constellation

~~Constellatoin~~ Coverage Analysis

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ABSTRACT

The design of satellite constellations require an understanding of the dynamic global coverage provided by the constellations. Even for a small constellation with a simple circular orbit propagator, the combinatorial nature of the analysis frequently renders the problem intractable. Particularly for the initial design phase where the orbital parameters are still fluid and undetermined, the coverage information is crucial to evaluate the performance of the constellation design. We have devebped a fast and simple algorithm for determining the global constellation coverage dynamically using image processing techniques. This approach provides a fast, powerful and simple method for the analysis of global constellation coverage.

1. INTRODUCTION

Ever since the first satellites entered orbit, people have been concerned with their coverage. When can we observe the satellite from a point on the ground? What is the best location for a ground station? With the advent of satellite constellations, the complexity of the coverage problem increased combinatorially. Even for simple, highly symmetric constellations, coverage questions are not easily answered. Generally, the algorithms are not hard, it's just that the problems are time-varying and there are so many things to keep track of and to compare. As a result, planners rarely are able to obtain global system performance estimates that are rigorous and meaningful. What-if studies are difficult to perform if at all possible.

The difficulty of static discrete counting problems, also known as combinatorics, is well known; for example, the traveling salesman problem or the four color problem. Coverage analysis is inherently difficult because It is a dynamic infinitesimal "counting problem". One might think of this as "infinitesimal combinatorics". The idea of infinitesimal counting suggests that measure theory should be applicable to coverage analysis. This in turn suggests that averaging theory may be used to handle the dynamic combinatorics. These mathematical methods are by no means difficult, but may not be familiar to this community. Nevertheless, they are conceptually simple and provide rigorous metrics for global system performance which are easy to apply.

We present two measure-theoretic methods for global coverage analysis in this paper which are geometric in nature and simple in concept. By simple we mean the resulting algorithms are easier to implement than the standard brute-force approach which is computationally and data intensive. For our purposes simply consider "measure" as a measure of area or volume. Mathematically, we mean the Lebesgue measure on the plane and 3- dimensional space.

The first method we call "Visual Calculus" because it is based on the amputations associated with the visualization of the coverage problem. We demonstrate this method on the global performance of a bistatic SAR (synthetic aperture radar) system with a double constellation. In this analysis, in addition to providing the global coverage metric for the system, our method also provides the performance analysis of this new SAR concept. It clearly demonstrates the power of computer visualization for analytic purposes.

The second method is an application of ergodic theory to the satellite view period problem: Given a fixed point on the ground (such as a station or a user), how frequently is the satellite in view of this point on average? The standard approach to this question is to compute the actual view periods from propagating the satellite orbit with at least J2 perturbation assumed. Suppose now we wish to compare the coverage of a mission set of over 100 different orbits for 20 years. For planning purposes, most of the satellite orbits are ill defined. The combination of possibilities can quickly make this probfem intractable, especially if the ground element is mobile. The amount of raw view perfod data generated abne is considerable. Add to this the calculations of the statistics such as view period conflicts, the combinatorial problem quickly becomes intractable. Our approach, using ergodic theory, reduces this problem to solving definite integrals which is easily accomplished in packages such as Mathematical in just a few seconds. No large data set, no orbit propagation, but rigorous statistics and metrics.

2. THE VISUAL CALCULUS

The coverage of the 2-sphere by satellites is the key question for satellite coverage analysis. Figure 1 defines the satellite to ground station visibility geometry where we have assumed a minimum elevation angle alfa. Note, we use the term "ground station" loosely to mean any element on the ground which wishes to view the satellite. The figure shows that when the satellite groundtrack enters the circle of radius theta about the ground station, it is in view of the station. We call this circle the station mask. One way to visualize the coverage of a complex constellation is to piot the station masks on the gbbe with some map projection or in 3D. This enables the analyst to quickly get a sense of the performance of the constellation from the geometry.

It is well known that most projections greatly distorts the area or measure of the sphere. For example, in the cylindrical projection, the polar regions are artificially enlarged. In order to provide the analyst with a better sense of the coverage, an equal-area projection such as the sinusoidal or the Mollweide projection can be used. In this paper, we use the sinusoidal projection for simplicity to demonstrate this technique.

