

# Radio Frequency Measurements of Cloud Size in a Linear Ion Trap

John D. Prestage, Robert L. Tjoelker and Lute Maleki  
California Institute of Technology, Jet Propulsion Laboratory  
4800 Oak Grove Drive  
Building 298  
Pasadena, CA 91104

## Abstract

In this paper we describe a method for measuring the size of an ion cloud inside a linear ion trap. At  $10^{-12}$ , the 2nd order Doppler shift for trapped mercury ion frequency standards is one of the largest frequency offsets in these lamp based systems. Its measurement to the 1 % level would represent an advance in insuring the very long term stability of these standards to the  $10^{-17}$  or better level.

This measurement is implemented with a novel 12-rod linear ion trap which we have developed and describe *here*.

## Introduction

A small, portable clock with long term stability of  $10^{-17}$  or better would enable an autonomous deep space navigation system where ultra-light and ultra-stable clocks would be carried aboard a spacecraft to generate a two-way Doppler link from the spacecraft to the ground stations and transponded back. This system would allow the spacecraft to navigate using only a fraction of the ground station antenna time as is currently used. Such systems are now being planned at JPL for deep space missions where ground station antenna time allocation must dramatically shrink as small, inexpensive missions proliferate.

To maintain  $10^{-14}$  stability over time scales required for a spacecraft to travel to the outer reaches of the solar system an onboard atomic clock would need to be self-calibrating, assessing frequency *offsets* and taking corrective measures to hold such offsets constant to 1 %. The method described in this paper should enable a measurement of the 2<sup>nd</sup> order Doppler shift to the 1 %

level via a cloud size measurement. This method would be useful for all applications requiring long term stability.

## Improved Harmonic Linear Ion Traps

The harmonicity of a traditional four rod linear ion trap is a function of rod diameter and spacing. Improved harmonicity can be accomplished with variations to this geometry. For example, figure 1 shows a linear ion trap configuration based on a cylinder which has been cut along its length into eight sectors, four at 60° angular width and four at 30° angular width. The quadrupole requirement  $\Phi(\rho, \theta \pm \pi/2) = -\Phi(\rho, \theta)$  leads to the **expansion** for the potential inside the cylindrical linear trap

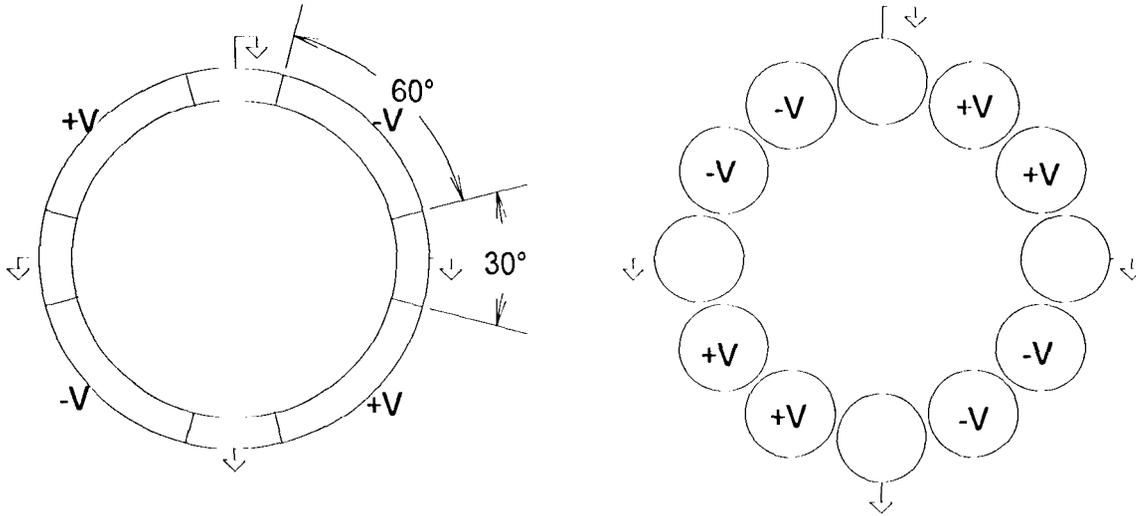
$$\Phi(\rho, \theta) = C_0 \rho^2 \sin(2\theta) + C_1 \rho^6 \sin(6\theta) + C_2 \rho^{10} \sin(10\theta) + \dots$$

If the 30° sectors are grounded and the remaining 60° sectors are biased in a quadrupole fashion as shown, the resulting field is very harmonic, i.e.,  $C1 \approx 0$ .

*One* simple implementation of this 60°/300 arrangement is shown in Figure 1. It consists of 12 circular rods with every 3rd rod grounded with the two intervening rods held at the same potential. This arrangement has the same 60°/300 symmetry of the sectorized cylinder of Figure 1.

