Effects of Non-Uniform Beam-Filling on Rainfall Retrieval for the TRMM Precipitation Radar

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Abstract

Introduction
be used, and a technique for obtaining such a measurement is the Surface Reference Technique (SRT), first proposed by Meneghini et al. (1983). In the SRT a radar measurement of the ocean surface in a clear area is compared with the measurement in the raining area. The difference in the measurements is assumed to be due only to the rainfall attenuation and is taken to be the two-way PIA. The SRT-measured PIA is then used as a constraint; the attenuation correction at the surface is made to equal that measured by the SRT. The SRT forms the basis of the TRMM PR algorithm (denoted 2A25); however, the TRMM algorithm also includes techniques for NUBF correction (described in part in Kozu and Iguchi 1996) and for handling light rain, where the relative error in the SRT-measured PIA can be large (Iguchi and Meneghini 1994). Marzoug and Amayenc (1994) have also considered algorithms similar to the the Hitchens-Bordan algorithm in combination with the SRT-measured PIA.

While details of the aforementioned algorithms vary, they are similar in principle. First, the SRT is used to estimate the PIA and develop corrections to the initial algorithm assumptions about the $k - R$ or $Z - R$ relations. Next, the retrieval process begins either at the top of the rain and works downward or at the surface and works upward. The measurement at each range bin is corrected for attenuation using the results from the previous range bins. All the algorithms were derived assuming that the rain within the resolution cell is uniform. When rainfall is not uniformly distributed, one would like to retrieve the rain rate averaged over the resolution cell. Obviously, this could be done using algorithms derived for the uniform case if the measured PIA and reflectivity profile were those corresponding to the profile of average rain rate within the beam. Unfortunately, because of the nonlinearity in the relations between reflectivity, attenuation, and rain rate, the measured reflectivity profile and PIA may deviate from those corresponding to the average rain rate. The method of Kozu and Iguchi (1996) uses the observed PIA and its variability to estimate the PIA that corresponds to the average rain rate.

To understand the effects of NUBF, we can thus compare not only the rain rate retrieved from a spaceborne system with the average rain rate but also the observed PIA and reflectivity profile with those corresponding to the average rain rate profile. To see how NUBF can effect the PIA and reflectivity profile observations, consider a two-dimensional slice of the atmosphere that has the width of a radar beam and the height $L$ of a typical storm (Figure 1). Assume that this region can be segmented into a uniform grid of boxes whose dimensions are small enough that the rain rate can be considered uniform within each box. As noted previously, when the rain rate is not distributed uniformly throughout the radar resolution volume, the quantity to be retrieved is the horizontally averaged rain rate, i.e., $\frac{1}{M} \sum_{j=1}^{M} R_{i,j}$. where $R_{i,j}$ is the rain rate within the $(i,j)^{th}$ box of the $M \times N$ grid $1 \leq j \leq M$, $1 \leq i \leq N$. The two-way PIA corresponding to the horizontally averaged rain rate is

$$A_t = 10^{-0.2 \log \left( \frac{1}{M} \sum_{j=1}^{M} R_{i,j} \right)} \quad (4)$$

where the subscript $t$ denotes “true”. However, if one divided the echo power received from the surface within this region by the power received from the surface under clear air (as done in the SRT), the apparent attenuation $A_a$ that one would obtain (assuming a rectangular antenna gain pattern) should be equal to

$$A_a = \frac{1}{M} \sum_{j=1}^{M} \left( 10^{-0.2 \log \left( \frac{1}{M} \sum_{i=1}^{N} a R_{i,j}^a \right)} \right) \quad (5)$$

where the subscript $a$ denotes “apparent”. Because of the non-linear dependence of attenuation on $R$, $A_t$ and $A_a$ are generally not equal. In a simple example in which the beam is half-filled with rain and half clear, Nakamura (1991) noted that the maximum observed PIA is 3 dB. However, the “true” PIA, which corresponds to the average rain rate, can be much larger. Thus, the PIA and the path-averaged rain rate (PARR) can be severely underestimated.

