A KALMANFILTER-BASED APPROACH ‘1’() COMBINING INDEPENDENT EARTH ORIENTATION SERIES

by


Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

*Now at:
US Naval Observatory
Washington, DC

Wednesday, January 8, 1997

To be submitted to Journal of Geodesy

(Short title: COMBINING EARTH ORIENTATION SERIES WITH A KALMAN FILTER)
ABSTRACT

An approach, based upon the use of a Kalman filter, that is currently employed at the Jet Propulsion Laboratory (JPL) in combining independent measurements of the Earth's orientation is presented. Since changes in the Earth's orientation can be described as being forced by a randomly excited stochastic process, the uncertainty in our knowledge of the Earth orientation parameters grows rapidly in the absence of measurements. The Kalman filter methodology allows for an objective accounting of this growth in the uncertainty between measurements of the Earth orientation parameters, thereby facilitating the intercomparison of measurements taken at different epochs (not necessarily uniformly spaced in time) and with different precision. The specific design of the JPL Kalman filter for Earth orientation was dictated to a large extent by the presence of degenerate data types. Lunar laser ranging observations from a single station, or VLBI observations from a single baseline, are only able to determine two independent components of the Earth's orientation. Since the elements of the covariance matrix associated with the degenerate component cannot be properly defined numerically, the Kalman filter was designed to use the information matrix (the inverse of the covariance matrix). Prior to combining the independent measurements of the Earth orientation parameters, corrections to their bias, rate, and stated uncertainty are determined and applied. In order to avoid subjectively choosing a reference series, these corrections are determined in an iterative fashion wherein each series is compared to a combination of all others. The corrected series are then combined and placed within a specific IERS reference frame. As an example of this approach to combining Earth orientation series, a description is given of a combination that has been generated recently at JPL, namely, SPACE95.

INTRODUCTION

A new field in the geophysical sciences has recently emerged, namely, that of space geodesy. An integral part of geodesy has always been the definition and realization of a terrestrial,
body-fixed reference frame, a celestial, space-fixed reference frame, and the determination of the Earth orientation parameters (precession, nutation, spin, and polar motion) that link these two reference frames together. But with the advent of space geodesy— with the placement of laser retro-reflectors on the Moon by Apollo astronauts and Soviet landers, the launch of the Laser Geodynamics Satellite (LAGEOS), and the development of Very Long Baseline Interferometry and the Global Positioning System—a quantum leap has been taken in our ability to realize the terrestrial and celestial reference frames and to determine the Earth orientation parameters.

Each of the modern, space-geodetic techniques of Lunar Laser Ranging (LLR), Satellite Laser Ranging (SLR), Very Long Baseline Interferometry (VLBI), and the Global Positioning System (GPS) is able to determine the Earth orientation parameters. But each technique has its own unique strengths and weaknesses in this regard. Not only is each technique sensitive to a different subset and/or linear combination of the Earth orientation parameters, but the averaging time for their determination is different, as is the interval between observations and the precision with which they can be determined. By combining the individual Earth orientation series determined by each technique, a series of the Earth’s orientation can be obtained that is based upon independent measurements and that spans the greatest possible time interval. Such a combined Earth orientation series is useful for a variety of purposes, including a variety of scientific studies, and as an a priori series for use in data reduction procedures. However, care must be taken in generating such a combined series in order to account for differences in the underlying reference frames within which each individual series is determined (which can lead to differences in bias and rate between the Earth orientation series), as well as to properly assign relative weights to each observation prior to combination. The issues and concerns surrounding the combination of Earth orientation series is the subject of this report [see also Gross, 1996a]. As a framework in which to discuss these issues and concerns, a description will be given of the determination of a particular combination, SPACE95 [Gross, 1996b], that has been generated recently at the Jet Propulsion Laboratory (JPL) from the space-geodetic Earth orientation series listed in Table 1. Other approaches to combining Earth orientation series have been discussed by Vicente and Wilson.
Satellite Laser Ranging

In the technique of satellite laser ranging, the round trip time-of-flight of laser light pulses are accurately measured as they are emitted from a laser system located at some ground-based observing station, travel through the Earth's atmosphere to some artificial satellite orbiting the Earth, are reflected by retro-reflectors tamed onboard that satellite, and return to the same observing station from which they were emitted [e.g., Lambeck, 1988, chap. 6]. This time-of-flight range measurement is converted into a distance measurement by using the speed of light and correcting for a variety of known or modeled effects such as atmospheric path delay and satellite center-of-mass offset. Although a number of satellites carry retro-reflectors for tracking and navigation purposes, the LAGEOS satellite is most commonly used for the determination of the Earth orientation parameters since it was specifically designed and launched to study this and other geodetic properties of the Earth [e.g., Christodoulidis et al., 1985; Cohen and Smith, 1985].

The Earth orientation parameters are recovered from the basic range measurements in the course of determining the satellite's orbit. The basic range measurement is sensitive to any geophysical process that changes the distance between the satellite and the observing station, such as displacements of the satellite due to perturbations of the Earth's gravitational field, motions of the observing station due to tidal displacements or plate tectonics, or a change in the orientation of the Earth (which changes the location of the observing station with respect to the satellite). These and other geophysical processes must be modeled when fitting the satellite's orbit to the range measurements as obtained at a number of globally distributed tracking stations. Adjustments to the a priori models used for these effects can then be obtained during the orbit determination.
procedure, thereby enabling, for example, the determination of the Earth orientation parameters [e.g., Smith et al., 1985, 1990, 1991, 1994; Tapley et al., 1985, 1993].

A number of organizations, including groups at Goddard Space Flight Center (GSFC), the University of Texas at Austin Center for Space Research (UTCSR), and the Delft University of Technology (DUT), currently determine the Earth orientation parameters from LAGEOS range measurements. Since each of these groups uses the same basic set of LAGEOS range measurements, their results are not completely independent of each other even though their subsequent data editing and processing procedures are different. For the purpose of combining Earth orientation series, it is desirable to combine only completely independent determinations of the Earth's orientation. Thus, in generating SPACE95, only one SLR dataset was used, namely, that determined at the University of Texas at Austin Center for Space Research.

The particular UTCSR SLR Earth orientation dataset used in generating SPACE95 is the series designated EOP(CSR) 95 L 01 [Eanes and Watkins, 1995]. This series consists of values for universal time (UT1) and the x- and y-components of polar motion (PMX and PMY, respectively) spanning May 19, 1976 to January 28, 1995. However, because the first few values of this series have larger than usual stated uncertainties and averaging intervals, only those values after October 2, 1976 were used in generating SPACE95. Also, only the polar motion values of this series have been used in generating SPACE95. The UT1 determinations were not incorporated into SPACE95 since their long period behavior has been constrained to that of the a priori series [Eanes and Watkins, 1995], and hence are not entirely based upon range measurements as are the PMX and PMY values. It is difficult to separate variations in UT1 from variations in the orbital node of the LAGEOS satellite due to the effect of unmodeled forces acting on the satellite [e.g., Lambeck, 1988, sec. 6.3.5], so that both these quantities cannot be simultaneously estimated without making additional assumptions. Solutions for these quantities are obtained by assuming that the effects of the unmodeled forces are such that they cause the orbital node to vary slowly, and that rapid variations therefore reflect UT1 behavior. Hence, the slow UT1 variations are not adjusted, but are constrained to those of the a priori series. The
Kalman filter used at JPL to combine Earth orientation series (see below) does not currently have the capability of separating the measured rapid UT1 variations from the constrained slow variations. Thus, to insure that only independent determinations are incorporated into the combined Earth orientation series, the UTC[SRS]-SLR UT1 values have not been used. The proper incorporation into the Kalman filter of the rapid SLR UT1 variations (but not the slow variations) is currently under study.

_Lunar Laser Ranging_

The technique of lunar laser ranging is similar to that of satellite laser ranging except that the laser retro-reflector is located on the Moon, rather than on some artificial satellite (e.g., Mulholland, 1980; Lambeck, 1988, chap. 71). Lunar laser ranging is technically more challenging than satellite laser ranging because of the need to detect the much weaker signal that is returned from the Moon than from the much closer artificial satellite. Larger, more powerful laser systems with more sophisticated signal detectors need to be employed in LLR, consequently, there are far fewer stations that range to the Moon than range to I AGI E OS. In fact, there are currently only two stations that regularly range to the Moon: McDonald Observatory in Texas and the CERGA station in Grasse, France.

The Earth orientation parameters are typically determined from lunar laser ranging by analyzing the residuals at each station after the lunar orbit (and other parameters such as station and reflector locations) has been fit to the range measurements from all the stations (e.g., Stolz et al., 1976; Langley et al., 1981; Dickey et al., 1985, 1994; Newhall et al., 1988; William et al., 1993; Whipple, 1993). From this single station technique, two linear combinations of UT1 and the polar motion parameters PMX and PMY can be determined, namely, UTO and the variation of latitude (VOL) at that station. A number of organizations, including groups at the University of Texas McDonald Observatory and at the Jet Propulsion Laboratory, currently determine UTO and the variation of latitude from lunar laser ranging. But all such determinations are based upon the same set of range measurements and are therefore not completely independent of each other, even though the subsequent data editing and processing procedures employed by each analysis center are
different. Since it is desirable to combine only independent Earth orientation results, only one I. LR solution has been included in SPACEI:95, namely, that determined at JPL.

The particular JPL I. LR series used in generating SPACEI:95 is an updated version of EOP(JPL) 95 M 01 [Newhall et al., 1995]. This series consists of values for 11'1'0 and the variation of latitude as determined from observations taken by the I. LR station at Haleakala Observatory in Hawaii (spanning February 10, 1985 to August 11, 1990), by the CIRGAS system in Grasse, France (spanning April 7, 1984 to December 14, 1995), and by the three I. LR stations located at McDonald Observatory in Texas (collectively spanning April 15, 1970 to December 28, 1995). However, in generating SPACEI:95, the I. LR U0 and VOI determinations made from observations taken prior to October 2, 1976 have not been used. Polar motion values are needed to convert the I. LR U0 measurements to UT1 [see equation (1) below]. Since regular SLR geodetic measurements are not available for all three Earth orientation components before the SLR series begins, and since only those SLR measurements since October 2, 1976 have been used in generating SPACEI:95 (see Table 1 and the above section on Satellite Laser Ranging), not all I. LR measurements made prior to this date have been used in SPACEI:95.

