Self-Organization of Zonal Jets in Outer Planet Atmospheres: Uranus and Neptune

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Abstract

The statistical mechanical theory of a two-dimensional Euler fluid is applied for the first time to explore the spontaneous self-organization of zonal jets in outer planet atmospheres. Globally conserved integrals of motion are found to play a central role in defining jet structure. Maximum-entropy jet structures are calculated for the limit where the constraint to conserve energy does not substantially prevent vorticity mixing throughout the atmosphere. Zonal wind profiles predicted for Uranus and Neptune in this limit agree quite well with available observations. We discuss how the theory might apply to Jupiter and Saturn, where vorticity mixing may be less efficient.

The atmospheres of Jupiter, Saturn, Uranus, and Neptune are dominated by strong eastward and westward winds, or "zonal jets", that alternate in direction between the poles (Fig. 1). Jupiter and Saturn possess strong eastward winds at their equators, flanked by several alternating jets arranged approximately symmetrically on either side of the equatorial plane. Uranus and Neptune possess westward jets at their equators and a single eastward jet at high latitudes in each hemisphere. We know that these motions must be driven by buoyancy forces arising from the density contrasts induced by heat transfer, but we do not yet understand how the winds ultimately organize themselves into the patterns we observe. *

Attempts to understand the origin of the zonal winds at the altitudes where clouds form have followed one of two paths. One class of models treats the outer planet atmosphere as a shallow weather layer, ignoring any influence from the deep fluid interior.[2] These models neglect forcing by solar heating or internal heat flow. Instead, the motions are initialized as a random velocity field and the flow' evolves through two-dimensional (2D) turbulent advection in a nondissipative environment. In some of the models the flow is maintained by baroclinic instability yin the weather layer.[3] Although highly idealized in their representation of the density stratification and thermodynamic effects, the shallow layer models have the advantage of allowing the effects of various processes and planetary physical parameters to be isolated in a relatively simple calculation. More rigorous calculations have involved 3D numerical simulations of thermal convection in rapidly rotating, deep spherical shells.[4] Both shallow weather layer models and thermal convection models have been successful in generating alternating zonal jets with approximately the correct amplitude and latitudinal width, but the computed latitude profiles of longitude-mean zonal wind are far from those observed on the outer planets .[2,3,4]

It is intriguing that both weather layer and convection models yield alternating jets despite the fact that their motions are forced in very different ways and in different geometries. Perhaps the answer lies in what the models have in common: anisotropic turbulent flow. In the convection models, anisotropy of the velocity field is engendered primarily by rapid rotation in a weakly stratified environment. whereas in the weather layer models it enters as a consequence of rapid rotation and a small aspect ratio of vertical to horizontal length scales. Long-lived coherent structures, such as jets and vortices, are often seen to emerge spontaneously from anisotropic turbulence in a wide variety of geophysical flows.[5] '1'best structures behave as self-organizing, attracting patterns. Is it possible that anisotropic turbulent advection assumes the defining role in organizing the zonal jets? Applying this view' to the outer planets, one could hypothesize that the jets self-organize out of a statistical equilibrium of advective processes. while weak thermodynamic forcing, from the Sun and/or from internal heat flow, enters only to maintain the differential rotation against weak dissipation. This is not a new idea; in fact, it W'as the underlying motivation for shallow-water model simulations performed by Cho and Polvani.[2]

To explore the consequences of this hypothesis, it is desirable to calculate the steady, alternating jet structures toward which inviscid flow would evolve on the four outer planets, in the absence of competing processes, and compare the results to the observed wind patterns. To do so, we take advantage of recent advances in the statistical mechanical theory of the 2D Euler equation.[6] Application of this theory to the problem of jet self-organization has the benefit of defining and underscoring the significance of the globally conserved integrals of motion for characterizing turbulent advection's effect on the global circulation. Solutions under the theory take the form of latitude profiles of mean zonal wind that maximize a suitably defined entropy, subject to all the constraints imposed by the conserved quantities.