An equal area projection is also known as a measure-preserving transformation. In other words, it preserves the area of the sphere onto the plane. Given any region in the sinusoidal map, if we transform it back to the sphere, we should get a region with exactly the same area. In particular, this is true at the pixel level, ignoring the discretization error for the moment. Therefore, we can compute area simply by counting pixels.

What are the advantages of this approach? This provides a very simple algorithm for the discretization of the sphere and for computing coverage.

To compute the coverage, for each instant in time, one simply draws the instantaneous limb of the planet with an minimum elevation angle of alfa. One then performs a polygon fill centered on the satellite nadir for the limb circle (radius theta) which is also the station mask at the nadir. This represents all points of the ground which can see the satellite. -Notice, no visibility verification is necessary. Suppose we add 1 for each pixel in the fimb circle of the satellite and add O everywhere else. Suppose we do this for each satellite. The resulting map provides a tabulation of the instantaneous coverage of the constellation. The value of each pixel is precisely the number of satellites in view at that point on the map. In this way, not only have we quickly generated the coverage information, but we have also generated an image which can be displayed for visualization purpose by appropriately defining the cobr map to reflect the number of satellites in view at each point.

Now begins the fun and games we can play with these data. Let I(t) denote the coverage image produced in the above fashion at the t-th time step. Define

total(S(I)) = total number of pixels in image I where the statement S(1) is true.

Then total(I(t) > O) provides the total number of pixels > 0 in the image i(t). This counts the number of points on the map which can see the satellite. Let N denote the total number of pixels of the map. Then, since we are using an equal area projection,

$$C(n) = 100x\ \mathrm{total}(I(t)>0)/N$$

gives the percentage of total coverage at time t.

Now suppose we wish to compute the total coverage over iand. Let LAND denote the image of the map obtained by setting the pixels over land to 1 and over sea to O. Similarly, we can define SEA to be the {0,1} matrix for the oceans. Let A*B denote pixel-wise multiplication, i.e. if we think of the images A and B as matrices, this means:

$$A^*B(i,j) = A(i,j) \bullet B(i,j).$$

Then, I(t)*LAND gives the coverage over land, and I(t)* SEA gives the coverage over sea. For the coverage over any particular region of the planet, simply create the (0,1 }- image, R, for the region (where R is 1 over the region and O otherwise). I(t)*R defines the coverage over the region defined by R.

To compute the total coverage from time 1 to n, compute

$$C = 100^*\mathrm{total}(\{I(1) + I(2) + \ldots I(t)\} > 0)/N$$

which defines the total percent of the planet covered by the constellation over time t. To compute the total coverage of the planet by 3 satellites simultaneously, compute

$$C3 = 100*\mathrm{total}(I(t) > 2)/N.$$

For a few satellites, these calculations are easily computed within a package such as IDL. But for larger constellations, custom codes are required, but they are easily implemented. Care must be taken with the image resolution and transformations. To perform the calculus above, all images must be of the same resolution. Resolution can be bst during projection transformations since they are "not reversible" from the pixel point of view.

3. SARCON: APPLICATION OF MEASURE PRE-serving MAPS TO CONSTELLATION DESIGN

This method can be apptied to instrument footprints of greater complexity than just the nadir looking limb described above. In this section we present an application to the bistatic SAR (synthetic aperture radar) defined by a double constellation. For this system, the first constellation emits the signals while the second constellation receives the signals. In general, the emitters are cheaper to build and the receivers are much more expensive. By dividing the two, one can provide 50 emitters and 4 receivers, for example, thereby creating an efficient and cost-effective SAR with global coverage. However, typical SARS frown to date have both emitter and receiver on the same satellite. in fact, the scatter geometry provided by this configuration is essential. When the path of the receiver satellite is orthogonal to the emitter satellite, the geometry for the SAR is very poor and the signals are greatly degraded and are useless.