Next, we consider the effect of non-uniformity on the reflectivity. Near the top of the rain column, attenuation can be neglected, and the apparent reflectivity, as measured by the spaceborne radar, would be

$$Z_a = \frac{1}{M} \sum_{j=1}^{M} a R_{i,j}^a \quad (6)$$
3 Theory of NURB Effects

Thus, it is primarily constructive in which effects of NURB are expected to cause problems. When the NURB effect is significant and the connection effect is not, the ARMA model can be effectively used for estimation and description. This approach is based on the idea that the NURB effect is a significant component of the system's behavior. The ARMA model is used to describe the system's behavior under various conditions and the NURB effect is used to modify the model's predictions. The ARMA model is a powerful tool for analyzing and predicting the behavior of complex systems. It can be used to identify and quantify the effects of NURB on the system's performance and to develop strategies for improving the system's behavior.
rate to also be slightly overestimated. Figure 2 (a) shows the PIA error versus rain rate; it can be seen that the PIA is always underestimated and that the error can become very large at high rain rates. The PARR derived from the PIA would also be underestimated. The error in the near-surface reflectivity, using equations 15 and 16, can be either positive or negative, as shown in Figure 2 (b). In evaluating 16, it was assumed that the means of \( R \) and \( R_m \) are equal. When the error in Figure 2 (b) is negative (underestimation), the surface rain rate will be underestimated, since it is derived from correcting the near-surface reflectivity by the apparent PIA, which is also underestimated. When the reflectivity is overestimated, the rain rate may be either underestimated or overestimated, depending on which error dominates (overestimation of reflectivity or underestimation of PIA). It can be seen in Figure 2 (b) the underestimation of reflectivity at high rain rates is more severe in the case where the near-surface and path-averaged rain rates are independent, given by 16.

4 Data Analysis Technique

To quantify the effects of NUBF on real radar data we need both spaceborne radar data and corresponding high resolution data for comparison. Currently, such data are not directly available, so we take the approach of simulating the spaceborne data using aircraft radar data, as was done by Amayenc et al. (1996). We simulate TRMM PR observations over these data in two steps. First, the ARMAR data is resampled to a uniform Cartesian grid; this is necessary because of aircraft motion, which cases the raw data to have non-uniform sampling. The resampling is performed by dividing the atmosphere in 60 m thick horizontal slices. The locations of the closest ARMAR measurements, along with the corresponding reflectivity are found for each slice. Thus, for each slice we have a set of reflectivity measurements specified by their reflectivity and location on the slice. We then construct a Delaunay triangulation and use linear interpolation to create a set of uniformly sampled reflectivities over each slice. The combination of all the horizontal slices gives a reflectivity volume with 60 m vertical spacing and 200 m horizontal spacing. The second step of the simulation involves convolving a multi-dimensional Gaussian function with the resampled data. The Gaussian is chosen so that it approximates both the range resolution (due to matched filtering in the receiver) and the two-way antenna pattern. One version of the simulation software uses a three-dimensional (3-D) Gaussian to simulate the TRMM PR resolution volume in the range direction and both the along-track and cross-track dimensions. The other version uses a two-dimensional (2-D) Gaussian to simulate the PR range and along-track resolution. The overall simulation approach taken here is similar to the 2-D approach described in Amayenc et al. (1996), and the 3-D approach described in Testud et al. (1996).

In performing the convolution, there are several details to be considered. As discussed in Testud et al. (1996), a general simulation method must compute the attenuation to each range bin in the simulated spaceborne radar data. However, in our case ARMAR and the TRMM PR have the same geometry and frequency. Hence, the attenuation experienced by a simulated PR bin is essentially the same as the attenuation already experienced by the ARMAR measurement. Consequently, we do not need to explicitly include attenuation in the simulation procedure. The convolution is thus implemented directly on ARMAR reflectivity to produce the PR reflectivity. The TRMM reflectivity \( Z_{pr} \) is given by

\[
Z_{pr} = \frac{\int Z(r)W(r)\delta(r)dr}{\int W(r)\delta(r)dr}
\]  

