varphi_j^T M(\theta)\phi_i)^2 \frac{(\omega_j^2 - \tilde{\omega}_i^2)^2}{\tilde{\omega}_i^4} \tag{5}$$

$$R_i = \beta_i K^{-1}(\theta) M(\theta) K^{-1}(\theta) \quad \Longrightarrow \quad J_i(\phi_i, \theta) = \sum_{j=1}^{N} \beta_i (\varphi_j^T M(\theta)\phi_i)^2 \frac{(\omega_j^2 - \tilde{\omega}_i^2)^2}{\omega_j^4} \tag{6}$$

Note that for the case of perfectly correlated expanded and model mode-shapes, i.e. the case $\phi_i = \varphi_i$, $i = 1, \cdots, m$, all but tile term corresponding to $j = i$ in the modal measure $J_i(\phi_i, \theta)$ in (5) and (6) disappears. The modal measure $J_i(\varphi_i, \theta)$ becomes proportional to the fractional difference between the squares of the model and measured modal frequencies for mode i, weighted by the scalar β_i. This equivalence between the measure $J_i(\varphi_i, \theta)$ and the more direct measure involving the difference between the squares of the model and measured modal frequencies was first reported in a recent study (Vanik and Beck, 1997). In the general case for which $\phi_i \neq \varphi_i$, all terms in the modal error measure (5) and (6) are present. In particular, the terms in the modal error measure (5) and (6) corresponding to $j \neq i$ involve the mass orthogonality condition between the expanded and model modeshapes, weighted by the factors $\beta_i(\omega_j^2 - \tilde{\omega}_i^2)^2/\tilde{\omega}_i^4$ and $\beta_i(\omega_j^2 - \tilde{\omega}_i^2)^2/\omega_j^4$, respectively. Note that for a model which is well-correlated with the measured data, the factors $(\varphi_i^T M(\theta)\phi_i)^2 \approx 1$ and $(\varphi_j^T M(\theta)\phi_i)^2 \approx 0$ for $j \neq i$. Therefore, in the process of selecting the optimal model, the mass orthogonality conditions are also enforced through the terms in $J_i(\varphi_i, \theta)$ corresponding to $j \neq i$.

The term in (5) and (6) corresponding to $j = i$ provides insight into the problem of specifying the weights β_i. Specifically, from a Bayesian statistical point of view the weights β_i reflect the magnitude of the measurement errors expected between the experimental and model frequencies for each mode (Beck, 1989; Vanik and Beck, 1997). The size of these errors can be obtained from measurement data taken from repeated modal test analyses.

It should be noted that the weighting matrix R_i given in (5) is applicable only if $M^{-1}(\theta)$ is non-singular. Thus, it is not applicable for structures with zero mass degrees of freedom. However, this problem can easily be resolved by applying Guyan model reduction to eliminate the degrees of freedom corresponding to zero mass. Similarly, the weighting matrix R_i given in (6) is applicable only if the matrix $K(\theta)$ is non-singular. Thus, it cannot be applied to structures that are not supported or they are partially supported such as those employed in space or tested in the lab by suspending them by very soft springs.

The unknown quantities involved in the proposed error measure of fit (3) include, in addition to the model parameters θ, the components of the vector ϕ_i of the contributing modes at both measured and unmeasured model degrees of freedom. The optimal vector $\hat{\phi}_i, i = 1, \ldots, m$ resulting from the minimization can be viewed as the expanded modeshapes consistent with the measured modal data. One advantage of using an expanded modeshape approach is that there is no need to know the correspondence between the measured and model modes. The optimization in (3) and (4) can be performed using available inequality constaint optimization techniques. However, this is a complex and time-consuming nonlinear optimization problem. A more convenient two-step iterative procedure is proposed next which avoids some of the computational difficulties arising in the minimization of (3).