We developed a rapid prototype, SARCON (SAR Constellation Analysis Tool), to analyze this system and to demonstrate the utility of the Visual Calculus. SARCON provided the first global visual verification of the performance of the bistatic SAR. Fig. 2 is an image of an instantaneous coverage from SARCON using a double constellation consisting of 4 satellites each. We refer to the panels numbered 1 through 4, starting from the bottom ieft going clockwise, Panel 1 provides the raw coverage map without SAR performance. Panel 2 provides the intersection of the footprints where SAR calculations are to be made. In this figure, the "timde data" are also plotted. Panel 4 provides the four SAR performance indices: Resolution, SNR, Area, and Time. Panel 3, in the right top corner, provides the image of the pass/fail index obtained from the sum of the indices of Panel 3. Here green means pass and red means fail.

As a rapid prototype, little care was used in the numerical and image processing algorithms in the development of SARCON. Aside from total coverage between user specified latitudes, no other analysis was provided in the current version which was mainly used as a first order proof of concept for the bistatic SAR. Once the coverage information is obtained, the statistical analysis outlined above can be used to obtain a great deal of information and metric on the performance of the system. A second generation tool using a distributed architecture with cluster computing is currently under development to provide just this kind of analytical capabilities. A multiresolution approach using wavelets to model the sphere is also being considered.

The performance of the bistatic SAR is extremely complicated to analyze. The SARCON visualization provided a quick, intuitive, and quantitative evaluation not only of the bistatic SAR, but of the entire system. The importance of visualization for the analysis of complex geometric problems cannot be over emphasized. The use of measure preserving projections can further leverage the visualization process to provide quantitative information such as in the bistatic SAR probiem.

4. CONCLUSIONS

This paper presented a new approache to coverage analysis which yields quantitative measures of the constellation performance. The use of a measure preserving map projection enabled the use of simple image processing techniques to compute the coverage of complex missions while providing the visualization at the same time. The SARCON prototype demonstrated the utility and power of this concept..

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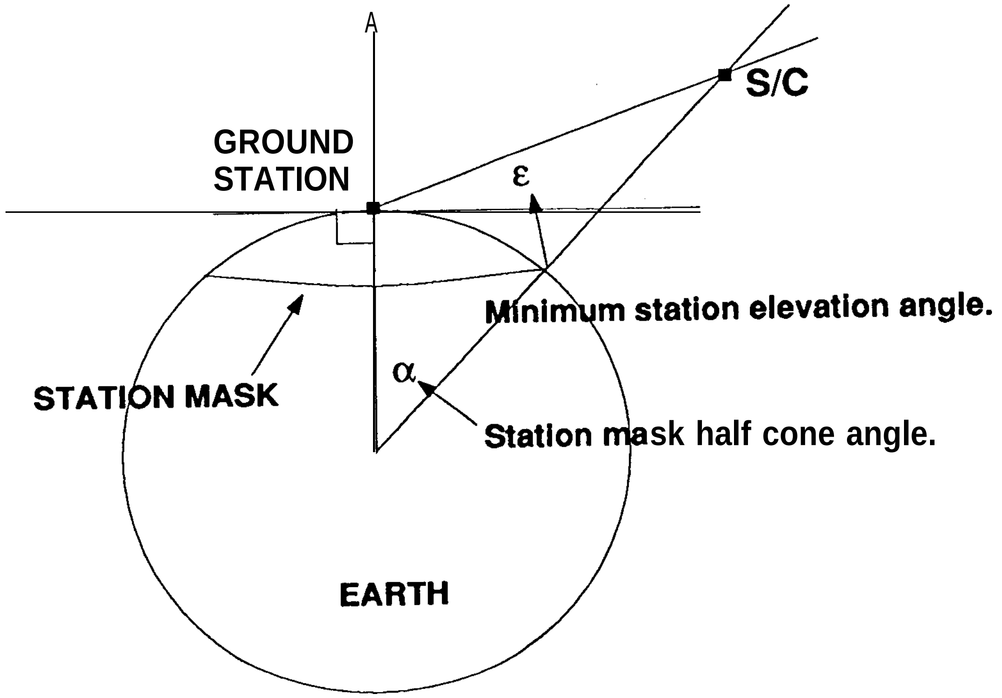