The two Earth orientation parameters U0 and VOI are related to 11’1’1 and the polar motion parameters PMX and PMY by the well-known expressions [e.g., Moritz and Mueller, 1988, p. 425]:

\[ \Delta \phi_i(t) = x_i(t) \cos \lambda_i - y_i(t) \sin \lambda_i \] \hspace{1cm} (1a)

\[ UTO_i(t) - TAI(t) = U(t) + x_i(t) \sin \lambda_i \tan \phi_i + y_i(t) \cos \lambda_i \tan \phi_i \] \hspace{1cm} (1b)

where \( \Delta \phi_i \) is the variation of latitude observed at the station \( i \) that is located at nominal latitude \( \#i \) and east longitude \( \lambda_i \); the observed UTO at that station is designated \( UTO_i(t) - TAI(t) \) where \( TAI(t) \) is a reference time scale based upon atomic clocks, the variable \( U(t) \) is defined by \( U(t) = UT1(t) - TAI(t) \), and \( x_i \) and \( y_i \) are the polar motion parameters PMX and PMY, respectively, with \( y_i \) being positive towards 90° W longitude. A third linear combination \( D_i(t) \) of
the UTPM parameters (PMX, PMY, UT1), representing that component of the Earth's orientation that cannot be determined from L.I.R observations at the single station \( i \), is given by:

\[ D_i(t) = U(t) \sin \phi_i - x_p(t) \sin \lambda_i \cos \phi_i - y_p(t) \cos \lambda_i \cos \phi_i. \]  \hspace{1cm} (1c)

This third, degenerate, component of the Earth's orientation represents changes in the orientation of the Earth resulting from a rotation of the Earth about an axis defined by the location of the station and the origin of the terrestrial reference frame (that is, about the position vector of the station).

In the Kalman filter it is necessary to be able to readily transform between the observed \( \text{UT0}, \text{VOL} \) values and the corresponding UTPM values. This is most conveniently accomplished by means of a transformation matrix. However, the system of equations (1) does not lend itself to this since the transformation matrix that results is not orthogonal. A proper (that is, orthogonal) transformation matrix is obtained upon defining a new parameter \( \text{UTF}_i \) related to the observed \( \text{UT0}_i(t) \cdot \text{TAI}(t) \) by:

\[ \text{UTF}_i(t) = \cos \phi_i \left[ \text{UT0}_i(t) - \text{TAI}(t) \right] \]  \hspace{1cm} (2)

The transformation between the UTPM parameters and the \( \text{VOL} \) parameters (\( \Delta \phi_i, \text{UTF}_i, D_i \)) can then be written in matrix form as:

\[
\begin{pmatrix}
\Delta \phi_i(t) \\
\text{UTF}_i(t) \\
D_i(t)
\end{pmatrix}
= 
\begin{pmatrix}
\cos \lambda_i & -\sin \lambda_i & 0 \\
\sin \lambda_i \sin \phi_i & \cos \lambda_i \sin \phi_i & \cos \phi_i \\
-\sin \lambda_i \cos \phi_i & \cos \lambda_i \cos \phi_i & \sin \phi_i
\end{pmatrix}
\begin{pmatrix}
x_p(t) \\
y_p(t) \\
U(t)
\end{pmatrix}
\]  \hspace{1cm} (3)

where it is easily shown that the resulting 3x3 transformation matrix is orthogonal. Unit vectors in the \( \Delta \phi, \text{UTF}, \) and \( D \) directions of Earth orientation parameter space form a triad of orthonormal base vectors that span this parameter space. Note that in the system of equations (1) or (3) the Earth orientation parameters are all assumed to have the same units. If the measured values \( \text{e.g., UT0}_i(t) \cdot \text{TAI}(t) \) of \( \Delta \phi_i(t) \) have different units, then they must be converted to the same units prior to application of the transformation represented by (3).
In the Kalman filter, besides transforming between the UTPM and VUD parameters themselves, it will also be necessary to transform their respective covariance matrices. This can be accomplished using the 3x3 transformation matrix defined by (3) once the stated uncertainties and correlations of the measurements have been converted from those appropriate for the measured $\Delta \phi$ and $\phi_{\text{UT0}} - \text{TAI}$ to those appropriate for $\Delta \phi$, and $\phi_{\text{UT0}} - \text{TAI}$. To accomplish this (once the units of the measured $\phi_{\text{UT0}} - \text{TAI}$ values and uncertainties have been changed to those of $\Delta \phi$), it is only necessary to convert the stated uncertainties of the measured $\phi_{\text{UT0}} - \text{TAI}$ to those appropriate for $\phi_{\text{UT0}}$ by multiplying them by the same scale factor used in (2) to convert $\phi_{\text{UT0}} - \text{TAI}$ to $\phi_{\text{UT0}}$, that is, by $\cos \phi$. The correlation between $\Delta \phi$ and $\phi_{\text{UT0}}$ is the same as that between $\Delta \phi$, and $\phi_{\text{UT0}} - \text{TAI}$ (correlations are unaffected by scale factor differences), and thus no conversion of the stated correlations need be made.

Very Long Baseline Interferometry

Radio interferometry is routinely used to make highly accurate measurements of changes in LTT1 and polar motion with observing sessions lasting from about an hour to a day. The VLB1 technique measures the difference in the arrival time of a radio signal at two or more radio telescopes that are simultaneously observing the same distant source (e.g., Shapiro, 1983; Lambeck, 1988, chap. 8). This technique is therefore sensitive to processes that change the relative position of the radio telescopes with respect to the source, such as a change in the orientation of the Earth in space or a change in the position of the telescopes due to, for example, tidal displacements or tectonic motions. If just two telescopes are observing the same sources, then only two components of the Earth’s orientation can be determined. A rotation of the Earth about an axis parallel to the baseline connecting the two radio telescopes does not change the relative position of the telescopes with respect to the sources, and hence this component of the Earth’s orientation is not determinable from VLB1 observations taken on that single baseline. Multibaseline VLB1 observations with satisfactory geometry can determine all three components of the Earth’s orientation.
A number of organizations, including groups at Goddard Space Flight Center, the Jet Propulsion Laboratory, the US Naval Observatory, and the Geodetic Institute of the University of Bonn currently determine Earth orientation parameters from VLBI measurements using their own independent data processing and reduction procedures. However, since many of the resulting Earth orientation series are based upon the same set of VLBI measurements they are not entirely independent of each other. Since it is desirable to combine only independently determined Earth orientation parameters, only those series described below have been incorporated into SPACE95.

**SGP/CDP** The VLBI group of the National Aeronautics and Space Administration (NASA) Space Geodesy Program (SGP: formerly the Crustal Dynamics Project (CDP)) at Goddard Space Flight Center conducts VI, BI observation campaigns in order to study the Earth's deformation and rotation [Clark et al., 1985, 1987; Ryan et al., 1986, 1993; Ma et al., 1993, 1994, 1995]. Using independent processing techniques they also reduce data taken under other observing programs, including the National Earth Orientation Service (NEOS) and the IRIS/POLARIS programs described below. The particular SGP/CDP Earth orientation series used in generating SPACE95 is designated by thomasGl.B973f [Ma, personal communication, 1995], spans August 4, 1979 to December 28, 1994, and consists of Earth orientation parameters determined from both single baseline and multibaseline VLBI observations.

**USNO.** As part of its participation in the National Earth Orientation Service [Eubanks et al., 1994], the US Naval Observatory (USNO) conducts VI, BI observing sessions in order to regularly monitor changes in the Earth's orientation. Series of Earth orientation parameters determined from these observations, as well as from observations conducted under other observing programs such as the SGP/CDP and 1)1S/1'01.ARIS (see below) programs, are produced by the US Naval Observatory using their own data editing and reduction procedures [Eubanks et al., 1994]. The particular USNO Earth orientation series incorporated into SPACE95 is their "n9604 cop" series dated February 15, 1996. However, since this USNO Earth orientation series is not entirely independent of the SGP/CDPGl.B973f series because they are both largely based upon the same set of VLBI measurements, only that portion of the USNO series that does not...
overlap with the SGP/CDP GLB973f series was incorporated into SPACE95, namely only that portion spanning December 29, 1994 to February 7, 1996.

**IRIS/POLARIS.** The POLar motion Analysis by Radio Interferometric Surveying (POLARIS) project was originally organized to monitor Earth rotation and orientation using VLBI observations within the United States [Carter, 1979; Carter and Strange, 1979; Carter et al., 1979, 1984; Robertson and Carter, 1982]. The POLARIS network was later expanded to include further European involvement in the International Radio Interferometric Surveying (IRIS) project [Carter and Robertson, 1984, 1986a, 1986b; Robertson and Carter, 1985; Carter et al., 1985, 1988; Robertson et al., 1988]. In addition to these 24-hour sessions, regular single baseline observing sessions of 1-hour duration, the “Intensive” sessions originally using the Westford (Massachusetts) and Wettzell (Germany) radio telescopes but currently using those at Green Bank (West Virginia) and Wettzell [Robertson et al., 1985], are conducted at quasi-daily intervals for the purpose of determining UT1 (the “Intensive” sessions are not currently conducted on Sundays, nor on the day of the 24-hour multibaseline NEOS session). Since the above SGP/CDP GLB973f and USNO “n9604.cop” Earth orientation series include values determined from the IRIS/POLARIS 24-hour VLBI measurements, they are automatically included in SPACE95 and no separate IRIS/POLARIS series needs to be incorporated. However, since neither the SGP/CDP GLB973f nor the USNO “n9604.cop” Earth orientation series include the “Intensive” [J1’1 values, they have been included in SPACE95 by incorporating series of them determined at both the USNO and the National Oceanic and Atmospheric Administration (NOAA). The particular NOAA “Intensive” UT1 series chosen for incorporation into SPACE95, F:OP(NOAA)95R02 [Ray et al., 1995], spans April 2, 1984 through December 31, 1994. In addition, that portion of the USNO “Intensive” UT1 series "n9604.cop.int" dated February 14, 1996 that does not overlap with the NOAA “Intensive” UT1 series, namely that portion spanning January 4, 1995 through February 10, 1996, was incorporated into SPACE95. Note that in principle the IRIS “Intensive” measurements, being derived from single baseline VLBI observations, should be treated as measurements of the transverse (T) and vertical (V) components of the Earth’s orientation (see
below) rather than as measurements of UT1. However, as released by both NOAA and the USNO, the IRIS “Intensive” measurements are given as measurements of UT1 without the necessary ancillary information (see below) that would allow them to be properly treated as measurements of T and V. Hence, in SPACE95 they have been treated as UT1 measurements.