2.1. Step 1: Mode-Shape Expansion

Given the current model of the structure at the k-th iteration step, corresponding to the optimal value of the parameter set θ designated by $\hat{\theta}^{(k)}$, expanded modeshapes are computed by solving the constrained minimization problem:

$$\min_{\phi_i} \sum_{i=1}^{m} \left\| (K(\hat{\theta}^{(k)}) - \hat{\omega}_i^2 M(\hat{\theta}^{(k)}))\phi_i \right\|_{R_i} \tag{7}$$

subject to

$$\left\| P\phi_i - \hat{\phi}_{ai} \right\|^2 \le \alpha_i^2 \left\| \hat{\phi}_{ai} \right\|^2, \quad i = 1, \cdots, m \tag{8}$$

The minimization is performed with respect to the modeshape components at both measured and unmeasured degrees of freedom while holding the values of the model parameters θ fixed at their current values $\theta^{(k)}$. It can easily be seen that the above constrained minimization problem is equivalent to minimizing the i-th modal measure of fit $\|(K(\hat{\theta}^{(k)}) - \hat{\omega}_i^2 M(\hat{\theta}^{(k)}))\phi_i\|_{R_i}$ subject to the i-th inequality condition in (8), i.e. each modeshape is computed independently from the other modeshapes. Both the objective function and the inequality constraints are quadratic in the set of unknown parameters. It can be shown that a unique optimum exists (Levine-West, Milman and Kissil, 1997), denoted herein by $\hat{\phi}_i^{(k+1)}$, $i = 1, \ldots, m$. The algorithm for obtaining the unique solution is described in the work by Levine-West, Milman and Kissil (1996, 1997) for $R_i = 1$. Extension of this algorithm to handle a general weight R_i is also straightforward.

2.2. Step 2: Updating of Model Parameters

In this step, the parameters of the model are updated using the latest estimates $\hat{\phi}_i^{(k+1)}$, $i = 1, \cdots, m$ of the complete modeshapes. The optimal values $\hat{\theta}^{(k+1)}$ corresponding to the $k+1$ iteration are obtained by the solution of the following unconstrained minimization problem:

$$\min_{\theta^{(k+1)}} J(\theta^{(k+1)}) = \min_{\theta^{(k+1)}} \sum_{i=1}^{m} \left\| (K(\theta^{(k+1)}) - \hat{\omega}_i^2 M(\theta^{(k+1)}))\hat{\phi}_i^{(k+1)} \right\|_{R_i} \tag{9}$$

This is a nonlinear optimization problem which can be solved using available iterative schemes such as teh modified Newton's method. For the case for which $K(\theta)$ and $M(\theta)$ are linear functions of" θ and R_i is independent of θ, the objective function $J(\theta)$ is quadratic in θ and the unique solution $\theta^{(k+1)}$ can be obtained without iterations by solving a linear algebraic system in θ.

The updated finite element model obtained at the $k+1$ iteration may contain inaccuracies due to the fact that the expanded modeshapes are based on an inaccurate model obtained at the previous iteration k. Thus, the two step procedure has to be repeated using the new updated finite element model until convergence is reached. Specifically, the iterative process is terminated when $\hat{\theta}^{(k+1)} - \hat{\theta}^{(k)} \;/\; \hat{\theta}^{(k+1)} < tol_1$ where tol_1 is a user-specified threshold level. Finally, it can be shown that the optimal solution θ and ϕ_i obtained from the iterative two-step procedure is also the optimal solution of the original constrained minimization problem described by equations (3) and (4).