Figure 1. Ground Station to Satellite Visibility Geometry

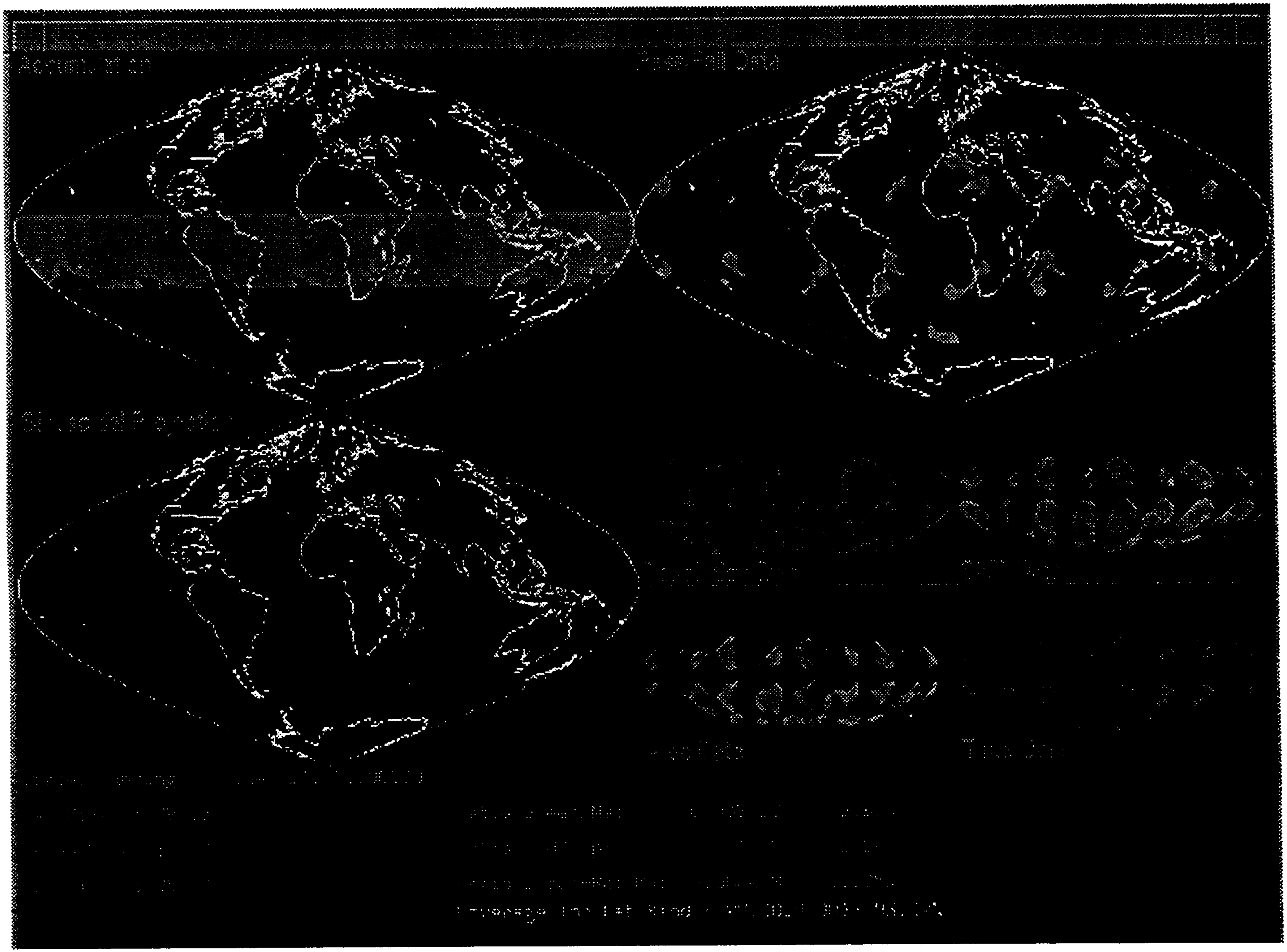

Figure 2. SARCON Coverage Images