Tracking Performance of the Soft Digital Data Transition Tracking Loop

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Abstract

This paper evaluates the steady-state tracking performance of the soft DTTL symbol synchronizer which is a low SNR approximation of the hyperbolic tangent, nonlinearity in the in-phase channel. The normalized s-curve, slope of the s-curve, and normalized noise density are derived and compared to the hard DTTL which is a high SNR approximation of the nonlinearity. Moreover, the symbol SNR region where the steady-state timing jitter of the soft DTTL outperforms that of the hard DTTL is determined.

(Technical Subject Area: Synchronization, and Satellite, and Space Communications)

‘Designated presenter should the paper be. accepted

1 Introduction

The symbol synchronizer is the heart of a digital communications system as it provides symbol timing to many essential components of a receiver. A commonly used symbol synchronizer is the digital data transition tracking loop (DTTL) [1-7] and it is used in various receivers such as the Advanced Receiver used by the Deep Space Network [8] and the TDRSS satellite receivers [9]. Using the maximum a posteriori (MAP) theory as motivation, the MAP DTTL structure consists of a hyperbolic tangent nonlinearity in the in-phase channel [10]. The DTTL analyzed in [1-7], however, is the special case where the hyperbolic tangent, nonlinearity in the in-phase channel is approximated by a hard limiter device. The hard limiter is a good approximation of the hyperbolic tangent at high signal-to-noise ratios (SNRs) and we will refer to this structure as the hard DTTL. At low SNRs, on the other hand. a good approximation to the nonlinearity in a linear device, and this structure will be referred to as the soft DTTL. This paper serves to evaluate the tracking performance of the soft DTTL. The salient information we seek to determine is the symbol SNR region where the steady-state timing jitter of the soft DTTL outperforms that of the hard DTTL. We are interested in the low symbol SNR region primarily due to the expected use of higher rate codes (1/4 and 1/6) [9] and Turbo codes with large interleaver size [10] in future space missions which, consequently, result in lower symbol SNRs.

The functional block diagram of the soft DTTL is shown in Fig. 1, and its operation is described below. The baseband input signal is first passed through two parallel channels: the in-phase channel (on top) monitors the weighted polarity of the actual transitions, and the quadrature channel (in the bottom) measures the timing error. Specifically, the in-phase channel accumulates over a symbol followed by a subtraction of two successive soft decisions at the output, of the transition detector. In contrast, for the hard DTTL , the output of the transition detector consists of only three possible numbers: O, 1, or -1. The quadrature channel, on the other hand, accumulates over the estimated symbol transition, and after an appropriate delay is multiplied to the in-phase channel out-put, I_k. The multiplication results in an error signal, e_k, that is proportional to the estimate of the timing error. Subsequently, e_k is normalized by the slope of the S-curve (which will be defined later), and then filtered with resulting output being used to control the timing generator.

Lets examine the output of the transition detector in the in-phase channel when no transition occurs for two successive symbols for both the soft and hard DTTL. In this case, the output of the transition detector should ideally be zero since no transition information exists in the quadrature channel. Nevertheless, with no transitions, the output of the soft DTTL transition detector in probability is non-zero. As such, this rnis-informs.tion results in greater symbol jitter as one would expect. In contrast, the hard DTTL makes a hard decision on each symbol which result in a symbol error rate (SER) equal to that of binary phase shift keying (BPSK): $P(E) = \frac{1}{2}\text{erfc}[\sqrt{R_s}]$ where R_s is the symbol SNR and erfc is the complementary error function. At low symbol SNR the BPSK SER is poor, and consequently, results in greater symbol jitter also as one would expect, Intuitively, a cross-over point should exist where the poor SER performance of the hard DTTL dominates the non-zero transition detector output of the soft DTTL. The goal of this report is to determine that cross-over point for the same loop parameters.

In the following sections, we determine 'the performance of the soft DTTL and the salient loop parameters are. then compared to the hard DTTL. In particular, section 2 illustrates the soft DTTL model, which is used in section 3 to derive the DTTL timing jitter. Afterwards in section 4, we conclude with the main points of the paper.