There is not sufficient space here to review the statistical mechanical theory in detail: rigorous accounts can be found in Robert and Sommeria (1991) and Miller et al. (1992), who provide different approaches to its formulation.[6] Although the literature on the subject has dealt entirely with strictly 2D flows, the theory can be readily generalized to certain approximate descriptions of 3D geophysical flows. What is needed is a set of approximations to the primitive equations that allow definition of a conserved potential vorticity, a streamfunction for the horizontal component of the flow, and an invertibility principle connecting the two, analogous to that connecting the streamfunction and potential vorticity in the quasigeostrophic system. The simplest approximation satisfying these criteria on the sphere corresponds to nondivergent barotropic flow. Although simple, this system will yield physically interesting results if baroclinic production of vorticity is weak in the neutrally stratified, fluid interiors of the outer planets. For simplicity. we limit our initial investigation to barotropic flow in a shallow spherical shell.

Geophysical flows tend to develop very complex potential vorticity (PV) filaments on small "fine-grain" scales while developing coherent structures at larger "coarse-grain" scales. The statistical mechanical theory rests on a separation of these scales and provides a means to calculate the flow structure on the scale of the domain size. The coarse-grain PV field is described in terms of local probability distributions $p(\sigma, \mathbf{x})$ of measuring a PV σ at position x. The distribution $p(\sigma, \mathrm{x})$ corresponding to the preferred statistical steady state is found by maximizing the Gibbs entropy $S = -\int d^3x \, d\sigma \, p \log p$ subject to the constraints to conserve global average energy E (kinetic energy in the barotropic model), angular momentum about the rotation axis L_z, and moments of the PV distribution Γ_n. Furthermore, if we search for motions that are symmetric about the equatorial plane, the solutions for stream function are required to have a definite, even or odd, parity. PV reduces in the barotropic model to absolute vorticity $q = \nabla^2 \psi'$, where ψ' is the coarse-grain streamfunction measured in the *inertial* frame. For barotropic flow, the globally conserved integrals are then given by

$$E = \frac{1}{2} \int_V \frac{d^3x}{V} \psi' q \qquad \frac{L_z}{a^2} = \int_V \frac{d^3x}{V} q \sin\phi \qquad \Gamma_n = \int_V \frac{d^3x}{V} q^n \tag{1}$$

where V is the domain volume, ϕ is latitude, and a is the planetary radius. The variational problem associated with maximizing S leads to the following equation[6,7] for a transformed streamfunction $\psi = \psi' + \eta \sin\phi$:

$$q = \nabla^2 \psi + \frac{2\eta}{a^2} \sin\phi = -\frac{1}{\beta} \frac{\partial}{\partial \psi} \log Z(\psi) \ ; \qquad Z(\psi) = \int d\sigma \, \exp[-\alpha(\sigma) - \beta\sigma\psi]. \tag{2}$$

β, the (positive or negative) "inverse temperature" and η are Lagrange multipliers associated with conservation of E and L_z, respectively, and $\alpha(\sigma)$ is associated with conservation of Γ_n. They are determined implicitly through the integral expressions for E, L_z, and Γ_n given above. The Lagrange multipliers measure the extent to which the integral constraints prevent the system from attaining its maximum possible entropy. The state with the largest entropy possible is the one where potential vorticity is completely mixed throughout the domain; such a state will have a relatively low value of β. Conversely. vorticity mixing is strongly inhibited by the energy constraint when β is large.

For the calculations presented here, we confine our attention to finding solutions only for a linearized version of the theory. the *strong mixing limit*, in which the constraint to conserve the global average energy does not substantially restrict mixing of potential vorticity throughout the atmosphere (small β).[7] In general, 2D Euler flow conserves an infinite number of global integrals of motion.[6] Solutions in the strong mixing limit, however, are found to depend parametrically on only two, the global average total energy and total angular momentum about the rotation axis. Previous studies, using weather-layer models to explore the effects of turbulent advection,[2,3] have implicitly constrained their initial conditions to give approximately correct values for the initial global-average energy but have generally neglected the global-average angular momentum. The calculations presented here demonstrate the indispensable connection that exists between an atmosphere's mean zonal wind profile and its global invariant of motion whenever 2D advection is the dominant organizing process for the flow. Unfortunately, the theory cannot be used to address the question of how the global invariants of motion came to have their present values, but once these invariants are specified, the maximum-entropy jet structure can be calculated.