**DSN The Deep Space Network (DSN) of NASA has conducted single baseline VLBI observations for the purpose of radio source catalog maintenance and enhancement (here called the CATM&E sessions) at irregular intervals since late 1978** [Fan et al., 1979; Sober et al., 1984; Steppe et al., 1995]. These sessions are nominally 24 hours in duration and are scheduled within a few days of each other on both the 8400 km-long baseline between the Goldstone (California) and Madrid (Spain) telescopes and the 10,700 km-long baseline between the Goldstone and Canberra (Australia) telescopes. Since July 1980, the DSN has also undertaken single baseline VLBI observations in support of spacecraft navigation [Eubanks et al., 1982; Steppe et al., 1995]. These semi-weekly Time and Earth Motion Precision Observations (TEIMPO) session generally last no more than 3 hours and are on the same baselines as are the CATM&E sessions. The particular DSN Earth orientation series chosen for inclusion in SPACE95, containing results determined from both the CATM&E and TEIMPO observing sessions, is a continuation of the series designated EOP(JPL) 95 R 01 [Steppe et al., 1995] and spans October 28, 1978 to January 9, 1996.

As mentioned above, only two components of the Earth’s orientation can be determined from VLBI observations taken on a particular single baseline. The Kalman filter must be able to transform between these two determinable components, each of which is a particular linear combination of the usual UTPM parameters, and the UTPM parameters themselves. In order to derive the transformation matrix needed to accomplish this, first consider a special coordinate system, the transverse, vertical, length system, that has been found to be useful in analyzing single baseline VLBI results [Eubanks and Steppe, 1988]. Figure 1 illustrates this coordinate system, the basis vectors of which lie along the three mutually orthogonal directions given by the baseline length, the baseline vertical, and the baseline transverse directions. Let \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) denote the
position vectors within some specified conventional terrestrial reference frame (with origin near the center of the Earth) of the two radio telescopes located at either end of the baseline:

\[ \mathbf{r}_1 = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k} \]  \hspace{1cm} (4a)

\[ \mathbf{r}_2 = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k} \]  \hspace{1cm} (4b)

where the hat (\(^\wedge\)) denotes a vector of unit length. Mutually orthogonal unit vectors \( \mathbf{\tau}, \mathbf{v}, \mathbf{b} \) in the baseline transverse, vertical, and length directions, respectively, can be defined from these position vectors by

\[ \mathbf{b} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{\| \mathbf{r}_2 - \mathbf{r}_1 \|} \]  \hspace{1cm} (5a)

\[ \mathbf{\tau} = \frac{\mathbf{r}_1 \times \mathbf{r}_2}{\| \mathbf{r}_1 \times \mathbf{r}_2 \|} \]  \hspace{1cm} (5b)

\[ \mathbf{v} = \mathbf{b} \times \mathbf{\tau} \]  \hspace{1cm} (5c)

The coordinate transformation matrix relating the components \( w_\tau, w_v, w_b \) of some position vector \( \mathbf{w} \) in the baseline transverse, vertical, length system to its components \( w_x, w_y, w_z \) in the usual \( \hat{i}, \hat{j}, \hat{k} \) system is then given by:

\[
\begin{pmatrix}
    w_\tau \\
    w_v \\
    w_b
\end{pmatrix} =
\begin{pmatrix}
    \tau_x & \tau_y & \tau_z \\
    v_x & v_y & v_z \\
    b_x & b_y & b_z
\end{pmatrix}
\begin{pmatrix}
    w_x \\
    w_y \\
    w_z
\end{pmatrix}
\]  \hspace{1cm} (6)

where expressions for the \( \tau_i, v_i, b_i \) elements of the coordinate transformation matrix in terms of the locations (4) of the radio telescopes are readily obtained from (4) and (5).

The transformation matrix needed by the Kalman filter is not the coordinate transformation matrix defined by (6), but rather the transformation matrix that relates the usual UTPM Earth orientation parameters to the linear combinations of them that are determinable from single baseline VLBI observations. As defined here, these two determinable components are the transverse \( \mathbf{T} \) component representing a right-handed rotation of the Earth about the \( \mathbf{\hat{v}} \)-coordinate direction (which perturbs \( \mathbf{b} \) in the positive \( \mathbf{\hat{\tau}} \) direction), and the vertical \( \mathbf{V} \) component representing a left-
handed rotation of the Earth about the \( \hat{\tau} \)-coordinate direction (which perturbs \( \hat{b} \) in the positive \( \hat{v} \) direction). The third independent component (not determinable from single baseline VLBI observations) is the degenerate \( D \) component which, as defined here, represents a left-handed rotation about the \( \hat{b} \)-coordinate direction (that is, about the baseline, thereby perturbing \( \tau \) in the negative \( \hat{v} \) direction). Note that all of the rotations being discussed here are first-order, small angle rotations of the solid Earth (and hence of the attached terrestrial reference frame) with respect to the celestial reference frame. With these definitions of the TVD Earth orientation parameters \( (\tau, V, D) \), and remembering that, e.g., a positive change \( \Delta v_0 \) in the \( x \)-component of the pole position is equivalent to a left-handed rotation of the body-fixed, terrestrial reference frame about the \( \hat{j} \)-axis, it is straightforward (albeit tedious) to show that the desired UTPM parameter space transformation matrix relating the usual UTPM parameters (PMX, PMY, UT1) to the TVD parameters is given by:

\[
\begin{bmatrix}
T(t) \\
V(t) \\
D(t)
\end{bmatrix} = \begin{pmatrix}
-v_1 & -v_2 & v_3 \\
\tau_x & \tau_y & \tau_z \\
b_1 & b_2 & -b_3
\end{pmatrix} \begin{pmatrix}
\lambda_p(t) \\
y_p(t) \\
u(t)
\end{pmatrix}
\tag{7}
\]

where the \( \tau_i, v_i, \) and \( b_i \) elements of the UTPM parameter space transformation matrix in (7) are the same as those elements of the coordinate transformation matrix in (6). Note that as with the VUD parameters, the Earth orientation parameters in the system of equations (7) are also all assumed to have the same units.

Apart from a sign difference (due to \( D \) being defined here as a left-handed rotation), the degenerate component of the Earth’s orientation that is not determinable from single baseline VLBI observations is related to the degenerate component that is not determinable from single station LLR observations. To show this relationship, (4) and (5) can be used to write the unit coordinate vector \( \hat{b} \) in the baseline length direction as:

\[
\hat{b} = \cos \phi \cos \lambda \hat{i} + \cos \phi \sin \lambda \hat{j} - \sin \phi \hat{k}
\tag{8}
\]

where the angles \( \phi \) and \( \lambda \) are defined by.
By (7), the degenerate component of the Earth's orientation that is not determinable from single baseline VI.BI observations can therefore be written as:

\[ D(t) = \cos \phi \sin \lambda x_p(t) + \cos \phi \cos \lambda y_p(t) - \sin \phi U(t) \]  

which, by (It) and apart from a sign difference, is formally equivalent to the expression for the degenerate component of the Earth's orientation that is not determinable from single station I.I.R observations.

"The angles \( \phi \) and \( \lambda \) defined by (8) and (9) specify the latitude and east longitude, respectively, of a vector parallel to the baseline, hill whose tail is located at the origin of the terrestrial reference frame '1'bus, in the case of single baseline VI.BI, the angles \( \phi \) and \( \lambda \) in (10) specify the orientation of the baseline vector, whereas in the case of single station I.I.R, the angles \( \phi_i \) and \( \lambda_i \) in (1e) specify the orientation of the station's position vector. If the VI.BI baseline and the I.I.R station position vectors are parallel, then single baseline VI.BI observations and single station I.I.R observations will have their degeneracies in the same direction of UTPM parameter space since their indeterminable components result from rotations of the Earth about parallel axes. In the case of single baseline VI.BI, the indeterminable component of the Earth's orientation results from rotations of the Earth about the baseline vector (whose tail can be considered to be located at the origin of the terrestrial reference frame), whereas in single station I.I.R the indeterminable component results from rotations about the station's position vector (whose tail is located at the origin of the terrestrial reference frame). In this sense, the baseline vector in single baseline VI.BI plays the same role as does the station position vector in single station I.I.R.

**Global Positioning System**

The Global Positioning System (GPS) has two major elements: 1) a space-based element consisting of a constellation of 24 satellites, and 2) a ground-based element consisting of a network
of receivers. The satellites are at altitudes of 20,200 km in orbits of 12-hour period and are located equi-distant from each other in three orbital planes each inclined at 55° to the Earth’s equator. Navigations signals are broadcast by the satellites at two L-band frequencies, thereby enabling first-order corrections to be made for ionospheric refraction effects. The ground-based multi-channel receivers detect the navigation signals being broadcast by those satellites that are above the horizon (up to the number of channels in the receiver). A variety of geophysical parameters including the position of each receiver, and by extension the orientation of the network of receivers as a whole, can be determined by analyzing the detected broadcast signals [e.g., Bock and Leppä et al., 1990, Biewer, 1993; Hofmann-Wellenhof, 1993; Butler et al., 1996].

A number of organizations, including groups at the Astronomical Institute of the University of Bern, the Geodetic Survey of Canada, the Scripps Institution of Oceanography (SIO), and the Jet Propulsion Laboratory currently determine Earth orientation parameters from GPS measurements. Since these groups use the same basic set of GPS measurements, their results are not completely independent of each other even though their subsequent data editing and processing procedures are different. For the purpose of combining Earth orientation series, it is desirable to combine only completely independent determinations of the Earth’s orientation. Thus, in generating SPACE95, three non-overlapping GPS Earth orientation series were used, two determined at JPL and one at SIO. The first JPL GPS series chosen is designated EOP(JPL)95 P 02 [Heflin et al., 1995] and consists of daily determinations of polar motion spanning June 1, 1992 to January 27, 1995. A second JPL series, spanning January 28, 1995 to February 10, 1996 and consisting of their routine international GPS Service for Geodynamics (IGS) polar motion determinations [Zumberge et al., 1995], was used to update the first series (although the two series were treated separately in the SPACE95 combination procedure described below). The particular SIO GPS polar motion series used is that portion of EOP(SIO)93 P 01 [Bock et al., 1993] that does not overlap with the JPL series, namely, that portion spanning August 25, 1991 through May 31, 1992. Note that because the Kalman filter used at JPL for combining Earth orientation series (see below) does not currently recognize length-of-day (LOD)
as an input data type, only the GPS polar motion determinations were used in generating SPACE95. The incorporation of LOD measurements into the Kalman Earth orientation filter is currently under study.