## Ion Cloud size Measurement via quadrupole shaped magnetic field

The 4 auxiliary grounded rods can be used to generate a quadrupole magnetic field inside the linear trap whose node line coincides with the node line of the rf trapping fields.



**Figure 1.** The trap on the left produces a very harmonic trap  $\Phi(\rho, 0) = C_0\rho^2\sin(20) + C_2\rho^{10}\sin(100) + \dots$ . The trap on the right is a simple implementation of the  $60^\circ/300$  symmetry constructed with 12 round rods.

The shift of the clock transition with applied magnetic field is quadratic,  $\nu = \nu_0 + a \mathbf{H}^2$  where  $\mathbf{H}$  is the total applied field and  $a$  is the sensitivity factor;  $a = 97 \text{ Hz/Gauss}^2$  for  $^{199}\text{Hg}^+$  clock transition. Note that  $97 \text{ Hz/Gauss}^2 = 97 \mu\text{Hz/mG}^2$ . If the field is the sum of a static, homogeneous field along the trap axis,  $\mathbf{H}_0$ , and the transverse quadrupole field from the four auxiliary trap rods,  $\mathbf{h}_\perp(\mathbf{r})$ , we find  $\nu = \nu_0 + a (\mathbf{H}_0 + \mathbf{h}_\perp(\mathbf{r}))^2 = \nu_0 + \alpha \mathbf{H}_0^2 + \alpha \mathbf{h}_\perp^2$ . Since  $h_\perp(\mathbf{r}) = h'(\hat{x}y + \hat{y}x)$  and therefore,  $\mathbf{h}_\perp^2 = (h')^2 \rho^2$ , the magnetic shift of the clock transition grows quadratically with distance from the node line. The clock frequency is the average of this spatially varying field over the ion cloud distribution,  $n(\rho)$ . Thus,  $\langle \nu \rangle = \nu_0 + \alpha \langle \mathbf{H}_0^2 \rangle + \alpha \langle \rho^2 \rangle (h')^2$  where the brackets,  $\langle \rangle$ , indicate average over the ion cloud distribution. A measurement of the frequency change of the clock transition when the transverse field  $\mathbf{h}_\perp(\mathbf{r})$  is applied can yield a measurement of the ion cloud radius. The quantities which determine  $\langle \rho^2 \rangle$  are ion number, ion temperature, and the trap rf level and its resulting secular frequency,  $\omega_{\text{sec}}$ . For a fixed secular frequency and buffer gas pressure, ion number and temperature are not independent thus a measurement of  $\langle \rho^2 \rangle$  could be used to servo the electron emission to hold the ion number (and temperature) constant.

To estimate the size of the shifts suppose  $\langle \rho^2 \rangle = 1 \text{ mm}^2$  and  $h' = 20 \text{ mG/mm}$ . The shift when the quadrupole field

of this strength is switched on is  $97 \mu\text{Hz/mG}^2 \times 1 \text{ mm}^2 \times 400 \text{ mG}^2/\text{mm}^2 \approx 40 \text{ mHz}$  which corresponds to a  $1 \times 10^{-12}$  shift of the clock transition. With a trap radius  $R \approx 5 \text{ mm}$  the field gradient produced at the center is  $h' \approx (\mu_0 I) / (\pi R^2) \approx 20 \text{ mG/mm}$  at a quadrupole excitation current of  $I = 125 \text{ mA}$ . Even with  $\langle \rho^2 \rangle = 0.1 \text{ mm}^2$ , a 400 mA current will produce a  $10^{-2}$  clock shift. A clock with  $10^{-13}/\sqrt{\tau}$  short term stability will measure this offset to about  $1^\circ\text{A}$  in a few minutes of averaging time,  $\tau$ .

### $T_2$ Relaxation in $\mathbf{h}_\perp$ field gradient

One problem that must be avoided with this technique is relaxation of the high Q clock transition in the field gradient of the quadrupole magnetic field,  $\mathbf{h}_\perp(\mathbf{r})$ . This method depends upon the clock transition shifting an amount proportional to  $\langle \rho^2 \rangle$  with no change in the signal size and Q. The two Zeeman states  $F = 1, m_F = \pm 1$  can be mixed with the upper clock state  $F = 1, m_F = 0$  and cause a rapid relaxation of the coherence between the two clock levels  $F = 1, m_F = 0$  and  $F = 0, m_F = 0$ . The frequency spacing to these Zeeman levels increases at  $\approx 1.4 \text{ kHz per mG}$  of applied field  $\mathbf{H}_0$ , which is typically about 50 mG.