2 The soft DTTL Model

Consider the soft DTTL shown in Fig. 1 with a Nonreturn-to-zero (NRZ) signaling format. Assuming that the carrier and subcarrier (if any) have been removed in an ideal fashion, the received baseband waveform is given by

$$r(t) = \sqrt{S}\sum_{k} d_k p(t - kT - \epsilon) + n(t) \tag{1}$$

where S is the data power, T is the symbol time, $n(t)$ is white Gaussian noise with one-sided power spectral density NO W/Hz, ϵ is the random epoch to be estimated, p(t) is the square-wave function having a value of 1 for $O \leq t < T$ and having value O elsewhere, and d_k represents the k-th symbol polarity taking on values 1 and -1 equally likely. Let the phase error λ (in cycles) be defined as

$$\lambda = \frac{\epsilon - \widehat{\epsilon}}{T} \tag{2}$$

where ϵ is the received symbol phase and $\widehat{\epsilon}$ is the estimated symbol phase. It is clear that the error signal is affected by λ, and in order to quantify this effect, we define the following variables shown in Fig. 2: T is the symbol time; λT is the fraction of the timing error; $\xi_o T$ is the quadrature window; and $R_s = \frac{ST}{No}$ is the symbol SNR. The error signal, e_k, shown in Fig. 2 can now be written as follows

$$\begin{aligned} e_k = \frac{1}{2}\Big\{\sqrt{S}[(0.5\xi_o + \lambda)Td_{k+1} \\ + (0.5\xi_o - \lambda)Td_k] + V_2 + N_1 + N_2\Big\}\ x \\ \Big[\sqrt{S}[(1 - \lambda)Td_{k+1} + \lambda Td_{k+2}] \\ + N_2 + N_3 + V_1] \\ \Big[\sqrt{S}[(1 - \lambda)Td_k + \lambda Td_{k+1}] \\ + V_1 + \mathbf{V^*} + N_1] \qquad \lambda \geq 0 \end{aligned} \tag{3}$$

where V_1 and V_2 are the noise components in the k-th symbol; N_1, N_2, and N_3 are the noise components in the (k+1)-th symbol; and W_1 is the noise component in the (k+2)-th symbol as shown in Fig. 2, and they arc all independent of each other.

3 Soft DTTL Tracking Performance

one of the key performance measures of the soft DTTL is the steady-state timing jitter of $\lambda; \sigma_\lambda^2$. Using linear theory, σ_λ^2 can be derived once the following two quantities are determined: 1) the loop S-curve $g(\lambda)$ as a function of' the normalized timing λ and 2) the two-sided spectral density $S(w, \lambda)$ of the equivalent additive noise n_λ (t).

The normalized S-curve $g_n(\lambda)$ is defined as follows

$$g_n(\lambda) = \frac{E_{n,s}[e_k|\lambda]}{ST^2} \tag{4}$$

where $E_{n,s}[\cdot]$ represents expectation over the signal and noise. The exact closed form solution of $g_n(\lambda)$ can be shown to be

$$g_n(\lambda) = \lambda\left(1 - \frac{\xi_o}{4}\right) - \frac{3}{2}\lambda^2 \tag{5}$$

Compared to the hard DTTL [1,7], the S-curve of the soft DTTL is independent of the symbol SN R, as we would expect, since the soft DTTL does not have a nonlinearity in the in-phase arm[1]. Figure 3 shows the normalized S-curve for both the hard and soft DTTL for window size of one as a function of symbol SNR. The first derivative of the S-curve at λ=0, termed the slope of the S-curve, can be shown to be

$$K_g = g_n'(0) = 1 - \frac{\xi_o}{4} \tag{6}$$

Figure 4 shows the soft and hard DTTL slope for various window sizes as a function of symbol SNR. As expected, the slope for the soft DTTL is independent of symbol SNR.

Assuming that $B_L T << 1$, it is sufficient to approximate $S(w, A)$, the spectrum of the additive noise $n_\lambda(t)$, at zero frequency; that is, S(O, λ). The normalized noise spectrum can be defined as

$$h(\lambda) = \frac{S(0,\lambda)}{\frac{1}{4}\xi_o N_o T^2} \tag{7}$$

[1] In fact, the complementary error function in the hard DTTL S-curve results because of the hard limiter.

where $S(0,\lambda) = R(0,\lambda) + 2R(1,\lambda)$; and $R(0,\lambda) = E_{n,s}[e_k e_k]$ and $R(1,\lambda) = E_{n,s}[e_k e_{k+1}]$. In addition, we consider the DTTL only at loop SNR greater that 10 dB, so $h(\lambda)$ is essentially the noise spectral density seen by the loop at $\lambda=0$; that is, h(0). It can be shown that

$$h(0) = \left(1 - \frac{\xi_o}{2}\right) + \frac{1}{2R_s}\left(1 - \frac{\xi_o}{4}\right) \tag{8}$$

Figure 5 shows the normalized soft and hard DTTL noise spectrum as a function of symbol SNR for window size of 1 and 1/~. The noise spectrum of the hard DTTL is constant over the low SNR region while the soft DTTL varies significantly.