(17)

where \( Z \) is the ARMAR reflectivity, \( W(r) \) is the gaussian resolution function, \( r \) is the 2-D or 3-D spatial location, and the integrals are computed over the 2-D or 3-D space corresponding to the PR resolution volume. The function \( \delta \) is an indicator function which takes on the value zero when ARMAR data is missing or invalid and unity when the ARMAR data is valid. This is useful in both the 2-D and 3-D codes when there are along-track gaps in the data due, for example, to a change in the radar mode. The indicator function is also needed in the 3-D code since ARMAR acquired data only over a relatively narrow \( \pm 20^\circ \) range of scan angles. At the surface the ARMAR swath is more than twice the size of the PR footprint. However, at higher altitudes, the simulated PR footprint in the 3-D software may partially lie in areas with no ARMAR measurements.

When using a 3-D gaussian, it must be remembered that the ARMAR data used for a single TRMM PR footprint was acquired over approximately a 10° variation in the ARMAR antenna scan direction. For
All the other extreme, if the surface rain rate and PRR are independent, the correct form for the apparent reflectivity is

$$\rho_{\text{H}}^{\text{PRR}} = \rho$$

The apparent reflectivity depends on the correlation between the surface rain rate and the PRR. For the case where PRR are perfectly correlated (i.e., $\rho_{\text{H}} = 0$ and $\rho_{\text{PRR}} = 1$), the equation is simplified to

$$\rho_{\text{H}}^{\text{PRR}} = \rho$$

Near the surface, the true reflectivity is

$$\rho_{\text{H}} = \rho$$

can then be written as

$$\rho_{\text{H}} = \rho$$

To perform a more quantitative analysis, the above equations are used to derive the apparent and true attenuation reflectivities, as defined by Eq. 9, to different.

As was the case for the PIA and multiattenuated reflectivity, horizontal variability of $\rho_{\text{H}}$ within the range bin

$$\rho_{\text{H}}^{\text{PRR}} = \rho_{\text{H}}$$

while the modeled reflectivity factor using the horizontally-averaged rain rate is given by

$$\rho_{\text{H}} = \rho$$

For rain near the surface, attention cannot be neglected. In this case, the apparent reflectivity factor

$$\rho_{\text{H}}^{\text{PRR}} = \rho_{\text{H}}$$

larger than $\rho_{\text{H}}$.

However, the "true" reflectivity factor, corresponding to the horizontally-averaged rain rate, is

$$\rho_{\text{H}}^{\text{PRR}} = \rho_{\text{H}}$$
rate is 21 mm/h. Figure 4 (b) shows the error (apparent minus true) in the PIA, near surface reflectivity, and 4 km reflectivity. The largest error is in the PIA, which is underestimated by nearly 4 dB in the convective cell. This is the primary source of error in the surface rain rate estimate. The reflectivity near the surface is slightly overestimated (0.4 dB), while the 4 km altitude reflectivity is overestimated by 1.7 dB.

Although studies of individual cases are useful, we choose, rather, to focus on statistics of NUBF biases over TOGA COARE, shown in Figure 5 and summarized in Table 2. Figure 5 (a) shows histograms of the errors in the radar observables, while Figure 5 (b) shows histograms of the rain rate errors. The number of occurrences for each bin is plotted on a base-10 logarithm scale, since there a large number of small errors and only a small number of large errors. The total number of simulated PR footprints was 1779. The PIA error is $\left(\frac{A_{PR}}{A_{R}}\right)$, while the PARR error is the apparent rain rate minus the true rain rate. The apparent quantities are always smaller than the true; i.e., a spaceborne radar would always underestimate PIA and PARR. The maximum error is 12 dB for the PIA and 21 mm/h for the PARR. If we compute the average rain rate over all 1779 footprints, we find that the apparent average is underestimated by 4.6%. The reflectivity and rain rate at 4 km altitude are always slightly overestimated. The maximum overestimations are 5.5 dB and 2.3 mm/h. In most cases, the overestimation is small; the average is overestimated by only 3.2%. The near-surface reflectivity errors are both negative and positive, extending from -12.2 dB to +10.4 dB; the mean is zero. The near-surface rain rate errors are more negative than positive and there are a few cases with errors as large as -79 mm/h. The maximum positive error is much smaller at approximately 18 mm/h. The average rain rate at the surface, computed over all 1779 footprints, is underestimated by 11%.

