SYNTHESIS OF NOVEL AL L-DIELECTRIC GRATING FILTERS USING GENETIC AI. GORITHMS

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1. Introduction

We are concerned with the design of inhomogeneous, all dielectric (lossless) periodic structures which act as filters. Dielectric filters made as stacks of inhomogeneous gratings and layers of materials are being used in the optical technology, but are not common at microwave frequencies. The problem is then finding the periodic cell's geometric configuration and permittivity values which correspond to a specified reflectivity/transmittivity response as a function of frequency/illumination angle. This type of design can be thought of as an inverse-source problem, since it entails finding a distribution of sources which produce fields (or quantities derived from them) of given characteristics. Electromagnetic sources (electric and magnetic current densities) in a volume are related to the outside fields by a well known linear integral equation. Additionally, the sources are related to the fields inside the volume by a constitutive equation, involving the material properties. Then, the relationship linking the fields outside the source region to those inside is non-linear, in terms of material properties such as permittivity, permeability and conductivity.

The solution of the non-lines inverse problem is cast here as a combination of two linear steps, by explicitly introducing the electromagnetic sources in the computational volume as a set of unknowns in addition to the material unknowns. This allows to solve for material parameters and related electric fields in the source volume which are consistent with Maxwell's equations. Solutions are obtained iteratively by decoupling the two steps. First, we invert for the permittivity only in the minimization of a cost function and second, given the materials, we find the corresponding electric fields through direct solution of the integral equation in the source volume. The sources thus computed are used to generate the far fields and the synthesized filter response. The cost function is obtained by calculating the deviation between the synthesized value of reflectivity/transmittivity ancl the desired one. Solution geometries for the periodic cell are sought as gratings (ensembles of columns of different heights and widths), or combinations of homogeneous layers of different dielectric materials and gratings. Hence the explicit unknowns of the inversion step are the material permittivities and the relative boundaries separating homogeneous parcels of the periodic cell.

The inversion step to compute materials and geometric boundaries is performed using the genetic algorithm package PGAPACK [1]. Several advantages are offered by genetic algorithms over gradient-based or other methods, for electromagnetic design problems where the solution space has many extrema, and some applications have been reported in the literature for the design of thinned phased-arrays [2], of multilayer radar absorbers [3], multilayer optical filters [4], and loaded wire antennas [5]. In particular, the ability of genetic algorithms to sample the parameter space globally not only avoids the common pitfalls of local minimization algorithms, but holds the promise of finding novel solutions perhaps not thought to exist. This, in fact, has been the primary objective of this work; specifically the investigation and evaluation of dielectric filter solutions whose geometry is simpler or more convenient than existing designs, and the understanding of the ensuing trends in required numbers and values of materials and their configurations.

It has been remarked that genetic algorithms carry a considerable computational cost. Naturally the most expensive part of the computational cycle is the 'forward module' that performs the evaluation of a candidate solution to determine its fitness. In our case this consists of solving a set of integral equations for the electric fields in the cell, one for each design frequency/illumination angle. Although the impedance matrix depends on the solution vector of materials and boundaries candidates, it can be formed as a product of a solution-independent matrix and a vector. This procedure allows us to fill the set of frequency-dependent impedance matrices only once. Additionally, the number of design frequencies at which the integral equations are actually solved is a small set of values within the frequency range of the filter desired response. The reduction is afforded by approximating the desired filter response by a rational function, through the procedure for transfer function parameter estimation described in [6]. Furthermore, full advantage has been taken of the parallel implementation of PGAPACK for the Cray T3D. The parallelization scheme used for the genetic algorithm is an intuitive, simple master-slave configuration, where the expensive evaluation cycles are distributed among the processors.

2. **Scattering from a Dielectric Grating**

Consider a two-dimensional dielectric grating, periodic in the x dimension, infinite in y, and having a thickness *t(x)* variable over the cell. The incident plane wave direction lies in the plane xz. The cell material is characterized by complex permittivity and permeability. The polarization of the electric field parallel to the strip will be considered; a formulation for the perpendicular polarization can be similarly developed. To properly pose the scattering problem for a periodic structure, the excitation field must be a function with constant amplitude and linear phase. The incident field is defined as

