

TOTAL VARIANCE: A PROGRESS REPORT ON A NEW FREQUENCY STABILITY  
CHARACTERIZATION

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ABSTRACT

We give results of recent work on a newly developed frequency stability characterization, called Total variance, whose main advantages are improved confidence at and near the longest averaging time of  $\tau=T/2$ , where  $T$ =data duration, and lower sensitivity to drift removal. By theory and simulation we compute its mean, variance, and probability distribution; in particular, we report its variance at  $\tau=T/2$  for the FM noises, and express the result as degrees of freedom. In the presence of white FM noise modulation, Total variance is an unbiased estimate of the Allan variance for all  $\tau$ , and has three degrees of freedom at  $\tau=T/2$ . We also find that simple bias formulas can be used in the presence of flicker FM and random walk FM.

We discuss Total variance's properties as a random variable that is functionally dependent on both  $\tau$  and  $T$ , as contrasted with the theoretical Allan variance, which depends only on  $\tau$  and whose conventional estimators are limited to  $\tau \leq T/2$  for a phase record of duration  $T$ . In principle, Total variance can report values beyond the usual Allan variance last- $\tau$  value of  $\tau=T/2$ ; its values at  $\tau > T/2$  might be used to augment the normal last- $\tau$  value of frequency stability reported at  $\tau=T/2$ .

We compute the frequency response of Total variance as a function of  $\tau$  by averaging the squares of the Fourier transforms of Total variance sampling functions, and find that it resembles the frequency response of Allan variance. The results of these investigations indicate that Total variance, while it has an interpretation like that of the Allan variance, also has lower variability and less sensitivity to drift removal.

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