3. Structural Parameterization

An important issue in finite element model updating is the selection of the type and number of properties to be included in the parameter set θ. Different choices are likely to give different model updates and subsequently affect model predictions. To avoid non-uniqueness and ill-conditioning problems, there is a need for limiting the number of parameters to as few as possible. Such efforts, on the other hand, may lead to the undesirable effect of excluding important uncertain properties from the parameter set θ. Specifically, the parameterization scheme depends on the purpose of the model updating analysis. For the purpose of calibrating the finite element model using test data obtained from the structure after it has been built, one would like to exploit possible symmetries and/or similarities present in the structure in order to reduce the number of parameter to be updated. For example, consider the three-dimensional structure shown in Fig. 1. Suppose that the ten bays or substructures have been manufactured with the goal of being identical, although deviations in the properties of the members and connections from one substructure to the other should be expexted due to errors in the manufacturing process. Then in a model correlation methodology, it would be appropriate to assume that any of the properties of the finite element model for one substructure will be the same as the corresponding properties for all other substructures. To limit the type and number of model parameters, efforts s} iould be concentrated on parameterizing one substructure only. Changes in the properties of all substructures will thus be regarded as fully correlated with changes in the properties of one substructure. For example, one uncertain parameter could be chosen to represent the axial stiffness of the diagonal elements for all bays.

The above parameterization becomes inefficient in identifying possible errors in the properties of the finite element model arising due to significant localized inaccuracies during the manufacturing process of the structure or due to localized damage from severe environmental events during the operational lifetime of the structure. Thus, for the purpose of detecting localized model faults or structural damage, alternative parameterization schemes should be explored which are able to locate faulty or damaged elements.

3.1. Application to Structural Damage Detection

In structural damage detection and health monitoring applications, damage is usually localized to one or a few structural elements or sub-structures of the super-structure. It is assumed herein that local decrease in stiffness is indicative of the location and size of damage. The purpose of the model updating methodology will be the identification of both the location and size of damage. In this case, the choice of the parameterization or substructuring scheme is critical for the effectiveness of the model updating algorithm. For example, a coarse parameterization of the type described previously will fail to locate where the damage has occurred. Alternatively, a sub-structuring scheme may be employed under which an uncertain parameter is assigned to each substructure. Changes in the value of this parameter will indicate damage in one or more of the elements comprising the substructure. A fine Sill)-strlt(tlllilig at the element 1('vel would be more appropriate for locating damage but it will lead to non-uniqueness due to the large 11(11111) ('1 of uncertain parameters to be updated. A course sill)-stIll(tl]firlg scheme for which each sub-structure consists of several finite elements may fail to reliably detect stiffness reductions if these are localized in one of the elements of the substructure.

An algorithm for identifying potential damage locations and subsequently updating only the properties of the finite element model at the identified locations of damage is proposed next. Specifically, the iterative procedure used in the proposed model updating methodology provides guidance in identifying the locations of damage and limiting the number of the parameters to be updating to only a few, thus reducing the problem of ill-conditioning and non-uniqueness expected when a large number of parameters is updated. For this, the expanded modeshapes predicted in the first step of the proposed methodology are used to identify possible locations of damage by examining, for each finite element (or substructure), the difference in element strain energy between the expanded mode-shapes and the model mode-shapes. The modal element (or substructure) strain energy for a finite element (or substructure) designated by < e > is defined as

$$S^e(\phi_i) = (1/2)\phi_i^T K^e \phi_i \tag{10}$$

where K^e is the stiffness matrix assembled from the element or substructure. The following measure of modal strain energy error is used

$$\Delta S_i^e = \frac{S^e(\hat{\phi}_i^{(k+1)}) - S^e(\varphi_i^{(k)})}{S^e(\hat{\phi}_i^{(k+1)})} \tag{11}$$

where $\varphi_i^{(k)} = \varphi_i(\theta^{(k)})$ is the modeshape computed from the structural model at iteration step k. It is expected that sufficiently large ΔS_i^e will be due to modeling errors in the particular element (or substructure) and will be indicative of probable damage in the element (or substructure). The properties of these elements (or substructures) are chosen to be updated if $|\Delta S_i^e| > tol_2$ for any mode i, where tol_2 is a user-specifid threshold. The thresshold values for each element can be obtained from statistical analysis of the quantities ΔS_i^e based on repeated modal test data carried out for the undamaged structure. Finally, only the properties of the identified probable damaged elements are included in the parameter set $\theta^{(k+1)}$ to be updated during the second step of iteration $k+1$. The properties of the finite element mode] included in the set $\theta^{(k+1)}$ may differ from those in the set $\theta^{(k)}$ obtained from the previous iteration.

