

Close Encounters of the Symplectic Kind: Integration Methods and Chaotic Dynamics.

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Abstract: We have developed a new symplectic close approach scheme which embodies the perturbation picture underlying the Wisdom-Holman method, but which employs a different integrable force model for each time step. This force model is intrinsically more accurate – namely, a Kepler potential for the nearby planet and a uniform force representing all other objects over one time step. The constant background force is a feature that the Wisdom-Holman technique does not include, hence a fundamentally more accurate symplectic splitting method is possible. The integrability feature which this methodology exploits derives from the Stark effect, in which a Coulomb force embedded in a uniform force field has three integrals of motion or, in the quantum mechanical case, three good quantum numbers. Like the Kepler problem, the Stark problem solution can be expressed exactly in terms of incomplete elliptic integrals. We can use the Baker-Campbell-Hausdorff Theorem to show that the Hamiltonian underlying the new method is intrinsically closer, in some sense, to the true Hamiltonian than is the equivalent Wisdom-Holman Hamiltonian. We compare the performance of the two close approach methodologies, and present some exactly soluble test cases.

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