KALMAN EARTH ORIENTATION FILTER

Kalman filters are commonly used for estimating parameters of some system when a stochastic model of the system is available and when the data contain noise [e.g., Nahl, 1969; Gelb, 1974; Bierman, 1977]. For the purpose of combining Earth orientation series, the system consists of a series of the usual UTPM parameters, their excitations, and full covariance matrices. The data consist of series of observed Earth orientation parameters, which may be incomplete and/or degenerate, along with the data measurement covariance matrices. Hernquist et al. [1984] and Eubanks et al. [1985a] have described a Kalman filter and smoother developed at JPL for use with UT1 measurements. The approach currently taken at JPL evolved from the earlier UT1 Kalman filter and now estimates all three UTPM parameters, along with their excitations, wherein the polar motion parameters are treated as individual real-valued quantities, rather than as a single complex-valued quantity [Morabito et al., 1988]. Other approaches that use Kalman filters to estimate the polar motion excitation functions [e.g., Barnes et al., 1983] from polar motion measurements have been described by Brzeziński [1990; i 992, 1994] and Preisig [1992].

The particular design of the current JPL Kalman Earth Orientation Filter (KFOF) was dictated to a large extent by the nature of the Earth orientation series being combined, and in particular by the presence of degenerate data types (single station LLR and single baseline VLBI observations). This section describes aspects of KEFO that are driven by the characteristics of the input space-geodetic data sets. A brief description of the stochastic models employed in KEFO may be found in Morabito et al. [1988; see also Eubanks et al., 1985 b]. No discussion is presented here on the ability of KEFO to recognize atmospheric angular momentum (AAM) data, which is used in KEFO as a proxy length-of-day measurement, since the emphasis of this report is
on combining space-geodetic Earth orientation series. The incorporation of AAM data into KEOF is described by Freedman et al. [1994].

The incorporation into KEOF of observed data series that consist of all three UTPM parameters (PMX, PMY, J'J'I) is straightforward. The 3x1 measurement vector \( x_m \) is simply filled with all three observed quantities, and the 3x3 measurement error covariance matrix \( C_m \) is filled with the measurement variances (the square of the measurement uncertainties) and, when correlations are available, the measurement covariances. When the correlations are not available, the off-diagonal elements of the measurement covariance matrix are filled with zero. When only a subset of the three usual UTPM parameters is available, as is the case with the IRIS "Intensive" series consisting of only UT1, or with the SIR series wherein the UT1 values have been discarded, then only that subset of the UTPM parameters is used to fill the relevant elements of \( x_m \) and \( C_m \). The other entries of the measurement vector and covariance matrix are filled with zero (actually, any value could be used here since, as discussed below, KEOF never accesses those elements of \( x_m \) and \( C_m \).

Complications arise when the observed quantity is some linear combination of the usual UTPM parameters. In this case, the measurement vector and measurement error covariance matrix are filled in the natural frame of that data type, and then rotated to the usual UTPM frame. (The use of the word "frame" here is not meant to denote a coordinate reference frame, but is rather meant to denote different sets of independent, linear combinations of the Earth orientation parameters that span the UTPM parameter space, such as the VUD and TVD sets.) In the case of single station IJLR observations, the observed \( UT0_i - TAI \) values (and their uncertainties) are first converted to \( UFF_i \) values (and their uncertainties) by (2). Nominal values for the locations of the IJLR observing stations are used to evaluate the transformation matrix defined by (3), which is then used to transform the VUD parameters and associated error covariance matrix back to the usual UTPM frame. In doing this, zeroes are used to fill those elements of the measurement vector and error covariance matrix in the VUD frame corresponding to the degenerate, component of the Earth's orientation not determinable from IJLR observations taken at that single station. In the case...
of single baseline VLBI observations, the measurement vector and associated measurement error covariance matrix are filled in the TVD frame, with zeroes being used to fill those elements of the measurement vector and error covariance matrix corresponding to the degenerate component of the Earth's orientation that is not determinable from VLBI observations taken on that single baseline. The transformation matrix defined by (7) is then evaluated using nominal values for the locations of the two radio telescopes defining the baseline. This transformation matrix is then used to transform the TVD parameters and associated error covariance matrix back to the usual UTPM frame.

The use of zeros to fill the elements of the measurement error covariance matrix corresponding to the degenerate component is not meant to indicate that the degenerate component is perfectly known. In fact, in principle, the degenerate component is perfectly unknown, and therefore of infinite variance. However, for numerical reasons, these elements of the measurement error covariance matrix must be filled with some value other than infinity, and zero was chosen for this purpose. The fact that the measurement error covariance matrix associated with degenerate data types cannot be properly represented numerically has required that KEOP be formulated to use the information matrix, rather than the covariance matrix.

The measurement information matrix $C_m^{-1}$ is the inverse of the measurement error covariance matrix. Elements of the information matrix corresponding to the degenerate component are properly filled with zero, indicating that the observations contain no information about the degenerate direction in UTPM parameter space. The measurement error information matrix is generated by inverting that submatrix of the measurement error covariance matrix corresponding to the observed components (after rotating back to the natural reference frame for that data type, if necessary). The elements of the information matrix corresponding to the degenerate (or, in some cases, discarded) components are (now properly) set to zero. The resulting full $3 \times 3$ information matrix associated with degenerate data types will be rank deficient. However, the eigenvectors associated with the zero eigenvalues span the degenerate subspace of UTPM parameter space that is not determinable from observations by that data type (or that have been purposely discarded).
Thus, even in the usual UTPM frame, the information matrix is always appropriately defined, even for the degenerate data types.

The Kalman filter contains a stochastic model of the process which is used to propagate the state vector (and its associated state covariance matrix) forward to the time of a measurement. The state vector $\mathbf{x}_s$ contains values for the parameters that are being estimated by the Kalman filter. In the case of KEKF, the $11 \times 1$ state vector $\mathbf{x}_s$ consists of the UTPM parameters ($PMX, PMY, \ldots$) and their excitations, along with additional parameters associated with the model of the polar motion excitation and the use of AAM data. At the time $t$ of a measurement, both an initial estimate $\mathbf{x}_s(t)$ of the state vector [along with its associated $11 \times 11$ error covariance matrix $\mathbf{C}_s(t)$] and the $3 \times 1$ measurement vector $\mathbf{x}_m(t)$ [along with its associated $3 \times 3$ measurement information matrix $\mathbf{C}_m^{-1}(t)$] are available. The state vector is updated at the measurement time $t$ by forming the vector weighted average of its initial estimate with the measurement vector:

$$
\mathbf{x}_s(t) = \left[ \mathbf{I} \mathbf{C}_s^{-1}(t) + \mathbf{H}^\mathsf{T} \mathbf{C}_m^{-1}(t) \mathbf{H} \right]^{-1} \left[ \mathbf{I} \mathbf{C}_s^{-1}(t) \mathbf{x}_s(t) + \mathbf{H}^\mathsf{T} \mathbf{C}_m^{-1}(t) \mathbf{x}_m(t) \right]
$$

(11)

where $\mathbf{H}$ is a $3 \times 1$ matrix which relates the elements of the state vector to the elements of the measurement vector.

$$
\mathbf{x}_m(t) = \mathbf{H} \mathbf{x}_s(t) + \mathbf{n}_m(t)
$$

(12)

with $\mathbf{n}_m(t)$ being a $3 \times 1$ vector representing the measurement noise that is assumed to be white, Gaussian, of zero expectation, and uncorrelated with the stochastic process noise. Since (11) is formulated in terms of information matrices, it is valid and can be used even in the presence of degenerate data types. The state covariance matrix is updated at the measurement time $t$ by inverting the sum of the initial state and measurement information matrices

$$
\mathbf{C}_s(t) = \left[ \mathbf{I} \mathbf{C}_s^{-1}(t) + \mathbf{H}^\mathsf{T} \mathbf{C}_m^{-1}(t) \mathbf{H} \right]^{-1}
$$

(13)

Smoothed estimates of the Earth orientation parameters are obtained by running the Kalman filter forward in time, backward in time [e.g., Gell, 1974], and taking the vector weighted average of the results. ‘1’bus, the final output of the Kalman filter consists of a series of smoothed, interpolated estimates of the usual UTPM parameters ($PMX, PMY, LJ1$), their excitations, and
full covariance matrix that is based upon independent measurements, whether or not they are incomplete, degenerate, or irregularly spaced in time.

**AN APPROACH TO COMBINING EARTH ORIENTATION SERIES**

A Kalman filter has a number of useful properties that make it an attractive choice as a means of combining independent Earth orientation datasets. Changes in the Earth's orientation can be modeled as being forced by a randomly excited stochastic process. Consequently, between successive measurements of the Earth orientation parameters, the uncertainty in the knowledge of their values grows and rapidly becomes much larger than the uncertainty in the measurements. Thus, it is important to analyze each measurement at its measurement epoch, rather than at some "nearby", regularized epoch as is commonly done in normal-point methods of combining data sets. Kalman filters are an effective means of dealing with irregularly spaced data sets since the state vector and state covariance matrix can be propagated to the measurement epoch regardless of whether or not the measurements are equi-spaced.

Due to this growth in the uncertainty of the Earth orientation parameters between measurements, when intercomparing data sets in order to evaluate their relative accuracies (and hence to set the uncertainty adjustmentscale factors), it is important to compare independent measurements whose epochs are as close as possible to each other. This argues for comparing an individual data set against a combination of all other independent data sets (rather than against some other individual series) so that the difference in the epochs of the measurements being compared is minimized. Since it is unlikely that independent measurements will be given at exactly the same epoch, it is important that the interpolation procedure used in generating the combined series accounts for the growth in the uncertainty of the Earth orientation parameters between measurements. The Kalman filter does this in an objective manner (see previous section).