Linearizing (2) in the strong-mixing limit for the axisymmetric case, we obtain a Helmholtz equation for the transformed stream function of the maximum-entropy jet structure,

$$\frac{1}{\cos\phi}\frac{d}{d\phi}\cos\phi\frac{d\psi}{\partial\phi} + \hat{\beta}a^2\psi = -2\eta\sin\phi + \hat{\beta}\langle\psi\rangle. \tag{3}$$

Here, $\langle\psi\rangle$ denotes the global average of ψ, and $\hat{\beta} = \beta\Gamma_2$, where Γ_2 is the global average enstrophy (squared vorticity). Although (3) is linear, the global problem is still nonlinear, due to the nonlinearity of the integral constraints. Solutions are found by expanding the basic equations in powers of $\beta\sigma\psi$. The expansions are carried out to order 1 for streamfunction and order 2 for tile energy, entropy, and angular momentum[7]. Eq. (3) admits two classes of solution, depending on whether or not $\hat{\beta} = -\lambda_n$, where $\lambda_n = -n(n+1)/a^2$, $n > 1$, is one of the eigenvalues of the axisymmetric Laplacian in spherical geometry. For $\hat{\beta} \# -\lambda_n$, the solutions are a family of jet profiles with zonal wind $\propto U_0\cos\phi$, i.e., a family of rigid-body rotations. The more interesting case occurs when $\hat{\beta} = -\lambda_n$. For this case, (3) combined with the integral constraints for E and L_z leads to the solution

$$\psi_r = (\Omega a^2 - \frac{3}{2}L_z)P_1(\sin\phi) \pm \left[\frac{2(2n+1)|\Lambda|a^2}{n(n+1)}\right]^{\frac{1}{2}} P_n(\sin\phi), \qquad n > 1 \tag{4}$$

where P_m is the Legendre polynomial of degree m, ψ_r is the streamfunction measured in a frame rotating at frequency Ω, and $\Lambda \equiv \frac{3}{4}L_z^2/a^2 - L'$. The zonal wind as seen in the planet's rotating frame is given by $u = -\frac{1}{a}\partial\psi_r/\partial\phi$.

The rigid-body rotations ($\beta \# -\lambda_n$) have $\Lambda = 0$, which of course will not be satisfied for arbitrary E and L_z, and hence these solutions are trivial. Nontrivial solutions exist for $\Lambda < 0$. Symmetry across the equatorial plane demands solutions with a definite parity. We see that ψ_r must have *odd* parity because of the presence of $P_1(\sin\phi)$ in (4). Therefore, n is odd. The entropy for mode n in the strong-mixing limit is given by

$$S_n = S_0 - \frac{1}{\Gamma_2}\left[\lambda_n \Lambda + \frac{3}{2}\frac{L_z^2}{a^4}\right] \tag{.5}$$

where S_0 is a constant. S_n is maximized for minimum n. Since n is odd and greater than 1, maximum entropy solutions correspond to $n = 3$. Thus, solutions consist of a family of zonal jet structures that depend parametrically on the two global invariants E and L_z. The enstrophy Γ_2 and higher moments of the vorticity do not enter the solutions in the strong-mixing limit. We find that all strong-mixing jet structures possess either i) a subrogating jet at the equator and a single prograde, high-latitude jet in each hemisphere, reminiscent of the observed zonal wind profiles of Uranus and Neptune (Fig. 1), or ii) a wind profile with the same general shape but with opposite sign. This multiplicity of the solutions stems from the nonlinearity of the energy conservation constraint.