3.2. Implementation in Software

The finite element model updating technique has been implemented in Matlab to enhance the capabilities of the existing Integrated Modeling of Optical Systems (IMOS) software package developed at Jet Propulsion Laboratory. Efficient parameterization schemes have been integrated with the existing IMOS finite element code. These schemes provide the user with the capability of exploring different parameterization alternatives in the model updating evaluations. At the present time, the linear parameterization for the system matrices $M(\theta)$ and $K(\theta)$ as a function of the parameters θ is used, that is.

$$K(\theta) = K_0 + \sum_{i=1}^{p} K_i\theta_i \qquad \text{and} \qquad M(\theta) = M_0 + \sum_{i=1}^{p} M_i\theta_i \tag{12}$$

where K_0, K_i, M_0 and M_i are constant matrices independent of θ. This linear parameterization often arises in practical applications. The incorporation of a general nonlinear parameterization is also straightforward.

The software has been constructed in a modular way so that enhancements or modifications can be independently developed for the four main modules involved, namely, the finite element analysis module, sensitivity analysis module, evaluation module accounting for the measure of fit between model predictions and test data, and the parameterization module. The goal in the development of the software is to provide a general user-friendly model updating tool that will greatly enhance model updating evaluations by assisting the user to explore, revise and identify the best parameterization scheme and/or measure of fit among the different alternatives available in the software or specified independently by the user.

4. Example

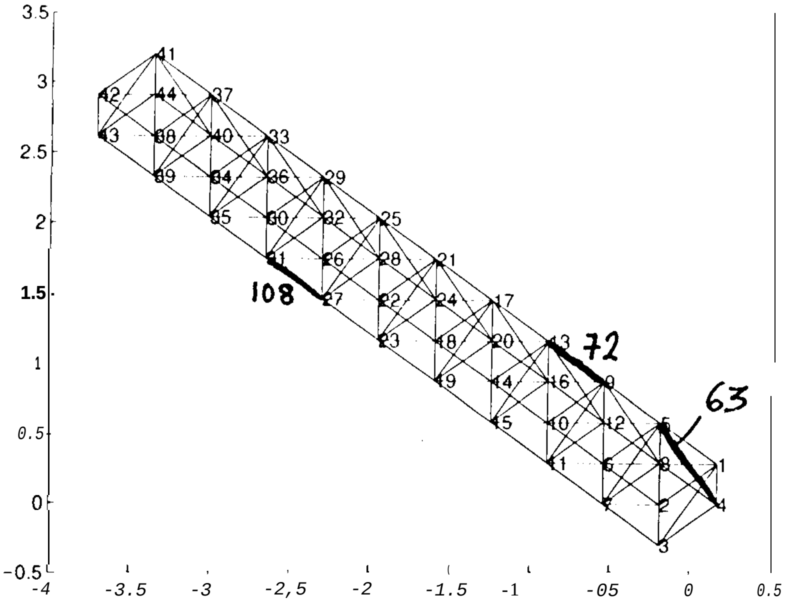

Figure 1: Three-dimensional truss structure

The methodology is assessed by applying it to the problem of damage detection of a structure. The model of the undamaged structure is a three-dimensional truss shown in Figure 1. It consists of 135 axial rod elements (1 per strut) with a total of 120 degrees of freedom (3 per node). The structure is supported by restraining all degrees of freedom at nodes 1 to 4. The modulus of elasticity, cross-sectional area and mass density are the same for all members. The values are chosen such that the first eight modal frequencies of the model range from 10 Hz to 200 Hz.

Simulated modal data are used to assess the performance of the proposed damage detection methodology. The elements 63, 72 and 108, located at different places on the structure as shown in Figure 1, are damaged by reducing the cross-sectional area of these elements by 50%. The modal test data are produced by calculating the modal data for the lower eight modes of the damaged model and by adding Gaussian white noise to simulate measurement and model errors encountered in practice. The standard deviation of the noise is taken as 1 % and 5% of the values of the modal frequencies and mode-shapes, respectively.

