ASYMPTOTICALLY GLOBALLY STABLE MULTIWINDOW CONTROLLERS

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Abstract

The Popov criterion is applied to control system analysis and design. Nonlinear dynamic compensators (NDC) are int reduced which ensure absolute stability without penalizing the available feedback.

1 Multiwindow Controllers and Nonlinear Dynamic Compensation

Multiwindow controllers consisting of linear controllers with nonlinear windows [1,2] perform superior to linear controllers. They can be designed using frequency domain methods, and their stability can be assured by using describing functions. However, the describing function methods are only approximate. It is convenient to use the methods to prove the system stability within each window, but rather difficult to use the methods to rule out the oscillations with the signal moving from window to window. It would be preferable to use design methods based on the strict methods of absolute stability. Such methods have been previously described in [3,4] but only with examples made to illustrate the achievable performance. While these examples are of theoretical significance, but use impractical very high-order controllers. In this paper, a technique for suboptimal but simple, low-order controllers with greatly improved performance is presented. Here, we consider absolutely stable two-window controllers with local nonlinear feedback producing the windows with the responses typical in a real application.

2 Absolute Stability

Many practical feedback systems consist of a linear link $T(s)$ and a memoriless (i.e. nondynamic) nonlinear link v(e) as shown in Fig. 1.

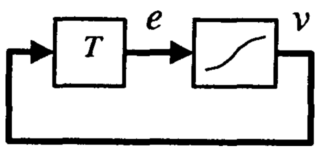

Figure 1: participation functions in composite controllers

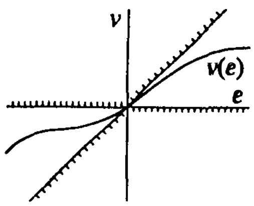

Figure 2: Characteristic of the nonlinear link

The system is Asymptotically Globally Stable (AGS) if it satisfies the Popov criterion [3 - 6], i.e., T(s) has no poles in the right half-plane, and at all frequencies

$$Re[(1 + jqw)T(jw)] > -1. \tag{1}$$

This system is said to be absolutely stable (AS) if it is asymptotically globally stable (AGS) with any characteristic $v(e)$ constrained by

$$0 < v(e)/e < 1 \tag{2}$$

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as illustrated in Fig. 2. Hard and soft saturation, dead zone, and three level relay belong to the claw of nonlinear characteristics defined by (l).

To check whether a system satisfies the Popov criterion, one needs to plot the Nyquist diagram for $(1+qs)T(s)$. If a q can be found such that the diagram stays to the right of the vertical line 1, the system is AS. The Popov criterion (which is sufficient but not necessary for AS) is more restrictive than the Nyquist criterion (which is necessary but not sufficient for AS).

3 Nyquist-Stable System with Nonlinear Dynamic Compensation (NDC)

Larger feedback and better disturbance rejection are available in Nyquist-stable systems. The Nyquist-stable systems are not AS when the compensators are linear. Still, they can be made AS by using nonlinear dynamic compensators. The NDC can be designed using linear and non-dynamic nonlinear links. The NDC can be represented in the form where the nonlinear links serve as nonlinear windows for the signal, and different linear links process the signal passing the nonlinear windows. This architecture is a particular case of multiwindow controller architecture discussed in [1,2].

In the system shown in Fig. 3, the nonlinear link $1 - v(e)$ in the local feedback of the NDC uses the same nonlinear function v(e) as the nonlinear link of the actuator. Typically, v(e) is a saturation link so that $1 - v(e)$ represents a dead zone. The rest of the links in the block diagram are linear. For the AS analysis, it can be assumed that the command signal is 0.

We denote by T_P the return ratio for the plant measured when the link v is replaced by 1 (and the link $1 - v$, by O). Then, the compensator transfer function for small level signals is expressed as T_P/P. When the signal level is very large, the return ratio in the NDC local loop becomes G.

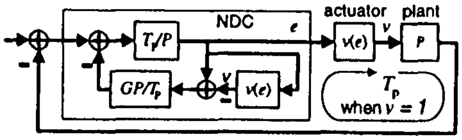

Figure 3: Feedback system with nonlinear link v in the actuator and link $1 - v$ in the feedback path of the NDC

4 Reduction to Equivalent System

The diagram shown in Fig. 3 depicts a system that has two identical nonlinear links $v(e)$, with the same input signal e and, therefore, the same output signal v. For the sake of stability analysis, the system can be modified equivalently into the one shown in Fig. 4, which contains only one nonlinear link v. The linear links within the dashed envelope form a composite linear link. We denote the negative of its transfer function T_E (equivalent return ratio), If T_E satisfies the Popov criterion, the system must be globally stable.