If we anticipate solutions relevant to Uranus and Neptune and choose them to give subrotation at the equator, we find that the barotropic strong-mixing limit yields zonal wind profiles showing remarkable agreement with the observed jets of Uranus and good agreement with Neptune's when appropriate values of global average kinetic energy and angular momentum are used. These can be estimated for Uranus, using a schematic interpolation/extrapolation of Voyager-2 cloud-tracked winds.[8] Defining $\epsilon \equiv (E - \frac{1}{3}\Omega^2 a^2)/(\Omega^2 a^2)$ and $l \equiv (L_z - \frac{2}{3}\Omega a^2)/(\Omega a^2)$, which represent nondimensional departures of E and L_z from their values for a thin spherical shell rigidly rotating with frequency Ω, we find $\epsilon = 0.012$ and $l = 0.010$ for the schematic profile. The barotropic, strong-mixing jet structure derived with these values is shown in Figure 2a. Agreement with the schematic fit is very good, within 7 m sec^{-1} at any latitude, suggesting that the strong-mixing regime is a good approximation for Uranus and that the self-organization hypothesis is valid for this planet. Results for Neptune are shown in Fig. 2b for a fixed value of ϵ and two values of l. The theory appears to underestimate the strength of the prograde jet at 70° S: a solution forced to recover this jet (having a slightly different value for l) forms too narrow and strong an equatorial jet. Perhaps the strong-mixing limit is a worse approximation for Neptune than for Uranus, or perhaps more than just anisotropic turbulent advection controls the wind profile. We need to apply the full theory to decide which is correct.[9]

The jet structures of Jupiter and Saturn (Fig. 1) cannot be represented by the smooth profiles obtained in the strong-mixing limit. This limit predicts every planet should have three jets like Uranus and Neptune, irrespective of its values of E, L_z, and Γ_n, but it breaks down at large β. The pronounced cyclonic-anticyclonic vorticity banding on Jupiter and Saturn suggests that the constraints to conserve the integrals of motion effectively impede vorticity mixing. If the self-organization hypothesis is valid for these planets, they must lie in the nonlinear regime. In Figure 3, we illustrate schematically how the theory might include these planets by plotting the entropy S against global average kinetic energy E, ignoring for simplicity any dependence of S on L_z or Γ_n. By definition, $(\partial S/\partial E) = -\beta$; hence, the slope of any curve on this diagram is equal to $-\beta$. Inspection of (3) shows that $\hat{\beta} = \beta\Gamma_2$ is a wavenumber squared and that the number of alternating jets a planet will have scales with $(\beta\Gamma_2 a^2)^{\frac{1}{2}}$. The strong-mixing solution plots as a straight line with $-\beta$ equal to the slope of the full nonlinear solution at E = O. The solid curve in Fig. 3 represents the general dependence that S *must* have on E for a full nonlinear solution to (1) when the global circulation Γ_1 = O (as it does in spherical geometry).[7] As E increases, β increases. Hence, the number of jets (for fixed $\Gamma_2 a^2$) scales with E, the global average kinetic energy measured with respect to an *inertial* frame. A larger, faster rotating planet will have larger E and more jets (for the same Γ_2). As Fig. 3 is drawn, Jupiter and Saturn would be predicted to have multiple alternating jets. In future work, we will be particularly interested to determine whether the nonlinear regime of the statistical mechanical theory predicts equatorial superrotation and multiple alternating jets for these giant planets.

References and Notes

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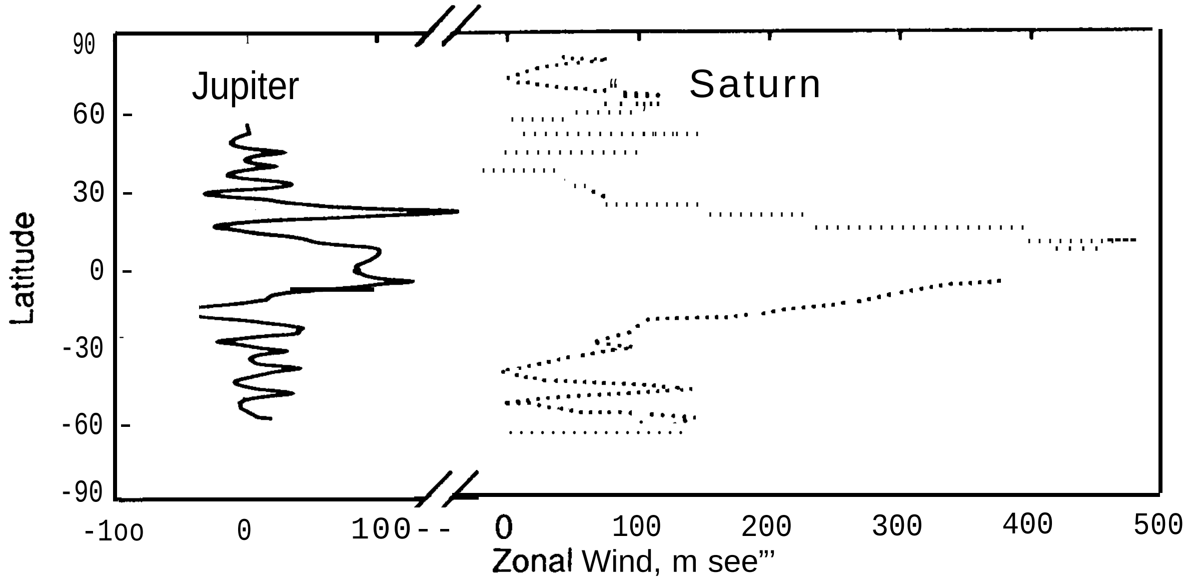
Figure Captions