Two case studies are used to assess the performance of the method in relation to the number and location of sensors. In the first case, designated by Case A, a large number of 99 sensors are used. These sensors are places at nodes 5 through 37 to provide measurements at all three degrees of freedom for each node. For the second case, designated by Case B, only 15 sensors are used which are placed at nodes 5, 13, 21, 29 and 37 to provide measurements at all three degrees of freedom per node. The properties in the parameter set θ to be updated are the cross-sectional area of each member. The methodology was slightly modified to consider as acceptable only changes in the values of the parameter set θ which correspond to reduction in the cross-sectional area of the members.

Multiple sets of simulated modal test data are generated and used to compute the mean and the standard deviation of the values of the parameter set θ. Multiple sets of modal test data are available from repeated modal test experiments usually carried out in the laboratory or obtained by monitoring over a period of time the dynamic modal characteristics of a structure. This idea of using the information from repeated modal measurements to establish the location and size of damage is similar to the one used by Vanik (1997) for structural health monitoring purposes. The use of repeated measurement have shown to filter out measurement error and thus can improve substantially the predictive accuracy of the damage detection methodology.

Using the weights w, given by (6) with $\alpha_i = 0.1$ and $\beta_i = 0.02$ for all modes, the predicted location

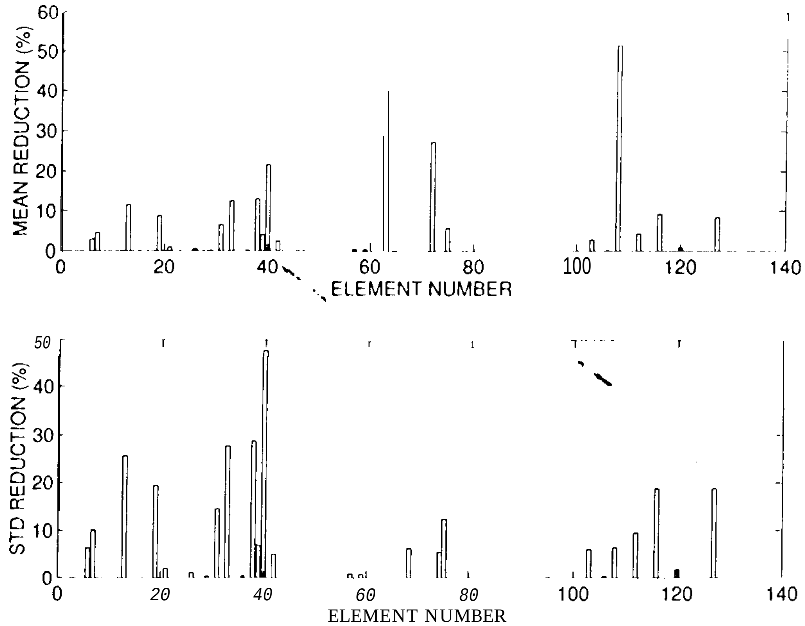

Figure 2: Predicted locations and magnitude of damage; Case A

and size of damage is shown in Figures 2 and 3 for the cases A and B, respectively. The results for the mean and the standard deviation of the predictions are based on five sets of simulated data. For case A, the predicted mean reductions in the cross-sectional area of members 63, 72 and 108 are 45% 28% and 52% with standard deviations 6%, 5% and 8%, respectively. These three members have correctly been identified as the damaged members with the highest mean reduction in cross-sectional area. The relatively small values of the standard deviation of these estimates is indicative of the relatively high confidence that damage has occurred in these members. In contrast, the standard deviation estimates of the rest of the members with non-zero mean reduction of Cross-sectional area are relative large. This is due to the fact that only a small percentage of the data sets have resulted in non-zero reduction in cross-sectional area of these members. Specifically, 3 to 4 out of the 5 data sets predicted no reduction or almost insignificant reduction in the cross-sectional area for these members. The use of a small size of simulated modal data sets has resulted in relatively high mean reduction values. As the number of modal data and therefore the number of modal tests increases, the mean values and the standard deviation for these members decreases. The results for the case B show a similar pattern. The predicted mean reductions in the cross-sectional area of members 63, 72 and 108 are 58%, 32% and 33% with standard deviations 5%, 11% and 7%, respectively. It is worth noting that the resolution of the size of damage at element 108 is not as good as for the Case A because sensors are not directly placed in the vicinity of the member 108. However, the elements 63, 72 and 108 have been correctly identified as the damaged elements.