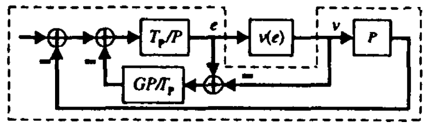

Figure 4: Equivalently transformed system containing a single nonlinear link v

To find the expression for T_E, the diagram in Fig. 4 is further redrawn as shown in Fig. 5.

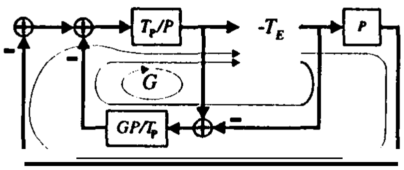

Figure 5: Calculation of T_E: two parallel paths and a loop tangent to both paths

From this diagram, using Masons rule *[7]*, the negative of the transfer function from the output of the nonlinear link to its input is

$$T_E = \frac{T_P - G}{1 + G} \tag{3}$$

Given T_E and T_P, the NDC linear link transfer function is

$$G = \frac{T_P - T_E}{1 + T_E} \tag{4}$$

From (4), the plant feedback is

$$1 + T_P = (1 + G)(1 + T_E) \tag{5}$$

5 Positive and Negative Feedback

Equation (5) states that the plant feedback in the linear mode of operation is the product of the feedback in the NDC for large signals, and the feedback in the equivalent system,

We follow the Bode and Black [4] definition of the negative feedback as the case when the feedback reduces the output signal, i.e. the modulus of the return difference is more than 1. Positive feedback is defined as regenerative feedback, i.e. the feedback increasing the closed loop gain, i.e. the feedback when the return ratio magnitude is less than 1.

Generally, as follows from Bode theorems, the integral of log $|1 + T_P|$ along the linear frequency axis is zero [4,5]. This means that the integral over the band where the integrand is positive (i.e. the feedback is negative) equals the area where the integrand is negative (the feedback is positive), The same is valid for G and T_E. Therefore, the areas of substantial positive feedback in the G and T_E loops should not overlap or else the posit ive feedback in the plant will be excessive and the phase stability margin in the plant loop, correspondingly, small. If positive feedback in each loop is substantial, the crossover frequency of $T_E(jw)$ must be eit hcr much smaller *or* much larger than the crossover frequency of $G(jw)$. In this case the area of positive feedback for *Tp* can be increased, and correspondingly, the area of negative feedback will increase,

Wc can scc the advantage of a system with an NDC as follows: in a conventional system, T_p must satisfy the Popov criterion, but in a system with an NDC, the only requirements arc that T_P satisfies the weaker Nyquist criterion, and that T_E, defined by (3), satisfies the Popov criterion. Exploiting this extra design flexibility leads to better performance as will be shown in the following design examples.

There still exists some freedom in choosing the responses for G and T_E. This freedom can be utilized for the provision of desired transient responses for large level signals, especially for homing systems where command feedforward [5] cannot be introduced.

In the most practical cases, the link v(e) is a saturation and 1 – v(e), a dead-zone link. If the thresholds in the actuator and in the nonlinear link within the NDC are different, a k-times scaling can be employed as shown in Fig. 6.

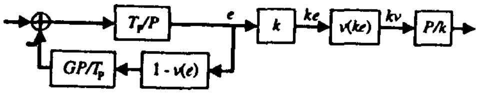

Figure 6: Scaling down the nonlinear link in the NDC by a factor of k

6 Design examples

A controller for a gimbaled actuator for Cassini spacecraft was designed following the block-diagram Fig. 3, and was briefly described in [2]. Here, we present several simple textbook-type examples [5] with rather low-order controllers that can be easily simulated. These examples demonstrate the advantage of using asymptotically stable multi-window controllers for the systems where the compensators are kept low-order. The performance can, however, be further improved by using higher order compensators.

Example 1

The plant transfer function is $P = 1/.9$. With the compensator function

$$\frac{T_P}{P} = \frac{2(s + 0.5)}{s(s + 2)} \tag{G}$$

the plant return ratio is

$$T_p = 2\frac{s + 0.5}{s^2(s + 2)} \tag{7}$$

The slope of the asymptotic Bode diagram is -12 dB/oct at lower frequencies and 6 dB/oct within onc octave to the right and to the left from the crossover frequency of 1, as shown in Fig. 7. Since the employed compensator is low order, stability margins in phase and gain are not well balanced and the disturbance rejection is not the maximum available.