Assuming linear theory, $g_n(\lambda)$ can be approximated as $K_g\lambda$ and the variance of λ becomes [1]

$$\sigma_\lambda^2 = \frac{h(0)B_L T\xi_o}{2RsK_g^2} \tag{9}$$

where B_L is the noise-equivalent loop bandwidth defined as [13]

$$B_L = \frac{1}{2T}\frac{1}{H^2(1)}\frac{1}{2\pi j}\oint_{|z|=1} H(z)H(z^{-1})\frac{dz}{z} \tag{10}$$

where $H(z)$ is the closed loop transfer function. Clearly, for same loop bandwidth, window size, and symbol time the performance of the hard and soft DTTL can be compared by determining the squaring loss, S_L, as a function of symbol SNR defined as follows

$$S_L = \frac{h(0)}{K_g^2} \tag{11}$$

The squaring loss for the soft and hard DTTL is shown in Figure 6 as a function of symbol SNR. The soft and hard DTTL have approximately the same tracking performance at symbol SNR of 2,5 dB and -1 dB for window size of 1 and 1/4, respectively. Moreover, at symbol SNR of -10 dB, the soft DTTL has a factor of 2 improvement in timing jitter over the hard DTTL.

4 Conclusion

This paper evaluated the steady-state tracking performance of the soft DTTL symbol synchronizer which is a low SNR approximation of the hyperbolic tangent nonlinearity in the in-phase channel. The normalized S-curve, Slope of the S-curve, and normalized noise density were derived and compared to the hard DTTL. The soft and hard DTTL have approximately the same tracking performance at symbol SNR of 2.5 dB and -1 dB for window size of 1 and 1 /4, respectively. Moreover, at symbol SNR of -10 dB, the soft DTTL has a factor of 2 improvement, in timing jitter over the hard DTTL. The acquisition performance of the soft DTTL will be a focus of a future paper.

Acknowledgments

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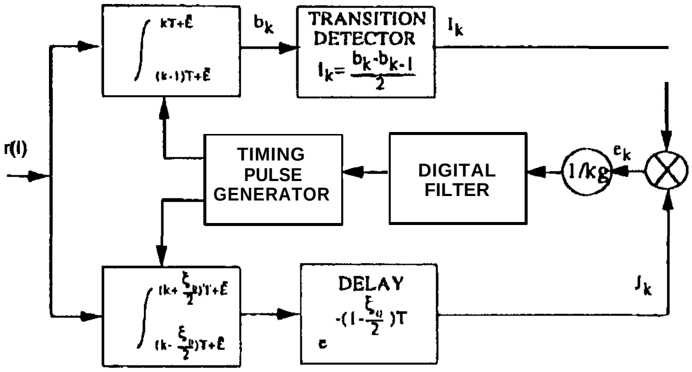


Fig. 1, Soft Digital Data Transition Tracking Loop (DTTL).

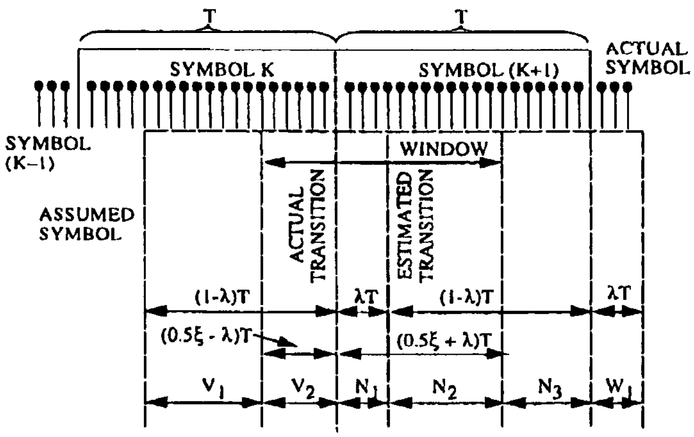
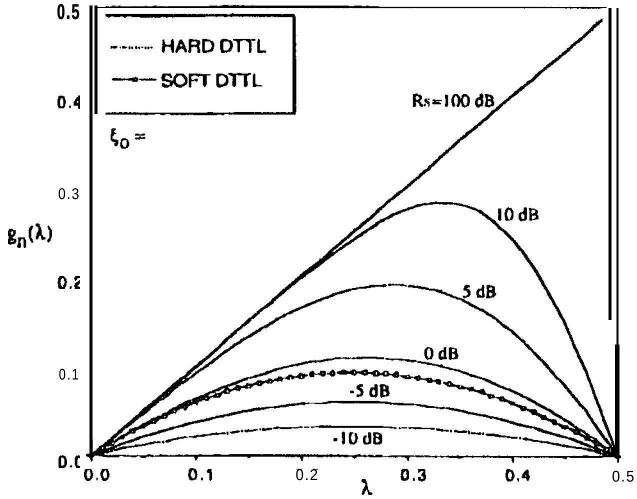