The importance of accounting for interpolating error between measurements is illustrated in Figure 2 which shows the uncertainty in the (J1-1) component of two different USNO IRIS
"Intensive" complementary smoothings formed by combining all but the "Intensive" series used in forming SPACF95 (see below) in generating the solid line the uncertainties of the measurements have been reduced by a factor of $10^4$ prior to combination in order to model the error growth between perfectly known measurements, whereas the dashed line was generated without modification of the measurement uncertainties. With perfectly known measurements the UT1 uncertainty of the complementary smoothing depends only on the error in interpolating between the measurements, whereas with imperfect measurements, the UT1 uncertainty at any time is a combination of interpolation and measurement error. The relative importance of these two sources of UT1 uncertainty can be assessed by comparing the solid and dashed lines. As can be seen, for example near MJD 50026 and MJD 50031, the UT1 uncertainty is oftentimes larger than the difference between the two curves, indicating that between measurements the uncertainty in the UT1 component of combinations of Earth orientation series is oftentimes dominated by interpolation, rather than measurement error. The curves in Figure 2 indicate the uncertainties and epochs of the USNOIRIS "Intensive" measurements taken during the 9-day-long time interval illustrated (note that the stated uncertainties of the "Intensive" measurements have been adjusted hereby the factor given in Table 7). The "Intensive" measurements are seen to generally occur between the epochs of the measurements in the complementary smoothing. Thus, the "Intensive" measurements frequently occur attimes when interpolation error dominates the UT1 uncertainty in the complementary smoothings. Therefore, accounting for interpolation error, and not just measurement error, is important when combining measurements since this is oftentimes the dominant source of uncertainty. This is particularly important when evaluating the measurement uncertainty of one series by intercomparison with other series. Kalman filters, by virtue of incorporating stochastic models of the process and measurement noise, are particularly well suited to this task.

Finally, in order to obtain the best possible combined series, the degree of smoothing that is applied to the measurements must vary with both the precision and the sampling interval of the measurements. As improvements have been made to the measurement systems (in both hardware
and software), the precision with which the measurements have been made has dramatically improved, as has the time resolution. With a Kalman filter, the degree of smoothing applied (see (11)) is a function of both the precision of the measurements and the time span over which the state vector and covariance matrix must be propagated (i.e., of the time resolution of the measurements). (Of course, the Kalman filter model for the growth in the uncertainty of the Earth orientation parameters between measurements is also important in this regard.) '1'bus, with a Kalman filter, no arbitrary changes in the applied degree of smoothing need be made. The degree of smoothing is automatically adjusted as the precision and time resolution of the measurements changes.

Because of these above considerations, a Kalman filter-based approach to combining Earth orientation series has been taken at JPL. But prior to combining independent estimates of the Earth's orientation, a number of corrections must be applied to the individual series. First, the series are likely to exhibit differences in bias and rate due to differences in the underlying reference frames within which they are given. A conventional terrestrial reference frame is realized in practice by specifying the positions and secular motions of a set of observing stations that are globally distributed on the surface of the Earth. When deriving the Earth orientation series, the locations and velocities of the observing sites must be specified, and are, in fact, usually estimated simultaneously with the UTPM parameters. '1'bus, in principle, each solution for the Earth orientation parameters is given within its own reference frame. Constraints are typically applied during the data reduction procedure in order to place a given solution within a particular reference frame, but different constraints are employed by the various analysis centers leading to differences in the underlying reference frames, and hence to differences in the bias and rate of the determined Earth orientation series. Also, the observing stations of the various techniques are located on different subsets of the set of mobile, deformable tectonic plates. A model of the plate motions is usually employed when reducing Earth orientation observations, but discrepancies between the plate model and the true tectonic motions of the stations as evidenced by their adjusted velocities can lead to differences in the rates of the Earth orientation series, especially when the observing stations are located on different subsets of the plates. Thus, Earth orientation series derived by
different techniques, or from independent observations by the same technique but using different station sets, can be expected, in principle, to exhibit differences in bias and rate, and therefore provision must be made to determine and apply bias-rate corrections prior to their combination.

Each of the observed Earth orientation series is a measure of the Earth's true orientation. Since the true orientation of the Earth is not known, it is impossible to know which of the observed series is the most accurate measure of the Earth's orientation. The stated uncertainties distributed with each data set indicate the internal precision with which the observations have been made, but do not necessarily reflect the accuracy of those observations. An estimate of the relative accuracy of the different data sets can be obtained through their intercomparison (Of course, the true accuracy of the data sets cannot be obtained through intercomparison studies due to the possible presence of systematic errors common to all the data sets.) In the absence of a priori knowledge about the relative accuracy of the different data sets, at the beginning of the intercomparison process they should all be treated as though they are potentially equally accurate, but, of course, not equally precise.

In generating SPAC1995, no referenceseries has been used for the purpose of determining the bias and rate corrections that must be applied to each series in order for them to be consistent with each other (in bias and rate) prior to their combination. First, none of the individual series listed in Table 1 arc appropriate choices for a reference series since none have the requisite time resolution, span a great enough time interval, and/or consist of the full set of Earth orientation parameters. Also, any available combined Earth orientation series is a dubious choice for a reference series since it may contain spurious variations due to inadequacies of the combination technique, thereby affecting the desired bias-rate corrections. Finally, a democratic treatment of the individual data sets precludes the use of a reference series. Inherent in the use of a reference series is the belief that the particular series chosen as the references more accurately represents the true orientation of the Earth than do any of the other available series. Since, as discussed above, the true accuracy of any observed series cannot be known, the decision about which series to use as a reference is, to some extent, a subjective one. In generating SPAC1995, care has been taken to
make the process be as free as possible from the need to make such subjective decisions, and, therefore, no reference series has been used. Rather, for the purpose of determining the bias-rate corrections, each data set has been iteratively compared to a combination of all other data sets. Thus, all data sets are treated equally, with no subjective decision being made about their presumed accuracies.

For SPACE95, the bias-rate corrections have been determined by an iterative scheme wherein, during a given iteration, each individual series is compared to a combination of all other series (this combination of all other series is hereafter called the complementary smoothing for that particular individual series). Ideally, in order to minimize interpolation error when subsequently forming the residual series (see below), the complementary smoothing should be produced by a Kalman filter that interpolates to and prints its estimates at the exact measurement epochs of the corresponding individual series. However, due to software limitations, this ideal case is currently approximated by using the Kalman filter to produce a regular smoothing without printed at one-day intervals and then linearly interpolating the output values and covariance matrices to the exact measurement epochs of the corresponding individual series.

Within each iteration, each series is analyzed in parallel with the analyses of each of the other series (i.e., the order in which each series is analyzed is completely immaterial in this procedure). Half the incremental bias-rate corrections determined during a given iteration are applied to the series and the process repeated until convergence is attained, convergence being indicated by the incremental bias-rate corrections approaching zero. The reason for applying only half the bias-rate corrections determined during any iteration can be understood by considering the application of this procedure to just two series, labeled A and B. By following this procedure, bias-rate corrections would be determined that make the bias and rate of series A agree with that of B, and vice versa. Upon applying the full corrections thus determined, the corrected series A would have the bias and rate of the original series B, and the corrected series B would have the bias and rate of the original series A. Thus, the bias-rate (difference between series A and B after correction) would be the same size as that before correction. This non-converging situation can be
avoided if only half the corrections that were determined are actually applied to each series. In this case, the corrected series will then agree with each other in bias and rate.

The bias-rate corrections are determined by a weighted least squares fit of a bias and rate to each individual Earth orientation component of the residual series, with the weights being based upon the uncertainties of the residuals, and with the residual series being analyzed in the natural Earth orientation reference frame of the individual series. The residual series is formed by differencing the given individual series with a combination of all other series (the complementary smoothing) after both the individual and combined series have been transformed (if necessary) to the natural Earth orientation reference frame of the individual series (that is, the difference is formed in the natural reference frame of the individual series). Since the complementary smoothing is independent of the individual series, the covariance matrices associated with the residual values are formed by simply summing the covariance matrices of the individual series, with those of the complementary smoothing, after the covariance matrices have also been transformed (if necessary) to the natural reference frame of the individual series. Since the bias-rate corrections are determined separately for each component, only the diagonal elements of the residual covariance matrices corresponding to that component being corrected are used in the weighted least squares fit. This single component (or single variate) approach to analyzing the residual series results from current software limitations that preclude the use of a multivariate approach. Future software upgrades will allow the use of a multivariate approach in which all components of the residual series are analyzed in unison, thereby allowing correlations between components to be taken into account in the analysis.

Along with determining bias-rate corrections, the stated uncertainties of each series are also adjusted during the iterative procedure by determining a scale factor which when applied to those uncertainties makes the residual of that series, upon differencing it with a combination of all other series, have a reduced chi-square of one. Note that by means of the Kalmanfilter’s stochastic model of the Earth rotation process, the Kalmanfilter’s estimate of the covariance matrix of the complementary smoothing at each epoch accounts for the interpolation error at that epoch due to the
stochastic character of the Earth rotation process. Of course, the covariance matrix of the complementary smoothing also depends upon the measurement uncertainties of the series being combined in the complementary smoothing. Thus, the covariance matrix of the residual, being the sum of the covariance matrices of the individual series and its complementary smoothing, accounts for the uncertainty of the interpolation as well as the measurement uncertainties of all the data series. The purpose of adjusting the stated uncertainties of the observations is to make them consistent with the scatter in the residual series. Such consistent estimates for the uncertainties of the observed values are important because the Kalman filter uses the uncertainty estimates as weights when combining the observations with the initial state estimate [see (13)].

Since the uncertainties are being adjusted along with the bias and rate of the series, and since the bias-rate corrections are determined by a weighted least squares fit with the weights being dependent upon the values of the adjusted uncertainties, it is important that during each iteration the uncertainty scale factors and bias-rate corrections are determined simultaneously. The uncertainty scale factor is determined separately for each component by analyzing the scatter in the fit of a bias anti rate to that component of the residual series, with the residual series being formed and the bias-rate fit being computed as described above. Once that value for the uncertainty scale factor has been found, which upon application leads to a value of one for the reduced chi-square in the scatter about the fit for a bias and rate to the residual series, then half those particular values for the bias and rate are taken as the desired bias-rate corrections that are to be applied to the individual series, and that value for the uncertainty scale factor is taken as the desired scale factor to be applied to the uncertainties.

During the iterative procedure for the determination of bias-rate corrections and uncertainty scale factors, individual data points are deleted whose residual values are greater than three times their adjusted uncertainties (the residual values being obtained from the difference of that data set with its complementary smoothing). Since the uncertainties are being adjusted during the iterative procedure, a sufficient number of iterations must first be completed in order to converge upon stable values for the adjusted uncertainties (convergence, here, being indicated by the incremental
uncertainty scale factors approaching one). During subsequent iterations, outlying data points can then be deleted. Currently, outliers are identified by separately examining each component of the residual series, although in the future outliers will be identified using a multivariate approach. All components (e.g., PMX, PMY and UT1) of the data point are deleted even if only one component is the outlier, and once a data point has been deleted it remains deleted for all subsequent iterations. Of course, during subsequent iterations, the incremental uncertainty scale factors determined for some data set that has had outliers deleted during previous iterations are usually found to be such as to reduce the adjusted uncertainties, thereby usually causing additional data points to be considered outliers and to be deleted. After a sufficient number of iterations, however, the incremental scale factors will converge to one, and no more data points will be found to have residual values greater than three times their adjusted (and converged) uncertainties.