With such loop transfer function and a single nonlinear element in the actuator, the system is prone to have windup in the response to large amplitude step commands. The NDC must make the system AS with large stability margins in T_E and improve the system transient responses in the nonlinear mode of operational ,2,4,5]. The large margins can also eliminate the process instability [4,5].

We will start the design by a guessed response for T_E as

$$T_E = \frac{2}{S(s+2)} \tag{8}$$

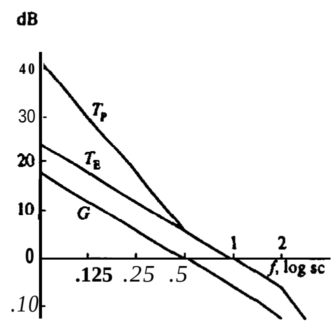

Figure 7: Asymptotic Bode diagrams for AS systems in Example 1.

This response merges with T_P at higher frequencies but has less slope at lower frequencies and, correspondingly, less phase lag. With such T_E, from (4), the NDC local loop return ratio

$$G = \frac{1}{s(s^2 + 2s + 2)} \tag{9}$$

The plots for T_P, G and T_E on the logarithmic Nyquist plane (L-plane) arc shown in Fig. 8.

Since G and T_E enter the equations (3) and (5) symmetrically, if we interchange G and T_E, the

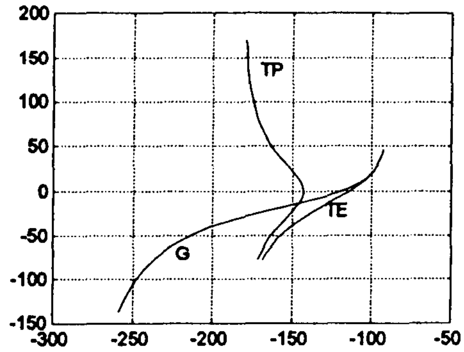

Figure 8: L-plane plots for Example 1

equations are still satisfied. With this replacement, i.e. with G = 2/[s(s + 2)], the transfer function $G(s)/[T_P(s)/P(s)]$ of the link in the feedback path of the NDC will be simplified:

$$\frac{2}{S(s+2)} \Big/ \frac{2(s+0.5)}{s(s+2)} = \frac{2}{2s+1} \tag{10}$$

The stability margin in T_E is now reduced, but the system still remains globally stable with rather large margins (although not process *stable),*

The system transient responses to the step command are shown in Fig. 9. To make the responses easier to compare, the step command value is kept the same, 1, and the threshold of the saturation varied so that small thresholds correspond to "large" commands, i.e. commands large relative to the actuator threshold. In (a), the output response shown in the linear mode of operation, i.e. simulated with setting the threshold of saturation and the dead zone to large values, The overshoot is close to 50%. In (b), the saturation threshold is 0.2, and the dead zone set to a large value so that the NDC local feedback does not pass the signal. It is seen that the system has a large windup. In (c), the dead zone is set equal to the saturation threshold (as must be), each of 0.2, The response has no overshoot. Similar responses, wit bout an overshoot, appear when the threshold and the dead zone are set to smaller value, 0.1 in (d) only the slew rate is correspondingly smaller. Fig. (c) shows the signal at the output of the actuator for the case (d). The actuator works nearly in time-optimal way, full power until the output approaches the command. In (f), the output is shown for the threshold and the dead zone set to 0.5. This case in intermediate between the linear case with 50% overshoot and the case with the threshold smaller than 0,2 when the overshoot disappears.