Fig. 1. Latitude profiles of mean zonal wind for the outer planets. Note that the velocity scale for Saturn is offset relative to that for Jupiter.

Fig. 2. Barotropic jet structures derived in the strong mixing limit of the statistical mechanical theory for Uranus and Neptune. a) Uranus: solid curve is the theoretical prediction from Eq. (4) with $n = 3$, $\epsilon \equiv (E - \frac{1}{3}\Omega^2 a^2)/(\Omega^2 a^2) = 0.012$ and $l \equiv [L_z - \frac{2}{3}\Omega a^2)/(\Omega a^2) = 0.010$. These values for ϵ and 1 were derived from the dashed curve. The dashed curve is a schematic interpolation/extrapolation of Voyager-2 cloud-tracked wind data.[8] Solid circles denote cloud-tracked wind measurements" and wind velocity inferred near 5° latitude from radio occultation data.[11] b) Neptune: solid curve is theoretical prediction from Eq. (4) with $\epsilon = -0.071$ and $l = -0.079$. Dotted curve is for same ϵ but 1 = –0.082. The solid circles are Voyager-2 cloud-tracked winds averaged in 1° latitude bins.[12]

Fig. 3. Schematic entropy *vs.* energy diagram for outer planet barotropic models. E is global-average kinetic energy measured relative to the inertial frame and normalized with respect to Jupiter. The solid curve represents the dependence of entropy on E expected for the full nonlinear solution, ignoring any possible additional dependence of the entropy on L_z or Γ_n. The dashed curve represents the strong mixing solution. The slope of a curve on this diagram is proportional to the square of the number of alternating jets comprising a zonal wind profile. For comparable global average enstrophies on the outer planets, the diagram suggests that Jupiter and Saturn should have more jets than Uranus and Neptune.