$$E^i(x,z) = E_0\psi_0(x)\, e^{jk_{z0}z} \tag{1}$$

where

$$\psi_m(x) = \frac{1}{\sqrt{T_x}}\, e^{jk_{x_m}x} \tag{2}$$

and

$$k_{x_m} = \frac{2\pi}{T_x} m + k_{x_0} \quad k_{x_0} = k\sin\theta^i$$

$$k_{z_m} = \begin{cases} \sqrt{k^2 - k_{x_m}^2}, & k \geq k_{x_m} \\ -j\sqrt{k_{x_m}^2 - k^2}, & k \geq k_{x_m} \end{cases}$$

and the $e^{j\omega t}$ time convention is used, Using the electric field integral equation, the unknown induced current J(p) is found from

$$E^i(\rho) = \frac{1}{j\omega\varepsilon_0\chi(\rho)} J(\rho) - E^s(\rho) \tag{3}$$

where all components are y directed, and χ is the contrast function $\chi(\rho) = \varepsilon_r(\rho) - 1$.
The scattered field is found from integrating the induced currents over the grating

$$E^s(x,z) = \frac{-\omega}{4}\int_0^{T_x}\int_0^{t(x)} \mu(x',z')J(x',z')G_p(x,z|x',z')dx', dz' \tag{4}$$

where the two-dimensional periodic Green's function G_P - the outgoing Hankel function of order zero – representing the field due to source points within each cel 1, is used. The integration area is then over one periodic cell. Equation (4) contains an integrable singularity as the source and observation points coincide. By isolating this singular point and performing the integration in an efficient manner, the result is

$$E^s(x,z) = \int_0^{T_x} dx' \int_0^{t} dz' J(x',z')\left[Z_m^P(x,z\,x',z') - Z_m^P(x,z|x',z'+\delta)\right]$$

$$+\sum_{m} \tilde{Z}_m \tilde{I}_m^{\pm} \psi_m(x) \; e^{\mp k_{zm}(z+\delta)} \quad \textbf{(5)}$$

where the integral is the contribution from the singular point, and the series represents the contribution from sorce points away from the observation point. A method of moment is used here to obtain a numerical solution of (3) which is found by first discretizing the current over a periodic cell in a pulse basis set

$$J(x,z) = \sum_{p'=1}^{P} \sum_{q'=1}^{Q} C_{p'q'} \pi_{p'}(x) \pi_{q'}(z) \; P'x \; Q' \;\; = \;\; P'Q' \quad (6)$$

In the method of moments solution, point matching is used, with the testing functions being δ functions. The matrix system for the unknown coefficients C is then

$$\left\langle E^i, T_{pq} \right\rangle = \sum_{p'q'} C_{p'q'} \left\langle \; \frac{1}{j\omega\varepsilon_0 \chi} B_{p'q'}, T_{pq} \; \right\rangle$$

$$- \sum_{p'q'} \left[\left\langle E^{ss}(x,z), T_{pq} \right\rangle + \left\langle E^{sn}(x,z), T_{pq} \right\rangle \right] \; pq \;\; = \;\; 1,2,...P'Q' \quad (7)$$

Assuming only the dominant (m= O) mode is propagating, the reflection and transmission coefficients are found from evaluating the total field at z >> t, and z <<0 respectively,

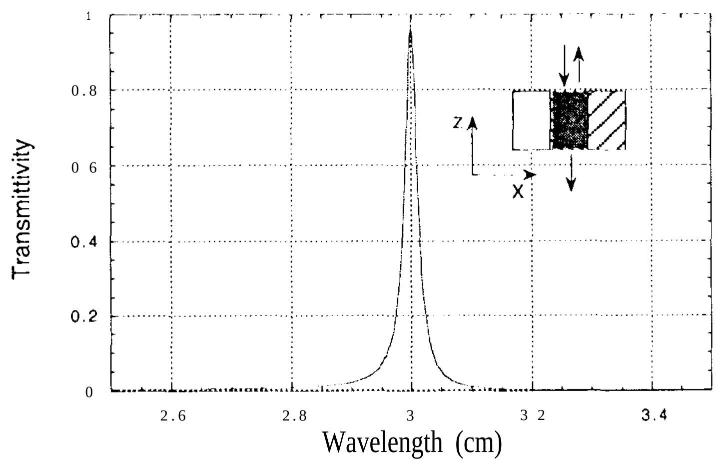
$$R = \frac{-\omega}{4E_0} \frac{1}{2jk_{z0}} \int_0^t dz' \mu \tilde{J}_0(z') \;\; e^{jk_{z0}z'} \quad (8a)$$

$$T = \left(\; 1 - \frac{\omega}{4E_0} \frac{1}{2jk_{z0}} \int_0^t dz' \mu \tilde{J}_0(z') \, e^{-jk_{z0}z'} \; \right) \quad (8b)$$

where the transform of the current is not explicitly reported here. Given a candidate solution for χ(x,z) and a set of locations describing the boundaries between regions of uniform contrast value, (3) is solved for the correspondent electric currents. Then, (8a) and (8b) yield the associated R and T. The cost function is the sum over all design frequencies of the differences between calculated R^2 (or T^2) and desired R^2. Candidate solutions which minimize the above cost function are provided by a genetic algorithm.