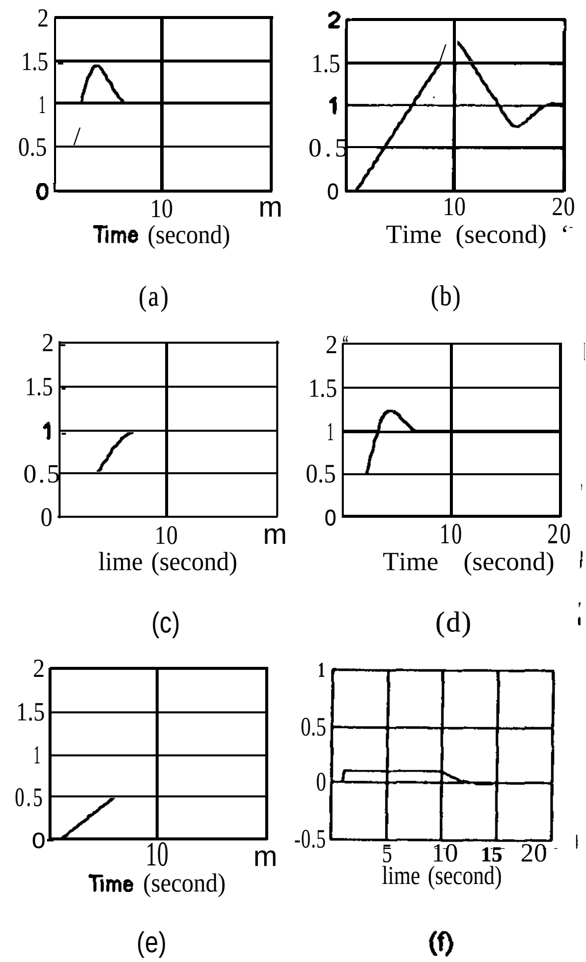

Figure 9: Output transient response of a system with a linear controller (a) and of AS systems (b)-(e), and of actuator output in Example 1.

This kind of performance is desirable for systems without prefilters or command feedforwarding where the output should not exceed the command by more than a certain specified value. Therefore, percentwise, rather large overshoots are allowed in responses to small commands and disturbances, but not for the responses to large commands and disturbances.

Example 2

The plant transfer function is P = 1/s. With a compensator

$$\frac{T_P}{P} = \frac{2(s + 0.5)(s + 0.1)}{s^2(s + 2)} \tag{11}$$

which has an extra pole at zero frequency compared with the previous example, the plant return ratio is

$$\frac{T_P}{P} = \frac{2(s + 0.5)(s + 0.1)}{s^3 (s + 2)} \tag{12}$$

The Bode diagram is shown in Fig. 10. The system is Nyquist stable.

With the same $T_E = 2/[s(s + 2)]$ as in the previous example, from (4),

$$G = \frac{(1.2s + 0.1)}{s^2(s^2 + 2s + 2)} \tag{13}$$

The asymptotic Bode diagrams for these functions are shown in Fig. 10, and the plots on the L-plane, in Fig. 11.

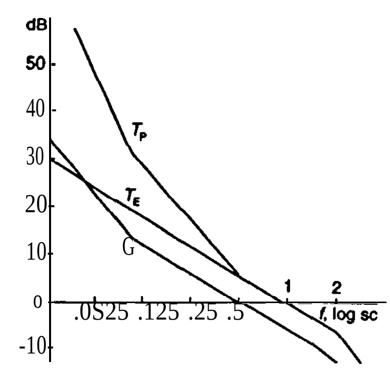

Figure 10: Asymptotic Bode diagrams for AS systems in Example 2

Example 3

If in the previous two examples, the plant transfer function is $1/s^2$, and the same loop responses are preserved by correspondingly changing the compensator, the system will remain absolutely stable and process stable, but the transient responses to large command step functions will exhibit large undershoot. This undershoot persists for a long time as

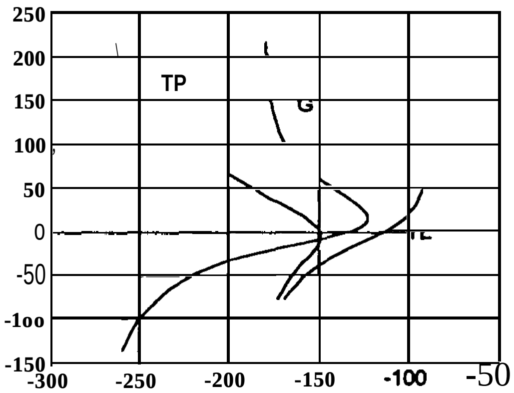

Figure 11: L-plane plots Example 2

the result of reduced gain of the nonlinear dynamic compensator for large signal levels.

It is impossible to simultaneously provide good transient responses for large level signals and preserve process stability. One of this desirable features (certainly the second one is less significant) for practical applications must be sacrificed.

Thus, for the systems with $1/s^2$ plants, the nonlinear dynamic compensator must only guarantee global stability. Although the process stability is not guaranteed, the errors caused by the process instability will be insignificant [4].

Acknowledgement

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