The histograms in Figure 5 show a large variability in the measurement errors. Presumably this is related to the variability of rainfall within the spaceborne radar's footprint. Figure 6 shows scatter plots of the measurement errors versus the standard deviation of the high resolution measurements within the footprint. In Figure 6 (a) the error in PIA is correlated with the standard deviation of the PIA with the PR footprint. Largest errors occur when the PIA variability is large. There is, however, significant scatter with large standard deviations sometimes producing fairly small errors, indicating that the error depends on other factors besides the standard deviation. One such factor is the mean PIA; in cases with large error, the mean PIA is also typically large. In Figure 6 (b) the error in the 4 km altitude reflectivity is correlated with the standard deviation of the high resolution 4 km altitude reflectivity. The near-surface reflectivity error (not shown) was found to be poorly correlated with both the PIA and near-surface reflectivity standard deviations.

6 Discussion

The observations presented in the previous section showed that the PIA and PARR are always underestimated, while the reflectivity and rain rate at 4 km altitude are always overestimated. This was also found in the model calculations presented in Section 3. The model calculations showed that for 50% variability in rainrate, the PIA could be underestimated by 7 dB at 50 mm/h and 25 dB at 100 mm/h. The largest observed PIA error is 12 dB, which is within the range predicted by the model for heavy rain. In contrast, the observed maximum error in rain-top reflectivity was 5.5 dB, as compared with the model calculation of 0.4 dB for a rain rate standard deviation of 50%. One reason that the predicted and observed errors differ more for rain-top reflectivity than PIA is that the PIA error is related to the variability of the PARR, while the reflectivity error is related to the variability of the rain-top rate. The variability of rain rate within individual layers can be greater than that for the PARR, as can be seen by considering rain with $N$ layers, all with mean rain rate $\mu$ and standard deviation $\sigma$. If the layers are perfectly correlated, we can consider the rain to be acting as one layer with mean $\mu$ and standard deviation $\sigma$. On the other hand, if all layers are independent, the PARR still has mean equal to $\mu$ but standard deviation $\sigma / \sqrt{N}$. Thus, if the rain is behaving as several independent layers, for example, due to vertical shear of the horizontal wind, the standard deviation of the PARR would be lower than that of the rain rate in the individual layers. Considering the data in Figure 6, the maximum relative standard deviation in PIA is about 125%. In contrast, the largest reflectivity standard deviation is about 400%.

Table 2 and Figure 5 (e) showed that the near-surface reflectivity can be under- or overestimated, with both types of error occurring frequently in the data. This is in agreement qualitatively with the model
5 OBSERVATIONS OF NURF EFFECTS

All SRT-based algorithms and to allow physical interpretation of the results.

observe the Nyquist-Sampling theorem to ensure that the data is sampled at least twice the frequency of the highest frequency component of the signal.

The SRT algorithm is used to estimate the power spectrum of a signal. The algorithm involves discretizing the input signal and applying the SRT to each segment of the signal.

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those corresponding to the horizontally averaged rain rate. We found that relative to the desired values, the PIA is always underestimated, the rain-top reflectivity is always overestimated, and the near-surface reflectivity is underestimated roughly as often as it is overestimated. The largest errors in magnitude occur for the near-surface rain rate; over TOGA COARE the average is underestimated by 11%. We found that the largest contributor to the rain rate error near the surface is the PIA error. A simple correction scheme for the PIA was found to reduce the bias.

