ERROR ESTIMATION AND h-ADAPTIVITY FOR OPTIMAL FINITE ELEMENT ANALYSIS

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INTRODUCTION

The objective of adaptive meshing and automatic error control in finite element analysis is to eliminate the need for the application engineer from re-meshing and re-running design simulations to verify numerical accuracy. The user should only need to enter the component geometry and a coarse finite element mesh. The software will then autonomously and adaptively refine this mesh where needed, reducing the error in the fields to a user prescribed value. The ideal end result of the simulation is a measurable quantity (e.g. scattered field, input impedance), calculated to a prescribed error, in less time and less machine memory than if the user applied typical uniform mesh refinement by hand. It would also allow for the simulation of larger objects since an optimal mesh is created.

The adaptive software package develops a set of refined meshes based on estimates of the error given as an initial solution over a very coarse mesh. These error estimates follow from the Cauchy convergence properties of finite element mathematical analysis and only require a small percentage (a few percent) of the time needed to solve the complete system over the previous steps mesh. Based on the error estimate generated, the mesh is adaptive refined—independent of the user—where needed and the fields are calculated over the new mesh by solution of the sparse matrix equation. This step is repeated until the error in the fields over the mesh is uniform and reduced below a user prescribed level. In general the adaptivity is divided into h-refinement—the breaking of elements into smaller elements as needed, p-enrichment—increasing the order of the element but keeping its size the same, and h-p adaptivity—the combination of the above two techniques.

ERROR ESTIMATION

Error estimates have been well developed in finite element structural and fluid analysis [1]. Estimates of error in the magnetic or electric field have been less developed in electromagnetic analysis. This work follows from the error estimates in [2] where p-enrichment was implemented. The error estimate is a local estimate of the error in the fields over each element. The estimate is computed element by element rather than over the entire mesh for computational efficiency. By defining u to be the exact solution to the wave equation (either E or H), and $\hat{u}$ to be the computed solution over a given mesh the estimate can be derived. The error in the field for a given solution is then $e = u - \hat{u}$. The derivation of the estimate e follows that of standard finite element analysis [1]. The result is an equation that estimates the error that would exist over the next refined mesh, based on the global calculated error over the coarse mesh. The equation for this error is

$$B_k(e,t) = L_k(t) - B(\hat{u},t) + \frac{1}{2}\int_{\partial V_k / \partial V} \hat{n}_k \bullet (a_k \nabla \hat{u}_k - a_j \nabla \hat{u}_j) t ds \qquad (1)$$

where

$$B_k(e,t) = \int_k (a \nabla e \bullet \nabla t - k_0^2 bet) dV \qquad (2)$$

and the surface integral is along edges of adjoining elements *k* and *j*, and the term $L_k(t)$ is a surface integral at the computational boundary where a radiation boundary condition is imposed. Equation (1) is an equation for e over the k^{th} element, given known right-hand-side terms found from a coarse mesh solution. The testing functions t are the fine mesh elements, thereby producing a solution for an estimate of the error in the coarse grid projected onto a refined mesh. If this error is above a tolerance value, the element will be refined; if the error is below the given tolerance the element need not be refined. This refinement decision based on the error estimate is the adaptive h-refinement process.

ADAPTIVE MESHING

The h-refinement procedure consists of two main components, an adaptive mesh adjustment step, and an adaptive mesh refinement step. The input to our adaptive mesh refinement procedure is a coarse mesh, an array containing local error estimates for the coarse mesh, and a prescribed error tolerance. The adaptive mesh adjustment step implements a scheme that, given a mesh with a subset of elements indicated to refined, generates information necessary for producing a consistently-refined global mesh. The adaptive mesh refinement step performs the actual refinement operations on the input mesh based on the refinement information stored in each coarse element.

We have currently implemented the adaptive mesh refinement algorithm for triangular meshes. The software was developed in Fortran 90 using many of its advanced features such as modules, user-defined structures, pointers, dynamic memory management and array operations. The software is therefore highly modular and robust. It has interfaces to both Fortran 77 and Fortran 90. The adaptive mesh refinement code has been tested on several triangular finite element meshes, given an error estimate for each element. Figure 1 (top) shows the coarse mesh with an initial set of elements to be refined based on local error estimates. The mesh is colored by the error estimate, and those elements with circles drawn have error estimates above the threshold and need refining. Figure 1 (bottom) shows the adaptively refined mesh produced by our mesh refinement code.

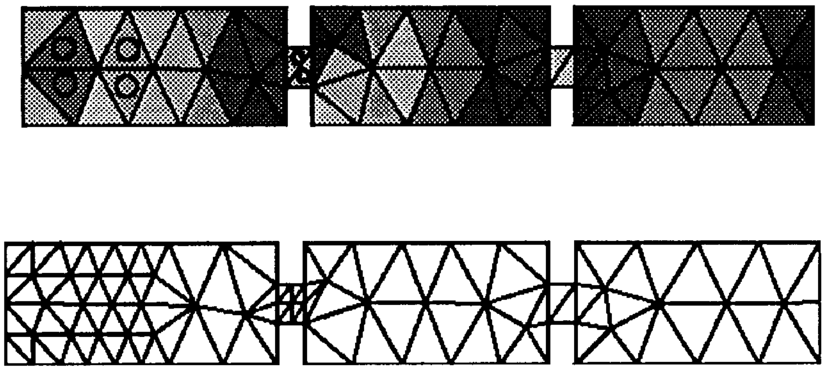

Figure 1. Example adaptive mesh refinement. Top) Error estimate over filter section. Circles indicate elements that are above error threshold. Bottom) Adaptive refined mesh.

DISCUSSION

This talk will expand on these issues, presenting further results on the accuracy of the error estimate and on convergence rates of the h-refinement adaptive procedure. A class of two-dimensional radiating and scattering examples will be shown.

REFERENCES

[1] M. Ainsworth and T. Oden, "A procedure for a posteriori error estimation for h-p finite element methods," *Comp. Meths. in Applied Mechs. and Eng.,* pp. 73-96, Vol 101, 1992.

[2] F. Meyer and D. Davidson, "Adaptive-mesh refinement of finite element solutions for two-dimensional electromagnetic problems," *IEEE Antennas and Prop. Magazine,* pp. 77-83, Vol. 37, No. 5, Oct. 1996.