**SPACE95**

Summary information about the particular data sets combined to form **SPACE95** is given in Table 1. Prior to their combination, corrections to the bias, rate, and stated uncertainties of each series were determined (and applied) by the above iterative approach (except for the GPS and USNO series which were treated separately as described below). But first, leap seconds and the effect of the solid Earth and ocean tides on the Earth's rotation (UT1) were removed. The model of Herring[1993; also see Herring and Dong, 1994] was used to remove the effect of the diurnal and semi-diurnal ocean tides from those UT1 observations that included them in their reported values, namely, the NOAA IRIS "Intensive" series. The model of Yoder et al. [1981] was used to remove the effect on J-1 of the solid Earth tides between the lunar nodal (18.6 yr) and fortnightly periods, with the model of Dickman [1993] being used to remove the effect on J'1 of the Mf, Mf', Mm, and Ssa ocean tides in this long-period tidal band. Since the Yoder et al.[1981] model already includes a contribution from the equilibrium ocean tides, just the values given in Table 2 for the Dickman [1993] oceanic tidal corrections to the Yoder et al.[1981] model were actually
employed when removing the effect on UT1 of the long-period solid Earth and ocean tides. The values given in Table 2 for the oceanic tidal corrections were obtained by first computing the tidal effect on UT1 of an oceanless Earth model by multiplying the negative of the tidal amplitudes given in Table 1 of Yoder et al. [1981] by \( k/C = 0.8073 \) (Williams, personal communication, 1992). To this was added the model results of Dickman [1993] for the effect on UT1 of the long-period ocean tides. The standard model of Yoder et al. [1981], formed by multiplying the negative of the tidal amplitudes given in their Table 1 by \( k/C = 0.94 \), was then subtracted from this total effect, thereby obtaining the entries given here in Table 2 for the Dickman [1993] oceanic tidal corrections to the Yoder et al. [1981] model.

In some cases, such as for the Bureau International de l’Heure (Bit I) optical astrometric series used in generating COMB94 [Grass, 1996a], the observations span a large enough interval of the tidal period that the tidal amplitude should be attenuated prior to being removed from the UT1 observations. The attenuation factor that should be applied to the amplitude of each of the tidal terms is a function of the tidal frequency as well as of the length of the observation window [e.g., Guinot, 1970]. Consider a tidally-induced UT1 signal of amplitude \( A \), frequency \( \omega_o \), and phase \( \alpha \).

\[
U(t) = A \cos(\omega_o t - \alpha) .
\]  

Observations of duration \( T' \) centered at epoch \( t_o \) effectively average this signal by:

\[
< u(t) > \equiv \frac{\int_{t_o - T/2}^{t_o + T/2} A \cos(\omega_o t - \alpha) \, dt}{\int_{t_o - T/2}^{t_o + T/2} dt} = \frac{\sin(\frac{1}{2} \omega_o T)}{\frac{1}{2} \omega_o T} A \cos(\omega_o t_o - \alpha)
\]  

\[ (15) \]
where the angular brackets \(<\>\) denote the averaging operation. Thus, the amplitude attenuation factor to be applied is \(\sin(\omega_f T/2)/(\omega_f T/2)\).

After removal of leap seconds and tidal terms (both solid Earth and oceanic) from the UT1 observations, bias-rate, corrections and uncertainties scale factors were determined for each series by the iterative procedure described in the previous section. In an attempt to reduce the required number of iterations, initial values for the bias-rate corrections (but not for the uncertainty scale factors) were obtained by the use of a reference series \(\text{SPACE94}[\text{Gross}, 1995, 1996a]\). Any inconsistencies introduced by use of a reference series for this initial bias-rate correction should be eliminated during the subsequent iterative procedure. The initial bias-rate corrections needed to align each series with the reference series are given in Table 3. The uncertainties of these initial bias-rate corrections (given in parentheses in Table 3) are the \(1\sigma\) standard errors in their determination. No bias-rate corrections are listed in Table 3 for those Earth orientation parameters that were either discarded (such as the UTCRS S1.R UT1 values) or not available (such as the PMX and PMY values of the NOAA IRIS “Intensive” series). Also, no rate corrections are given in Table 3 for those series (such as the SGP/CDP Westford-Mojave single baseline VLBI series) spanning such a short time interval that reliable rate corrections could not be determined for them.

After initial bias-rate alignment, final values for the bias-rate corrections and uncertainty scale factors were determined by the iterative procedure described in the previous section. During this iterative procedure, each data type was analyzed and corrections determined in its own natural reference frame. For single station \(L1\) observations, this is the VUD frame defined by the location of that particular station at which the observations are made, and for single baseline VLBI observations it is the TVD frame defined by the orientation of that particular baseline over which the observations are made. At each stage in this iterative procedure, every residual series was visually examined in order to check for the possible presence of systematic errors or problems with the procedure (none were found). Outlying data points (those whose residual values are greater than three times their adjusted uncertainties) were rejected during this process, but not until four iterations had been completed in order to converge upon initial estimates for the uncertainty scale.
t-actors A further five iterations were required to converge on final bias-rate corrections and uncertainty scale factors. During the final iteration, no data points had residuals greater than three times their adjusted uncertainties, the values for all the incremental bias-rate corrections were less than the standard errors in their determination, and the incremental scale factors changed the uncertainties by much less than 1%. The product of all the uncertainty scale factors determined and applied during the iterative procedure, along with the sum of all the bias-rate corrections applied (half those determined), are shown in Table 4. The uncertainties given in parentheses in Table 4 are the 1σ standard errors in the determination of the bias-rate corrections during the last iteration. A total of 253 data points (including the discarded USNO and GPS data points---see below), or about 2% of the available data points, were considered to be outliers and therefore discarded during the procedure to determine bias-rate corrections and uncertainty scale factors.

For the purpose of determining bias-rate corrections and uncertainty scale factors, the LLR stations at McDonald Observatory were cited, so that a common bias-rate correction and uncertainty scale factor was determined for all the McDonald LLR series. This was done so that a rate determination could be made for them. There is not enough overlap between any individual McDonald LLR series and the other independent series for a reliable rate determination to be made. But by clustering the McDonald LLR stations, there is then enough overlap that a common rate can be determined for all the McDonald LLR series. Similarly, the individual DSN radio telescopes in California were clustered, as were those in Spain, and, separately, in Australia, so that a common bias-rate correction and uncertainty scale factor was determined for all the California-Spain series, as well as for all the California-Australia series.

Neither USNO series was included in the iterative procedure for bias-rate correction and uncertainty scale factor determination since there is not enough overlap between their independent portions and the other series for reliable determinations of these corrections to be made (see Table 1). Instead, the bias-rate corrections and uncertainty scale factors for the two USNO series were determined by individually comparing them to two different reference series formed by two different special combinations of the other corrected series. In forming these reference series, the
GPS series were not included in either of them (see below), and in order for each reference series to be completely independent of its respective USNO series, only those portions of the SGP/CDP multibaseline and NOAA IRIS "Intensive" series that did not overlap in time with its respective USNO series were selected and included. That is, only that portion of the SGP/CDP multibaseline series before January 1, 1988 was selected and included (along with the entire NOAA IRIS "Intensive" series) in the reference series used to adjust the USNO multibaseline series; and only that portion of the NOAA IRIS "Intensive" series before December 21, 1993 was selected and included (along with the entire SGP/CDP multibaseline series) in the reference series used to adjust the USNOIRIS "Intensive" series. (In order to be able to determine reliable rate corrections for the USNO multibaseline series, all of its data points after January 1, 1988 were used, even though only those after December 29, 1994 ultimately get incorporated into SPACE95. Similarly, in determining the bias-rate correction for the USNOIRIS "Intensive" series, all of its values (beginning December 21, 1993) were used (i.e., even though only those after January 4, 1995 ultimately get incorporated into SPACE95.)

Thus, the two different reference series used to determine the bias-rate corrections and uncertainty scale factors of the two USNO series were formed by combining the respective truncated SGP/CDP multibaseline or NOAA IRIS "Intensive" series with all the other series (but not the GPS series) after the bias-rate corrections and uncertainty scale factors determined above for all the other series (Tables 3 and 4) had been applied to them. During the comparison for bias-rate correction and uncertainty scale factor determination, outlying data points (those whose residual values were greater than three times their adjusted uncertainties) were also discarded. Table 5 gives the bias-rate corrections and uncertainty scale factors thus determined for the USNO series. The values given in parentheses are the 1σ standard errors in the determination of these corrections.

The GPS series were similarly not included in the above iterative procedure for bias-rate correction and uncertainty scale factor determination. Rather, bias-rate corrections and uncertainty scale factors for the GPS series were determined by comparing them to combinations of all the
other corrected series, including the USNO series. The bias-rate corrections and uncertainty scale factors determined for all the other series (Tables 3-4 and Table 5 for the USNO series) were applied, the corrected series combined, and the results used as a reference against which the GPS series were compared for the purpose of determining their bias-rate corrections and uncertainty scale factors. No rate corrections were determined for the Scripps and JPL/IGS Analysis GPS series since their overlap with the other, corrected series was not great enough to allow reliable rate corrections to be determined. Table 5 gives the bias-rate corrections and uncertainty scale factors thus determined for the GPS series. The values given in parentheses are the 1σ standard errors in the determination of these corrections.

The final step taken prior to generating SPACE95 was to place the corrected 15 arth orientation series within a particular IERS reference frame. This was done by applying to them an additional bias-rate correction that is common to all of them. This additional common correction was determined by combining all the series, after applying to them the corrections that have been previously determined for them (Tables 3-5). This combination was then compared to the IERS combination designated F: OP(IERS) C 04 [IERS, 1995] during the interval 1984-1995 in order to obtain the bias-rate corrections that make it, and hence each of the corrected individual series, have the same overall bias and rate as the IERS series during 1984-1995. The bias-rate corrections thus determined are shown in Table 6, with the 10 standard errors in their determination given in parentheses.