The trajectory of an ion in the  $\rho, \theta$  plane determines the spectrum of variation of the quadrupole field,  $\mathbf{h}_\perp(\mathbf{r})$ . The coherence in the clock transition will relax rapidly as this

spectrum overlaps with the Zeeman states at  $\omega_0 = \pm \gamma H_0$ , where  $\gamma/2\pi \cong 1.4$  kHz/mG. The harmonic motion through the trap center leads to a magnetic field variation at the secular frequency of the trap,  $\omega_{sec}$ . These are the 'free particle' limits. When space-charge repulsion as for a large cloud is important, these frequencies move down. Thus, it would appear that to avoid relaxation via mixing with the Zeeman states we must run the static field  $H_0$  high enough so that  $\gamma H_0 > \omega_{sec}$ . As the static field is increased the clock transition grows more sensitive to field variations and the clock is potentially less stable.

Relaxation rates to the Zeeman states from the upper clock state can be estimated from NMR relaxation rates given in the Redfield theory [1,2]. The time dependent magnetic field seen by an ion moving in the field gradient of  $\mathbf{h}_J(\mathbf{r})$  can have spectral overlap with the frequency splitting to the Zeeman states  $\omega_0 = \pm \gamma H_0$  thereby transferring atoms into this state. In this estimate, the rate at which this occurs is assumed to be the same as the rate of coherence loss in the clock transition. The transfer of population occurs at a rate  $T_1^{-1} \approx \gamma^2 (\hbar')^2 (S_x(\omega_0) + S_y(\omega_0))/2$  where we have used  $|\vec{\nabla} H_x|^2 = |\vec{\nabla} H_y|^2 = (\hbar')^2 |\nabla|$  the notation of ref [1]. The spectrum of the time variation of the field gradient seen by the moving ion  $S_x(\omega) = S_y(\omega) = (2k_B T/m\omega_{sec}^2)(t_c(t_c^{-2} + (\omega - \omega_{sec})^2))^{-1}$  is assumed to be a Lorentzian shape centered on the ion secular frequency,  $\omega_{sec}$ , with width determined by the ion collision rate,  $t_c^{-1}$ , which changes the phase of the ion secular motion within the trap. The spectrum is derived from a harmonic oscillator which is randomly re-phased at an average time interval of  $t_c$ . These collisions could be with other atoms or ions, or could be with the trap end confinement fields where each turn-around at the end cap field will disrupt the secular frequency phase. The collisions must be of sufficient strength to randomize the phase of the harmonic secular motion. The mean square amplitude of the transverse motion is  $2k_B T/m\omega_{sec}^2 = \langle P^* \rangle$  determined by the secular confinement and the ion temperature, T. We re-write the population rate transfer in terms of the frequency shift when the quadrupole field is applied,  $\delta\nu = \alpha \langle \rho^2 \rangle (\hbar')^2$ , as  $T_1^{-1} \approx \gamma^2 (\delta\nu/\alpha)(t_c(t_c^{-2} + (\omega_0 -$

$\omega_{sec}))^{-1}$ . Taking  $t_c = 1$  msec,  $\omega_0 - \omega_{sec} = 27 \times 50$  kHz, and  $\delta\nu = 40$  mHz, we find that  $T_1^{-1} \approx 300$ /sec, a very rapid loss of coherence in the clock transition.

One possible solution to this near resonance relaxation is to apply the quadrupole field at a frequency,  $\Omega$ , much higher than the secular frequency,  $\omega_{sec}$ . Since along the path of the ion trajectory,  $\mathbf{h}_J(\mathbf{r}(t)) = \hbar'(\hat{x}\mathbf{y} + \hat{y}\mathbf{x}) = \hbar' \cos \Omega t (y_0 \sin \omega_{sec} t \hat{x} + x_0 \sin(\omega_{sec} t + \phi) \hat{y})$ , the frequencies of the quadrupole field seen by the moving ion are now up-shifted to  $\Omega \pm \omega_{sec}$  which can be 10 or more times higher than  $\gamma H_0$  to avoid the mixing to the Zeeman states and loss of coherence in the clock transition. In this case the dominant frequency seen by the moving ion is  $-\Omega$  so that  $T_1^{-1} \approx \gamma^2 (\delta\nu/\alpha)(t_c(t_c^{-2} + (\omega_0 - [-\Omega])^2))^{-1}$ . If the quadrupole field is applied at 2 MHz the relaxation rate is  $T_1^{-1} \approx 0.2$  /sec, much slower than with the dc quadrupole current and thereby preserving the line Q and signal size.

### Summary

We have proposed a method for measuring the size of an ion cloud confined in a linear trap. This method involves measuring the shift in the clock transition frequency when a quadrupole rf magnetic field is applied. The node line of this field coincides with the node line of the rf trapping field. The method should allow stabilization of the 2<sup>nd</sup> order Doppler shift to 1% enabling 2<sup>nd</sup> order Doppler instabilities to be held below 10<sup>-6</sup> of the clock output frequency.

### References

- [1] D. D. McGregor, *Transverse Relaxation of Spin-Polarized <sup>3</sup>He gas due to a Magnetic Gradient*, Phys. Rev. A 41,2631 (1990).
- [2] C. P. Slichter, *Principles of Magnetic Resonance* (Harper and Row, New York, 1963), Sec. 5.7.