3. An Application: Guided-Mode Resonance Filters

It has been demonstrated that planar dielectric layer diffraction gratings exhibit sharp resonances due to the coupling of exterior evanescent diffractive fields to the leaky modes of dielectric waveguides. This is exploited in design of filters whose arbitrarily narrow linewidths can be controlled by the choice of modulation amplitude and mode confinement. In optics the guided-mode resonance effect has been combined with classical antireflection properties of thin film structures, leading to symmetric reflection filters, with low sidebands over wavelength ranges related to the number of films and their dielectric constants [7]. Additionally, a combination of guided-mode resonance and high-reflection layer design has been demontrated to yield transmission bandpass filters [8]. In the present work we start from a desired filter response and determine the grating/layer configuration and dielectric constants of the solution in a more general parameter space. Some important questions can be addressed with our inverse approach. The first relates to the shape of the resonance response of the filter; in particular, we have investigated the possibility of synthesizing a transmission response described by a Lorentzian lineshape with controllable half-width. A second issue concerns the actual number and configuration of gratings/layers required; specifically, we have been looking for the existence of the 'simplest' solutions, i.e. those which involve only one grating. We have also investigated the possibility of using more than two materials to make one grating, to achieve symmetric responses, exhibiting a sharp resonance and low side-bands. To examine the possibility of yet unknown designs, we have not restricted the choice of dielectric constants to a small discrete set of familiar values, but instead have considered the materials that can in principle exist. By allowing the dielectric constants to span the range between 1 and 10, we have obtained a solution for a three-material waveguide-grating transmission filter with bandwidth 0.007% of the central wavelength of 3 cm. To our knowledge, this has not been demonstrated before. The geometry of the cell and illumination condition is reported in the figure below, together with the filter response. The cell period is T_x= 2.2 cm, its thickness t = 0.9 cm. The dielectric constants of the materials are 2.296, 7.024 and 10.0. The cell boundary between the first and the second material is at x_{12}= 1.068 cm, and that between the second and the third is at x_{23}= 1.554 cm.



4. **References**

[1] D. Levine, "Users guide to the PGAPACK parallel genetic algorithm library," *Argonne National Laboratory - 95/16'*, January 1996.

[2] R.L. Haupt, "Thinned arrays using genetic algorithms," *IEEE Trans. Antennas and Propagation,* vol. 42, pp.993-999, July 1994.

[3] E. Michielssen, J.-M. Sajer, S. Ranjithan, and R. Mittra, "Design of lightweight, broad-band microwave absorbers using genetic algorithms," *IEEE Trans. Antennas and Propagation,* vol. 41, pp. 1024-1030, June/July 1993.

[4] T. Eisenhammer, M. Lazarov, M. Leutbecher, U. Schoffel, and R. Sizmann," Optimization of interference filters with genetic algorithms applied to silver-based heat mirrors," *Applied Optics,* vol. 32, pp. 6310-6315, Nov. 1993.

[5] A. Boag, A. Boag, E. Michielssen, and R. Mittra, "Design of electrically loaded wire antennas using genetic algorithms," *IEEE Trans. Antennas and Propagation,* vol. 44, pp. 687-695, May 1996.

[6] S. Chakrabarti, K. I?. Demarest, and E.K. Miller, "An extended frequency-domain Prony's method for transfer function parameter estimation," *Intl. J. Numerical Modeling:Electronic Networks, Devices and Fields, vol. 6, pp.269-281, 1993.*

[7] S.S. Wang and R. Magnusson, "Multilayer waveguide-grating filters," *Applied Optics,* vol. 34, pp. 2414-2420, May 1995.

[8] R. Magnusson and S.S. Wang, "Transmission bandpass guided-mode resonance filters," *Applied Optics,* vol. *34,* pp. 8106-8109, Dec. 1995.