Acknowledgment

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Conclusions

After reviewing the literature, it is evident that the low-resolution and high-resolution deflection are correlated. However, the magnitude of the correlation is not as strong as expected. This suggests that the low-resolution method may be a reasonable alternative for estimating the deflection.

application of this equation to estimate the deflection of a single Toga Coarse reduced the bias in this paper:

\[ A = A_0 + 0.1 A_0 + A_0 + 0.02 A_0 \]

The effect of the deflection on the accuracy of the results is shown in Figure 5, where the deflection correction factors range from 1% to 2%. This confirms that the PIA application of this equation to estimate the deflection of a single Toga Coarse reduced the bias in this paper.

The results of the experiment show that the accuracy of the deflection is improved when using the low-resolution method. However, the correlation between the low-resolution and high-resolution deflection is not as strong as expected. This suggests that the low-resolution method may be a reasonable alternative for estimating the deflection.

The conclusions of this paper are based on the data collected and analyzed. Further research is needed to confirm these findings.


References
Figure Captions

1. Two-dimensional spaceborne radar beam as it intersects rain. The vertical axis is altitude and the horizontal axis is distance across the radar footprint.

2. Calculated NUBF errors (apparent minus true) for a lognormal rain rate. The error is shown as a function of the mean rain rate; the standard deviation of the rain rate within the footprint is equal to the mean rain rate. (a) PIA, and (b) near surface reflectivity. Solid line is perfect correlation, dashed line is independence.

3. ARMAR data from TOGA COARE, acquired January 18, 1993. Vertical axis is altitude (8km) and horizontal is along track distance (40 km). Reflectivity ranges from 10 dBZ (black) to 50 dBZ (white). White horizontal line is return from ocean surface. Upper image is original high resolution data; lower is result of TRMM PR simulation. Alignment between upper and lower is approximate.

4. Plots of quantities computed for the case in Figure 3. (a) True rain rate (solid), apparent rain rate (dashed). (b) Errors in PIA (solid), rain-rop reflectivity (dashed), and near-surface reflectivity (dotted).

5. Histogram of NUBF errors over the TOGA COARE experiment. Errors are always apparent relative to true; positive errors indicate that use of the apparent quantities causes overestimation relative to the true quantities. (a) PIA (solid), reflectivity at 4 km altitude (dashed), reflectivity near surface (dotted). (b) PARR (solid), rain rate at 4 km altitude (dashed), and rain rate near surface (dotted).

6. (a) Scatter plot of the PIA error versus the standard deviation of the high resolution path attenuation within the footprint. (b) Scatter plot of the error in reflectivity at 4 km altitude versus standard deviation of high resolution reflectivity at 4 km within the footprint.

7. Scatter plot showing relation between low-resolution observables and high resolution standard deviation.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near-surface R (mm/h)</td>
<td>7.8</td>
<td>9.6</td>
<td>4.9</td>
<td>0.8</td>
</tr>
<tr>
<td>Near-surface Z (dBZ)</td>
<td>1.2</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Rain-drop R (mm/h)</td>
<td>2.3</td>
<td>1.4</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Rain-drop Z (dBZ)</td>
<td>5.0</td>
<td>4.9</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>PRF (GHz)</td>
<td>0.0</td>
<td>1.7</td>
<td>0.0</td>
<td>0.0</td>
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Table 1: Statistics of NUBR Errors (Apparent Minus True)

<table>
<thead>
<tr>
<th>Samples</th>
<th>Noise floor (at surface)</th>
<th>Range resolution</th>
<th>Surface horizontal resolution</th>
<th>Scan angle</th>
<th>Scanning swath</th>
<th>Frequency</th>
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<tbody>
<tr>
<td>&lt; 100</td>
<td>23 DBZ</td>
<td>100 m</td>
<td>800 m</td>
<td>90°</td>
<td>220 km</td>
<td>13.8 GHz</td>
</tr>
<tr>
<td>900</td>
<td>17°</td>
<td>200 m</td>
<td>400 m</td>
<td>80°</td>
<td>220 km</td>
<td>13.8 GHz</td>
</tr>
</tbody>
</table>

Table 2: Comparison of ARMAR and TMMR PR Parameters
Z ESTIMATION ERROR

EFFECT OF Z VARIABILITY WITHIN CELL