Extensive numerical studies has also been carried out which show that the accuracy of the predictions increases as the number of measured modes increases, or as the level of the measurement error decreases. Location and number of sensors also play a role in the resolution of location and size of damage.

Finally, the effect of the choice of the weight R_i on the results was also explored by repeating the numerical studies using the weight R_i defined by (5), as well as using the weight $R_i = I$ for all i, where I is the identity matrix. For both weights it was found that the iterative model updating methodology has much slower rate of convergence. Moreover, for most cases examined, the location and size of damage was correctly identified at element 63. However, no significant damage was predicted for members 72 and 108, although both members were correctly identified as faulty elements.

5. Conclusions

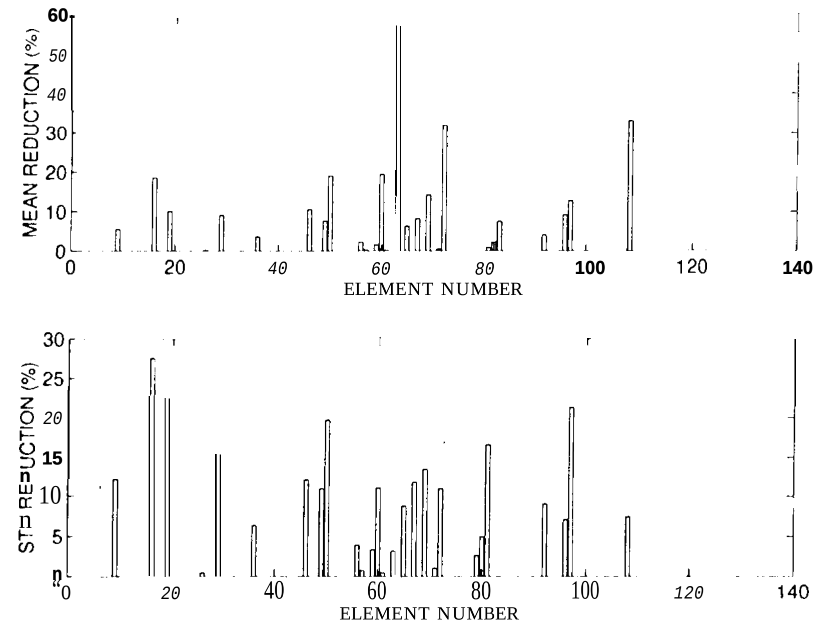

Figure 3: Predicted locations and magnitude of damage; Case B

The proposed modal-based model updating methodology is suitable for both model correlation and damage detection purposes. It is based on an iterative scheme which provides estimates of the expanded mode-shapes of each measured mode and predictions of the probable locations and size of errors in the properties of the finite element model of a structure. The expected measurement and modeling errors are directly accounted for in the methodology. In particular, applications to structural damage detection and structural health monitoring are addressed. It is suggested that the predicted accuracy of a model updating methodology can be significantly improved by jointly analyzing multiple sets of test data obtained from repeated laboratory or field tests or during structural health monitoring. A study using simulated data demonstrated that the methodology is promising for reliably predicting both the location and the size of damage in a structure. Measurement error was incorporated in the data by adding noise in the simulated data. The noise levels considered are similar to those expected in practical applications. Although the method suggested herein works well with simulated modal data and simulated measurement error, the practical use of this method with real data requires further study.

6. Acknowledgement

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