A New Method of Determining Orbit Lifetime Probabilities for use in Planetary Protection Analysis

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In order to fulfill the planetary protection requirements for the Mars Global Surveyor (MGS) mission a capability to predict the probability of various orbital lifetimes was needed. This pap-er presents a new method of solving the inherent problem of modeling the long-term behavior of the Martian atmosphere. The simple case of one solar cycle will be discussed before tackling the more complicated approach to the many n-year solar cycles situation. The former has the benefit of establishing the relationships between solar flux, atmospheric density and orbital lifetime requirements.

The nominal solar flux incident upon Mars has been previously modeled as a combination of several sinusoidal functions. The basis of this study is the statistical nature of the variations about the nominal behavior of the dominant Ii-year term. Prediction curves indicate the larger variations from nominal for this term are at the solar maximums. However, due to the structure of the software used in this study, the variation was assumed to be a constant (higher) offset throughout the entire Ii-year cycle. Note that this adds some small amount of conservatism to the study since the offset densities used at solar minimums were thus higher than their expected values.

The offset over a cycle is assumed to be a normally distributed variable. Ifs is defined to be the number of sigmas that the solar flux is away from its nominal value, the corresponding change in the atmospheric density is:

$$\rho/\rho_m = 10^{0.3s} \text{ or } \log(\rho / \rho_m) = 0.3\, s$$

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where ρ_m is the density profile when the solar flux follows its nominal pattern. The origin of this equation will be discussed in the paper.

The sequence of using the probability distribution of one solar cycle (the "uninormal method") for multiple cycles or the product of probabilities (the "binomial method") will be described. The excessive conservatism in using the common binomial method will be explained. The following gives a brief summary of this.

In the binomial method a probability (P) of the density being less than a certain value is picked such that integrating the orbit over n cycles at this value does not cause a crash and the product P" is equal to the probability requirement. Mathematically this can be represented by considering the probability of a variable α being either less than (or equal) to a value y or greater than y. The sum of these two possibilities is, of course, one. Further this holds for multiple trials. Specifically:

$$[P(\alpha<y) + P(\alpha>y)]^n = 1$$

Doing the normal binomial expansion yields:

$$P(\alpha<y)^n + n\,P(\alpha<y)^{n-1} P(\alpha>y) + \ldots\ldots + P(\alpha>y)^n = 1$$

where the first term represents the case where a is always less than y which we just used above and the second term contains some of the desired cases which have been omitted. For example, one cycle with *a* just above y and three cycles with very small α. However, for the lifetime analysis, these terms can not be included directly in this form because the term $P(\alpha<y)$ would imply running the density at infinity for one cycle.

This lead to the idea of using the trinomial method. In this case there are two parameters y and z, such that a can be less than (or equal) to y, greater than y but less than (or equal) z or greater than z. Expanding:

$$[P(\alpha<y) + P(y<\alpha<z) + P(\alpha>z)]^n = 1$$

gives, for n=4,

$$P(\alpha<y)^4 + 4P(\alpha<y)^3 P(y<\alpha<z) +$$

$$4P(\alpha<y)^3 P(\alpha>z) + 6P(\alpha<y)^2 P(y<\alpha<z)^2 + 12\,P(\alpha<y)^2 P(y<\alpha<z) P(\alpha>z)$$

$$\ldots + 4P(\alpha>z)^3 P(y<\alpha<z) + P(\alpha>z)^4 = 1$$

If the term $P(\alpha<y)$ represents a low density case (L) and $P(y<\alpha<z)$ represents a mid-density case (M) then the first term of the expansion represents four cycles of L and the second term the four cases of one M and three L's. The basic trinomial method sets the sum of these two terms equal to the desired probability. This gives the relationship between y and z so a single parameter search can be done to maximize the lifetime for a given starting altitude. Preliminary analysis showed that there was little difference between having the M cycle first followed by 3 L cycles compared to having the M cycle later.

As explained in the paper, the extended trinomial method includes portions of the remaining terms and thus precludes even more of the unneeded conservatism. It was used to determine the orbit to which the MGS satellite will be raised at the end of its mission (427 km) to insure satisfaction of the stricter Planetary Protection 50-year requirement, while the standard trinomial method sufficed for the less stringent 20-year requirement. The results demonstrated that these methods had to be used since the higher orbits determined by traditional methods incurred impossible requirements on the mission's propellant budget. For comparison, using the extended trinomial method gave a 95.7% chance of success from the chosen orbit while the binomial method predicted only a 20.4% probability. Besides all future Mars orbiters, this method has a very general application for worst-case analysis of multiple independent events.