Serial concatenation of interleaved codes: performance analysis, design and iterative decoding

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Abstract -- In this paper, upper bounds to the *average* maximum-likelihood bit error probability of serially concatenated coding schemes are derived, design guidelines for the outer and inner codes that maximize the *interleaver* gain are presented, and, finally, a highly-performing iterative decoding algorithm is proposed.[1]

1. Analysis and Design

Parallel concatenated codes (PCC) with interleave, known as " turbo codes" [I], can now be considered as a good trade-off between performance and complexity, yielding results close to the theoretical capacity limits at bit error probability in the raneg 10^{-5} — 10–7. As an alternative, a serially concatenated code with interleaver consists of the cascade of an *outer* encoder, an interleaver permuting the outer codewords symbols, and an *inner* encoder whose input words are the permuted outer codewords.

To compute an upper bound to the maximum likelihood (ML) performance, we use a *uniform interleaver* [2], i.e. a device that maps a given word at the output of the outer code into all possible permutations of it. Defining the *conditional weight enwner-sting function* (CWEF) $A^C(w, H)$ of a code as the weight distribution of the code words generated by an input word of weight w, the upper bound to the MI, bit error probability can be written in the form

$$P_b(e) \leq \sum_{w=1}^{k} \frac{w}{k} A^{C_S}(w,H)|_{H=e^{-R_c E_b/N_0}}$$

where $A^{C_S}(w, H)$ is the CWEF of the serially concatenated code (SCC), k is the size of the data word to the outer encoder, and R_c the code rate. Owing to the uniform interleaver properties, this CWEF can be easily evaluated from those of the CCs.

As an example, we show in Fig. 1 the upper bound applied to convolutional PCC and SCC using 4-state CCS and presenting the same input delay. 'l'he different behavior with respect to the interleave length (or, equivalently, input delay) is manifest. The PCC shows an interleave gain N-l, whereas the SCC shows a gain N^{-3}. In general, for convolutional SCC employing recursive inner encoders, the interleaver gain is $N^{\lfloor \frac{d_f^o+1}{2} \rfloor}$, where d_f^o is the free distance of the outer code. As a consequence, the design rules for SCC consist in using a recursive inner encoder, and an outer code with as large as possible free distance.

To obtain high coding gains, the size of the interleave must be large, and this makes the ML decoding impossible.

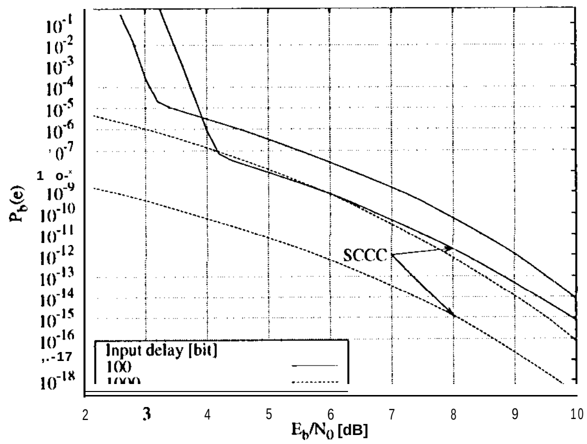

Fig. 1: Comparison of SCCC and PCCC with 4 states CCS.

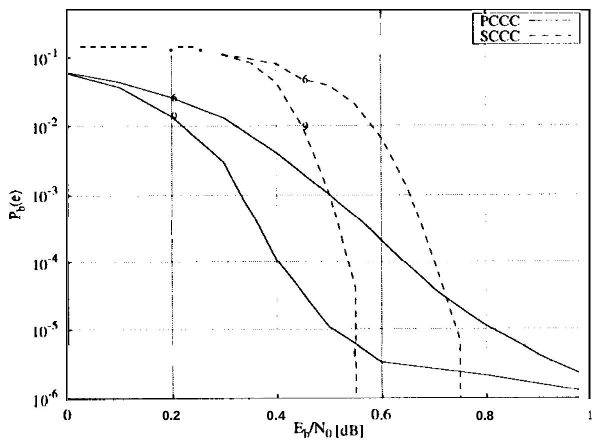

Fig. 2: Comparison of two rate 1/3 PCCC and SCCC.

A suboptimnm, yet highly performing alternative consits in the combination of two soft decoders that compute the a-posteriori probabilities of the inner and outer symbols. As an example of the obtainable performance, we report in Fig. 2 the simulated performance of the iterative decoder with 6 and 9 iterations applied to two concatenated rate 1/3 PCC and SCC with interleaving size 16,384. Serial concatenation does not present the typical *floor* effect of turbo codes, and permits to reach lower bit error probabilities.

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References

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