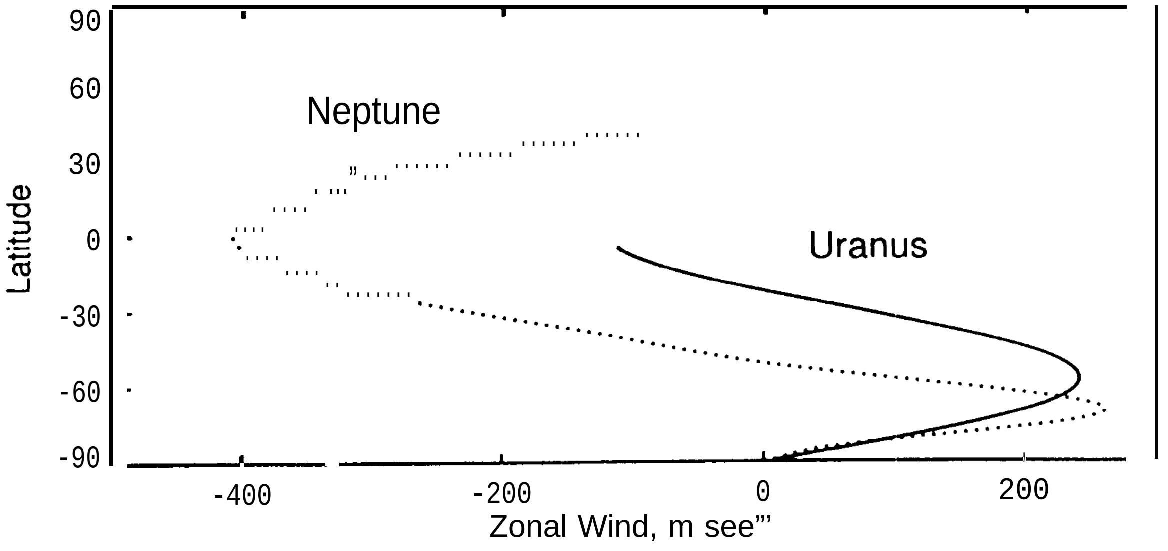


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Fig. 1

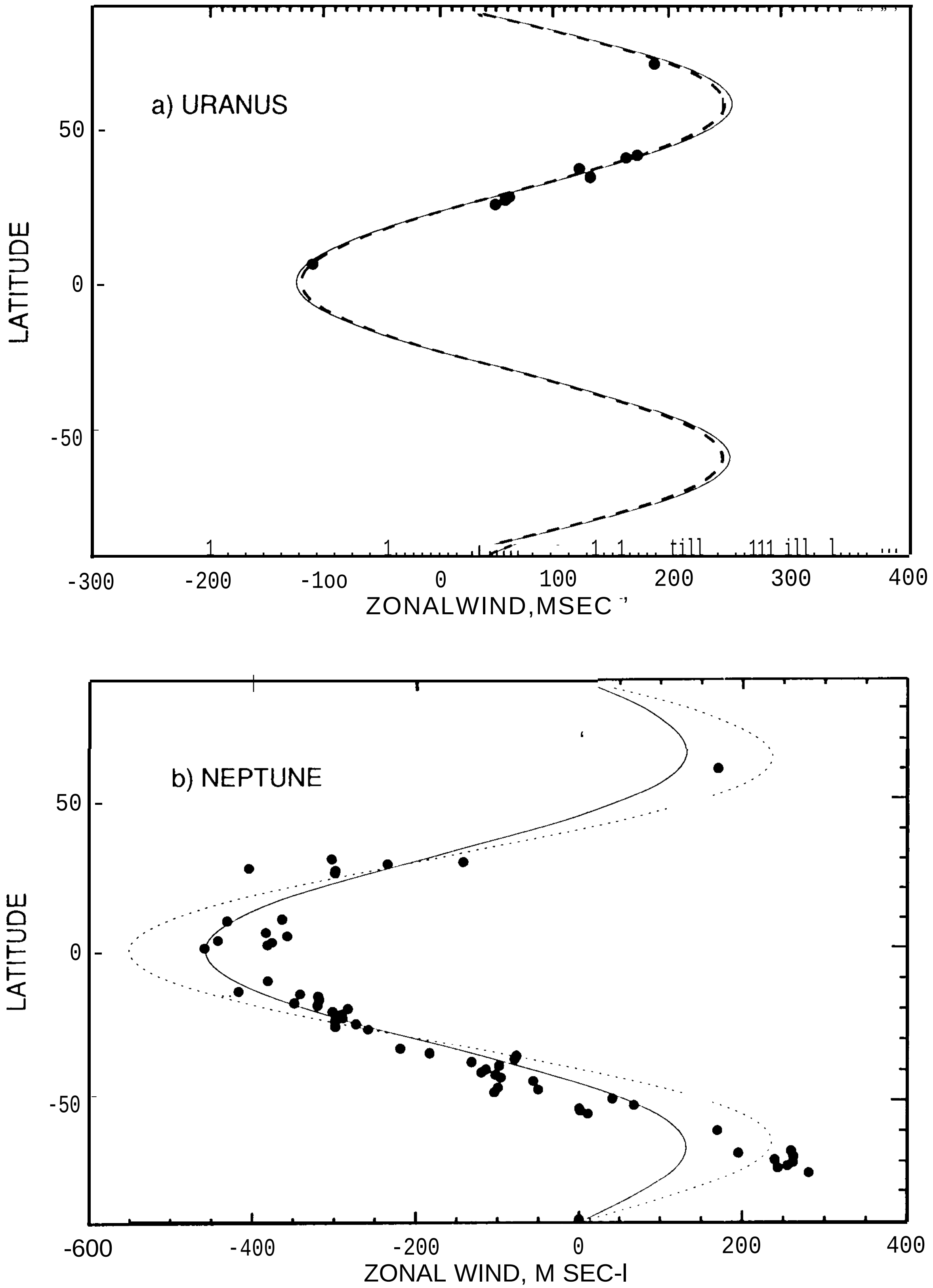