Fig. 2. Soft DTTL model



Fig, 3. Normalized S-curve for the Hard and Soft DTTL at&o = I

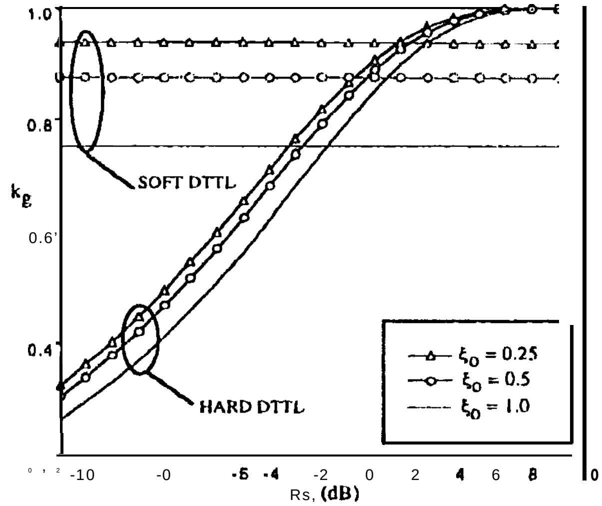


Fig. 4. Slope of the S-curve for the Hard and Soft DITL

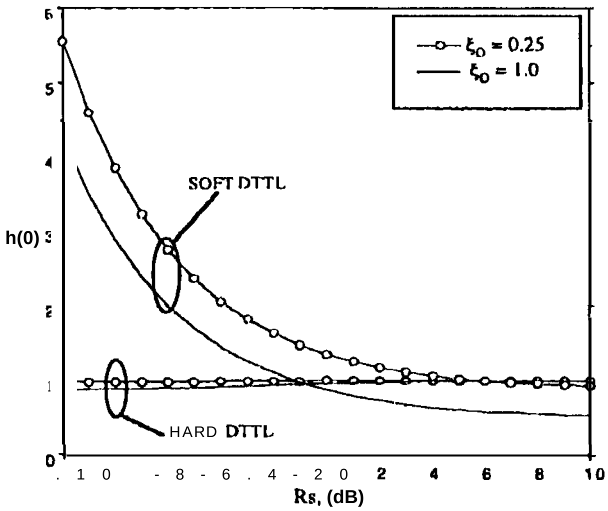


Fig. 5. Normalized Noise Density for Hard and Soft DTTL

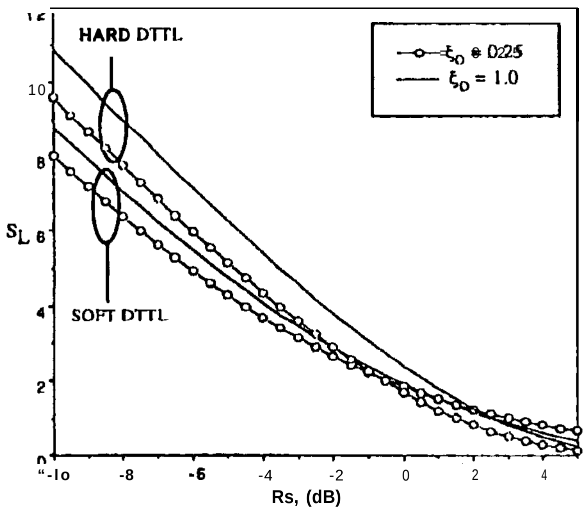


Fig. 6. Squaring Loss for the Hard and Soft DTTL