At this point, all the necessary corrections to all the series have been determined. It now remains to apply these corrections and combine the corrected series into SPACE95. Table 1 gives the actual number of data points and their span in time for each series that was used in generating SPACE95. Table 7 gives the corrections made to each of the raw series listed in Table 1 prior to their combination, with the bias-rate entries in Table 7 being the sum of the bias-rate entries in Tables 3-6. Note that the same IERS rate correction is applied to all of the series, including those (such as the Scripps and JPL/IGS Analysis GPS series) for which no relative rate correction could be determined. Thus, the rate correction given in Table 7 for those series for which no relative rate
correction could be determined is simply the IERS rate correction, but given, of course, in the
natural reference frame for that series. The resulting combination, SPACE95, consists of daily
values at midnight of PMX, PMY, and UT1, their 1σ formal errors and correlations spanning
October 6.0, 1976 to February 10.0, 1996. Figure 3 illustrates the resulting UT1 and polar motion
values, with Figure 4 illustrating the 1σ formal errors in the determination of the UT1PM values.
Leap seconds, and the effect on UT1 of the long period (fortnightly and longer) solid Earth and
ocean tides have been added back to the UT1 values using the same models for these effects that
were originally used in removing them from the raw series (Yoder et al. [1981] for the long period
solid Earth tides and Dickman [1993] for the long period ocean tides). However, no semi-diurnal
or diurnal tidal terms have been added back. Over time, improvements to the observing systems
(in both hardware and software) have led to increasingly precise determinations of the Earth's
orientation. This improvement is reflected in SPACE95 by the reduction of the formal errors from
about 2 mas in polar motion and 0.5 ms in Ut1 during the late 1970s to their recent values of
about 0.2 mas in polar motion and 0.02 ms in Ut1.

This combination, SPACE95, has been submitted to NASA's Crustal Dynamics Data
Information System and is available from them by anonymous ftp to the internet address
CDDIS.GSFC.NASA.GOV (128.183.10.141) where it can be found in the 1995 subdirectory of
the JPL directory.

**SUMMARY**

In this report, an approach currently used at JPL to combine Earth orientation series has
been presented. Many of the complications that arise under this approach are due to the presence
of degenerate data types, and to the desire for the process to be as free as possible from the need to
make any subjective decisions. in the absence of degenerate data types, a more conventional
Kalman filter based upon the state and measurement covariance matrices, rather than the
information matrices, could have been developed and used. However, the degenerate data types
provide important information about the Earth's orientation, especially before about 1984, and should therefore be included in any combined Earth orientation series.

The process of determining bias-rate corrections and uncertainty scale factors is made more complicated by not using a reference series. However, in the absence of any knowledge of the true accuracies of the series being combined, and with the desire to make the combination process be as free as possible of the need to make any subjective decisions, an approach such as that presented here based upon an intercomparison of the available data sets seems reasonable. But since the uncertainty in the values of the Earth orientation parameters grows rapidly in the absence of measurements, intercomparing data sets should be done by a technique that can objectively account for this uncertainty growth. The Kalman filter is such a technique since it contains a model for the process, and in the absence of measurements uses this model to propagate forward in time the state vector and its covariance matrix. The use of complementary smoothings minimizes interpolation error. Furthermore, when using a Kalman filter to combine independent data sets, the time-varying nature of the precision and temporal resolution of the measurements is automatically taken into account, thereby circumventing the need for any subjective decision to be made as to how or when to change the degree of smoothing applied as the properties of the measured series change.

Acknowledgments. We thank K. Deutsch, D. Ierququist, D. Morabito, and S. Mulligan for their help in developing the KalmanEarthOrientationFilter. We also thank all those involved in taking and reducing the space-geodetic Earth orientation measurements that have been combined into SPACE95. This study would not have been possible without their considerable efforts. The work described in this paper was performed at the JetPropulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.
REFERENCES


FIGURE CAPTIONS

Fig. 1. Illustration of the definition of the baseline, transverse \( \hat{t} \), vertical \( \hat{v} \), length \( \hat{b} \) coordinate reference frame. The positions within some rotating, body-fixed, conventional terrestrial reference frame of the VLBI observing telescopes (that are located at either end of the baseline) are given by the position vectors \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \).

Fig. 2. Illustration of UT1 error growth between measurements in two different USNO/IRIS "Intensive" complementary smoothings. The solid line was generated by reducing the measurement uncertainties by a factor of 10\(^4\) prior to combination in order to model the error growth between perfect measurements; the dashed line was generated without modification of the measurement uncertainties. The crosses indicate the epochs and adjusted (by the scale factor given in Table 7) uncertainties of the USNO/IRIS "Intensive" measurements taken during this time interval.

Fig. 3. Plots of the values of the x-component of polar motion (3a), y-component of polar motion (3b), UT1-UTC (3c), and change in the length-of-day (3d) as given by the Earth orientation combination SPACE95. The discontinuous changes in the plot of UT1-UTC are caused by the presence of leap seconds. Note that the UT1-UTC series displayed in 3c includes the tidal variations, whereas the LJD series shown in 3d does not.

Fig. 4. Plots of the 1\( \sigma \) formal errors in the determination of the x-component of polar motion (4a), y-component of polar motion (4b), UT1-UTC (4c), and change in the length-of-day (4d) as given by the Earth orientation combination SPACE95. The insert within panels 4a, 4b, and 4c shows that component's post-1984 uncertainties on an expanded scale with the same units (milliarcseconds for PMX and PMY, milliseconds for UT1-UTC). The slight increase in the formal errors during 1995 are a reflection of the fact that a number of the individual series combined to form SPACE95 do not extend through the end of 1995, but terminate at the end of 1994 (see Table 1).
<table>
<thead>
<tr>
<th>Data Set Name</th>
<th>Data Type</th>
<th>Analysis Center</th>
<th>Data Span</th>
<th>Number Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>LLR (JPL-95M01; VOL. UT0)</td>
<td>LLR</td>
<td>JPL</td>
<td>05OCT76 - 28DEC95</td>
<td>485</td>
</tr>
<tr>
<td>McDonald Cluster</td>
<td>LLR</td>
<td>JPL</td>
<td>07APR84 - 14DEC95</td>
<td>538</td>
</tr>
<tr>
<td>CERGA</td>
<td>LLR</td>
<td>JPL</td>
<td>10FEB85 - 11AUG90</td>
<td>65</td>
</tr>
<tr>
<td>Haleakala</td>
<td>LLR</td>
<td>JPL</td>
<td>26NOV79 - 06JAN96</td>
<td>578</td>
</tr>
<tr>
<td>DSN (JPL-95R01; T, V)</td>
<td>VLBI</td>
<td>JPL</td>
<td>28OCT78 - 09JAN96</td>
<td>584</td>
</tr>
<tr>
<td>California–Spain Cluster</td>
<td>VLBI</td>
<td>JPL</td>
<td>26NOV79 - 06JAN96</td>
<td>578</td>
</tr>
<tr>
<td>California–Australia Cluster</td>
<td>VLBI</td>
<td>JPL</td>
<td>28OCT78 - 09JAN96</td>
<td>584</td>
</tr>
<tr>
<td>SGP/CDP (GLB973f; T, V)</td>
<td>VLBI</td>
<td>GSFC</td>
<td>25JUN81 - 01JAN84</td>
<td>103</td>
</tr>
<tr>
<td>Westford–Ft Davis</td>
<td>VLBI</td>
<td>GSFC</td>
<td>25JUN81 - 01JAN84</td>
<td>103</td>
</tr>
<tr>
<td>Westford–Mojave</td>
<td>VLBI</td>
<td>GSFC</td>
<td>21MAR85 - 06AUG90</td>
<td>13</td>
</tr>
<tr>
<td>SGP/CDP (GLB973f, UTPM)</td>
<td>VLBI</td>
<td>GSFC</td>
<td>04AUG79 - 28DEC94</td>
<td>1784</td>
</tr>
<tr>
<td>Multibase</td>
<td>VLBI</td>
<td>GSFC</td>
<td>04AUG79 - 28DEC94</td>
<td>1784</td>
</tr>
<tr>
<td>USNO (N9604 15FEB96; UTPM)</td>
<td>VLBI</td>
<td>USNO</td>
<td>29DEC94 - 07FEB96</td>
<td>106</td>
</tr>
<tr>
<td>Multibase</td>
<td>VLBI</td>
<td>USNO</td>
<td>29DEC94 - 07FEB96</td>
<td>106</td>
</tr>
<tr>
<td>NOAA (NOAA95R02; UT1)</td>
<td>VLBI</td>
<td>NOAA</td>
<td>02APR84 - 31DEC94</td>
<td>2356</td>
</tr>
<tr>
<td>IRIS Intensive</td>
<td>VLBI</td>
<td>NOAA</td>
<td>02APR84 - 31DEC94</td>
<td>2356</td>
</tr>
<tr>
<td>USNO(N9604.INT14FEB96; J'1'1)</td>
<td>VLBI</td>
<td>USNO</td>
<td>04JAN95 - 10FEB96</td>
<td>269</td>
</tr>
<tr>
<td>IRIS Intensive</td>
<td>VLBI</td>
<td>USNO</td>
<td>04JAN95 - 10FEB96</td>
<td>269</td>
</tr>
<tr>
<td>UTCSR (CSR95L01; PMX, PMY)</td>
<td>SLR</td>
<td>UTCSR</td>
<td>02OCT76 - 28JAN95</td>
<td>2025</td>
</tr>
<tr>
<td>Lageos</td>
<td>SLR</td>
<td>UTCSR</td>
<td>02OCT76 - 28JAN95</td>
<td>2025</td>
</tr>
<tr>
<td>GPS (SIO93P01; PMX, PMY)</td>
<td>GPS</td>
<td>SIO</td>
<td>25AUG91 - 31MAY92</td>
<td>265</td>
</tr>
<tr>
<td>Scripps</td>
<td>GPS</td>
<td>SIO</td>
<td>25AUG91 - 31MAY92</td>
<td>265</td>
</tr>
<tr>
<td>GPS(JPL-95P02; PMX, PMY)</td>
<td>GPS</td>
<td>JPL</td>
<td>01JUN92 - 27JAN95</td>
<td>817</td>
</tr>
<tr>
<td>JPL</td>
<td>GPS</td>
<td>JPL</td>
<td>01JUN92 - 27JAN95</td>
<td>817</td>
</tr>
<tr>
<td>GPS(IGS Analysis; PMX, PMY)</td>
<td>GPS</td>
<td>JPL</td>
<td>28JAN95 - 10FEB96</td>
<td>365</td>
</tr>
<tr>
<td>JPL</td>
<td>GPS</td>
<td>JPL</td>
<td>28JAN95 - 10FEB96</td>
<td>365</td>
</tr>
</tbody>
</table>
### TABLE 2. APPLIED OCEANIC CORRECTIONS TO THE SOLID EARTH TIDE MODEL