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Fig. 2

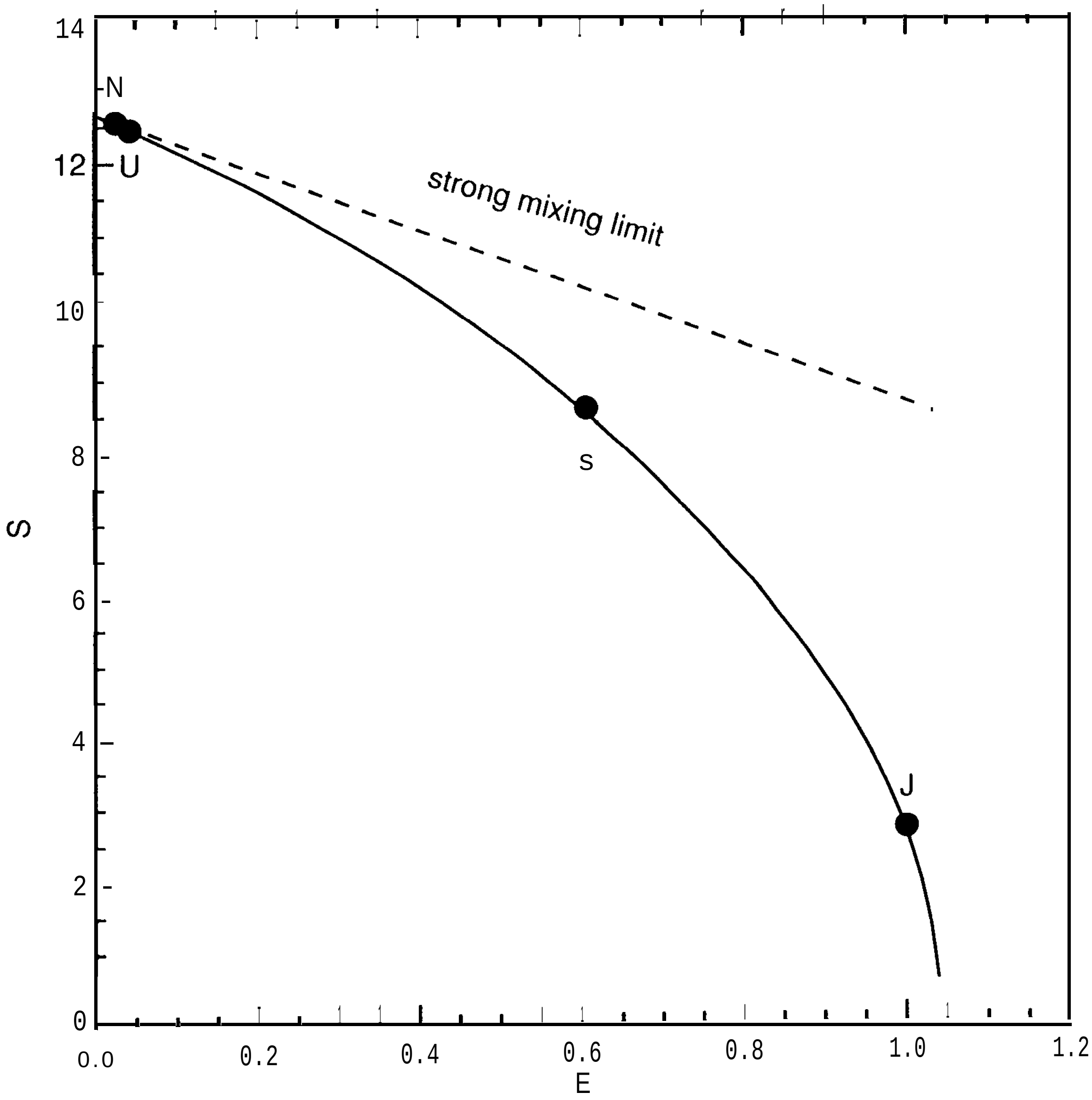

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Fig. 3