<table>
<thead>
<tr>
<th>Fundamental Argument</th>
<th>Period (solar days)</th>
<th>Correction sine (μsec)</th>
<th>Correction cosine (μsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Mf^*$</td>
<td>13.63</td>
<td>0.14</td>
<td>8.58</td>
</tr>
<tr>
<td>$Mf$</td>
<td>13.66</td>
<td>0.23</td>
<td>20.73</td>
</tr>
<tr>
<td>$Mm$</td>
<td>27.55</td>
<td>-9.13</td>
<td>12.09</td>
</tr>
<tr>
<td>$Ssa$</td>
<td>182.62</td>
<td>-75.21</td>
<td>14.97</td>
</tr>
</tbody>
</table>
### TABLE 3. INITIAL ADJUSTMENTS MADE PRIOR TO ITERATIVE PROCEDURE

<table>
<thead>
<tr>
<th>Data Set Name</th>
<th>Bias (mas)</th>
<th>Rate (mas/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRR (JPL95M01) Mc Donald Cluster</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.085</td>
<td>0.137</td>
</tr>
<tr>
<td></td>
<td>(0.127)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>CERGA</td>
<td>0.047</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Haleakala</td>
<td>0.875</td>
<td>0.438</td>
</tr>
<tr>
<td></td>
<td>(0.578)</td>
<td>(0.117)</td>
</tr>
<tr>
<td>DSN (JPL95R01) California-Spain Cluster</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.019</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>California-Australia Cluster</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.128</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>SGP/CDP (GLB973f) Westford-Ft. Davis</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.793</td>
<td>4.761</td>
</tr>
<tr>
<td></td>
<td>(0.841)</td>
<td>(0.083)</td>
</tr>
<tr>
<td>Westford-Mojave</td>
<td>0.623</td>
<td>0.432</td>
</tr>
<tr>
<td></td>
<td>(0.109)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>SGP/CDP (GLB973f) Multibaseline</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-1.992</td>
<td>0.469</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>NOAA (NOAA95R02) IRIS Intensive</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0729</td>
<td>(0.021)</td>
</tr>
<tr>
<td></td>
<td>(0.108)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>UTC/CSR (CSR951,01) Lageos</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.064</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.006)</td>
</tr>
</tbody>
</table>

Reference date for bias-rate adjustment is 1993.0
TABLE 4. TOTAL ADJUSTMENTS MADE DURING ITERATIVE PROCEDURE

<table>
<thead>
<tr>
<th>Data Set Name</th>
<th>Bias (mas)</th>
<th>Rate (mas/yr)</th>
<th>Uncertainty Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>JPL (JPL 95M01)</td>
<td>VOL, UTO</td>
<td>VOL, UTO</td>
<td>1.154</td>
</tr>
<tr>
<td>McDonald Cluster</td>
<td>0.1059 (.0130)</td>
<td>0.184 (.009)</td>
<td>0.036</td>
</tr>
<tr>
<td>CEGERGA</td>
<td>0.133 (.0068)</td>
<td>0.014 (.032)</td>
<td>0.021</td>
</tr>
<tr>
<td>Haleakala</td>
<td>0.418 (.915)</td>
<td>-0.01 (.186)</td>
<td>0.157</td>
</tr>
<tr>
<td>DSN (JPL 95R01)</td>
<td>I', 1 v</td>
<td>I', 1 v</td>
<td>1.094</td>
</tr>
<tr>
<td>CA- Sparr Cluster</td>
<td>-0.07 (.032)</td>
<td>0.042 (.074)</td>
<td>0.035</td>
</tr>
<tr>
<td>CA- Australia Cluster</td>
<td>0.051 (.027)</td>
<td>0.053 (.071)</td>
<td>0.031</td>
</tr>
<tr>
<td>SGP/CDP (GRB 973f)</td>
<td>T, v</td>
<td>T, v</td>
<td>1.354</td>
</tr>
<tr>
<td>Westford-Ft. Davis</td>
<td>3.148 (.729)</td>
<td>-1.230 (.6555)</td>
<td>0.904</td>
</tr>
<tr>
<td>Westford-Mojave</td>
<td>-0.233 (.210)</td>
<td>0.136 (.439)</td>
<td>2.954</td>
</tr>
<tr>
<td>SGP/CDP (973f)</td>
<td>PMX, PMY</td>
<td>PMX, PMY</td>
<td>2.192</td>
</tr>
<tr>
<td>Multibaseline</td>
<td>-0.043 (.015)</td>
<td>-0.036 (.013)</td>
<td>0.031</td>
</tr>
<tr>
<td>NOAA (95R02)</td>
<td>PMX, PMY</td>
<td>UT1</td>
<td>1.933</td>
</tr>
<tr>
<td>IRIS Intensive</td>
<td>-0.027 (.022)</td>
<td>---</td>
<td>0.933</td>
</tr>
<tr>
<td>UTC SR (951.01)</td>
<td>PMX, PMY</td>
<td>UT1</td>
<td>---</td>
</tr>
<tr>
<td>T agecos</td>
<td>-0.027 (.020)</td>
<td>---</td>
<td>0.743</td>
</tr>
</tbody>
</table>

Reference date for bias-rate adjustment is 1993.0
<table>
<thead>
<tr>
<th>Data Set Name</th>
<th>Bias (mas)</th>
<th>Rate (mas/yr)</th>
<th>Uncertainty Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PMX</td>
<td>PMY</td>
<td>UT1</td>
</tr>
<tr>
<td>USNO (N9604) Multibaseline</td>
<td>-0.143</td>
<td>1.249</td>
<td>-0.751</td>
</tr>
<tr>
<td>USNO (N9604.INT) IRIS Intensive</td>
<td>---</td>
<td>0.945</td>
<td>---</td>
</tr>
<tr>
<td>GPS (S10931'O1) Scripps</td>
<td>-1.081</td>
<td>-1.457</td>
<td>---</td>
</tr>
<tr>
<td>GPS (JPL.95P02) JPL</td>
<td>-0.184</td>
<td>-0.200</td>
<td>---</td>
</tr>
<tr>
<td>GPS (IGS Analysis) JPL</td>
<td>0.460</td>
<td>0.278</td>
<td>---</td>
</tr>
</tbody>
</table>

Reference date for bias-rate adjustment is 1993.0
TABLE 6. COMMON ADJUSTMENT MADE TO ALIGN SERIES WITH EOP(IERS) C 04

<table>
<thead>
<tr>
<th>Bias (mas)</th>
<th>Rate (mas/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMX</td>
<td>PMY</td>
</tr>
<tr>
<td>(0.011)</td>
<td>-0.059</td>
</tr>
<tr>
<td>(0.011)</td>
<td>(0.010)</td>
</tr>
</tbody>
</table>

Reference date for bias-rate adjustment is 1993.0
<table>
<thead>
<tr>
<th>Data Set Name</th>
<th>Bias (mas)</th>
<th>Rate (mas/yr)</th>
<th>Uncertainty Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.R (JPL95M01)</td>
<td>VOL 0.200 EVOL 0.568</td>
<td>VOL 0.134 EVOL 0.156</td>
<td>1.630 1.154</td>
</tr>
<tr>
<td>McDonald Cluster</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CERGA</td>
<td>0.624 0.055</td>
<td>0.136 0.012</td>
<td>1.788 1.423</td>
</tr>
<tr>
<td>Haleakula</td>
<td>-0.010 1.454</td>
<td>-0.283 -0.170</td>
<td>1.549 1.692</td>
</tr>
<tr>
<td>DSN (JPL95R01)</td>
<td>T 0.178 V 0.053</td>
<td>T 0.083 V 0.059</td>
<td>1.354 1.094</td>
</tr>
<tr>
<td>CA-Spain Cluster</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C- Australia Cluster</td>
<td>0.323 0.046</td>
<td>-0.072 -0.050</td>
<td>1.371 1.098</td>
</tr>
<tr>
<td>SGP/CDP (GLB973f)</td>
<td>T 11.357 V 0.576</td>
<td>T 1.055 V 0.090</td>
<td>0.904 0.870</td>
</tr>
<tr>
<td>Westford-Ft. Davis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Westford-Mojave</td>
<td>0.283 (0.791</td>
<td>0.028 0.008</td>
<td>2.326 0.954</td>
</tr>
<tr>
<td>SGP/CDP (973f)</td>
<td>PMX -1.203 PMY -2.040 UT1 0.480</td>
<td>PMX 0.154 PMY -0.072 UT1 -0.121</td>
<td>2.226 1.963 2.192</td>
</tr>
<tr>
<td>Multibaseline</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USNO (N9604)</td>
<td>PMX -0.153 PMY -1.190 UT1 -0.705</td>
<td>PMX 0.193 PMY UT1 0.099 0.043</td>
<td>2.012 1.602 1.783</td>
</tr>
<tr>
<td>Multibaseline</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NOAA (95R02)</td>
<td>PMX 0.057 PMY 0.193 UT1 0.043</td>
<td>PMX 0.043 PMY UT1 0.099 0.156</td>
<td>0.933 0.933</td>
</tr>
<tr>
<td>IRIS Intensive</td>
<td>--- ---</td>
<td>0.748</td>
<td>--- ---</td>
</tr>
<tr>
<td>USNO (N9604, INT)</td>
<td>PMX -0.027 PMY -0.156 UT1 ---</td>
<td>PMX 0.043 PMY UT1 0.099 0.156</td>
<td>1.840 1.840</td>
</tr>
<tr>
<td>IRIS Intensive</td>
<td>--- ---</td>
<td>-0.898</td>
<td>--- ---</td>
</tr>
<tr>
<td>UTC SR (951.01)</td>
<td>PMX -0.010 PMY -0.23 UT1 ---</td>
<td>PMX 0.106 PMY UT1 -0.049 --</td>
<td>0.849 0.743 ---</td>
</tr>
<tr>
<td>Lageos</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GPS (S1093POJ)</td>
<td>PMX -1.091 PMY 1.516 UT1 ---</td>
<td>PMX -0.027 PMY UT1 --0.049</td>
<td>1.956 1.903 ---</td>
</tr>
<tr>
<td>Scripps</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GPS (JPL95P02)</td>
<td>PMX 0.194 PMY UT1 -0.259</td>
<td>PMX 0.057 PMY UT1 -0.156</td>
<td>3.257 2.800 ---</td>
</tr>
<tr>
<td>JPL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GPS (IGS Analysis)</td>
<td>PMX 0.450 PMY UT1 0.219</td>
<td>PMX 0.027 PMY UT1 --0.018</td>
<td>12.273 3.542 ---</td>
</tr>
</tbody>
</table>

Reference date for bias-rate adjustment is 1993.0

---