Gravitational Wave Search With The Clock Mission

J. W. Armstrong

Jet Propulsion Laboratory, California Institute of Technology

Doppler tracking of distant spacecraft is the only method currently available to search for gravitational waves in the low-frequency (~0.0001-0.1 Hz) band. In this technique the Doppler system measures the relative dimensionless velocity $2\Delta v/c = \Delta f/f_0$ between the earth and the spacecraft as a function of time, where Δf is the frequency perturbation and f_0 is the nominal frequency of the radio link. A gravitational wave of amplitude h incident on this system causes small frequency perturbations, of order h in $\Delta f/f_0$, replicated three times in the observed record (Estabrook and Wahlquist 1975). All experiments to date and those planned for the near future involve only "two-way" Doppler--i.e., uplink signal coherently transponded by the spacecraft with Doppler measured using a frequency standard common to the transmit and receive chains of the ground station. If, as on the proposed Clock Mission, there is an additional frequency standard on the spacecraft and a suitable earth-spacecraft radio system, some noise sources can be isolated and removed from the data (Vessot and Levine 1978). Supposing that the Clock Mission spacecraft is transferred into a suitable interplanetary orbit, I discuss here how the on-board frequency standard could be employed with an all-Ka-band radio system using the very high stability Deep Space Network station DSS 25 being instrumented for Cassini. With this configuration, the Clock Mission could search for gravitational waves at a sensitivity limited by the frequency standards, rather than plasma or tropospheric scintillation effects, whenever the sun-earth-spacecraft angle is greater than 90 degrees.

I. Introduction

The general idea of combining a mission to measure gravitational redshift with one to search for low-frequency gravitational radiation is due to R. F. C. Vessot and colleagues (Vessot and Levine 1978; Smarr et al. 1983) and is at least 18 years old. Anticipating that phase scintillation due to wave propagation through the troposphere, ionosphere, and the solar wind would be a leading noise source, Vessot and Levine (1978) proposed an elegant symmetrical radio system to cancel or drastically reduce these noises. Here I propose gravity wave observations based on an alternate radio system involving 3 links, all at Ka-band (~32 GHz): an uplink driven by a high quality frequency standard on the ground, a downlink coherent with this uplink, and a second downlink referenced to a high-quality frequency standard on the spacecraft. One thus gets one "two-way" Doppler observable and one "one-way" Doppler observable, each associated with a very stable frequency standard. The use of Ka-band reduces the charged particle scintillation noise (ionosphere and solar wind) to below the residual uncalibrated tropospheric noise for observations in the anti-solar hemisphere. The discussion below shows how the one-way link can be used either to correct the two-way data for tropospheric scintillation or to contribute to the pattern-recognition for gravitational signals.

II. Doppler Technique and Low-Frequency Gravitational Waves

For a very thorough review of gravitational radiation, including detection techniques and expected wave strengths, see Thorne (1987). Briefly, in General Relativity gravitational waves (GWs) are propagating, polarized gravitational fields which change the distance between separated test masses and shift the rates at which separated clocks keep time. They are characterized by a dimensionless strain amplitude, $h = \Delta \ell / \ell \sim \Delta v/c$ where ℓ is the fiducial distance between the masses and Δv is the change in relative speed of the test masses associated with the GW. Like electromagnetism, GWs are transverse, have two independent polarization states and propagate at the speed of light. Unlike electromagnetism, GWs are *extremely* weak. This extreme weakness has two consequences. First, GWs are only generated at potentially detectable levels by extremely massive objects undergoing extremely violent dynamics, i.e. by astrophysical sources. Second, because of the extreme weakness there is negligible scattering or absorption by intervening matter. GWs thus preserve information about the *generation* of the waves in the deep interiors of astrophysical sources, not about the last scattering surface.

Detection methods depend on the time scale of the radiation (Thorne 1987). At high frequencies ($f > 10$ Hz), resonant bars and laser interferometers are used. In resonant bars the GW excites an acoustic wave which is read out with a transducer. In laser interferometers the test masses at the ends of the interferometer arms are perturbed as the GW passes, giving rise to a change in relative arm length and thus a fringe shift. For frequencies lower than about 10 Hz, it becomes prohibitively difficult to isolate any ground-based apparatus from seismic noise and time-variable environmental gravity gradient noise. To search for this longer wavelength radiation all the test masses must be put into space.

Currently the only method sensitive to low-frequency GWs is Doppler tracking of distant spacecraft (Estabrook and Wahlquist, 1975), although there are proposals to place very sensitive laser interferometers in space in the future. In the Doppler tracking method, the earth and spacecraft act as free test masses and the (two-way) Doppler tracking system measures the relative dimensionless velocity $2\Delta v/c = \Delta f/f_0$. A gravitational wave of amplitude h incident on the system causes Doppler perturbations of order $h \approx \Delta v/c$. The waveform is replicated three times in the two-way tracking record (Estabrook and Wahlquist, 1975). The sum of these three perturbations in the Doppler record is zero; thus the low-frequency band edge is set by pulse cancellation to be $\sim 1/T_2$ where T_2 is the two-way light time. (The low frequency band "edge" is soft, however; the Doppler response to a signal of duration $\tau > T_2$ is proportional to T_2/τ, giving degraded but non-negligible sensitivity even to very low frequency waves.) The high frequency band limit is set mainly by the stability of the frequency standard driving the link and by finite signal-to-noise ratio on the downlink. Thus to perform a low-frequency experiment, one needs a "reasonable" earth-spacecraft distance (for good response to the lowest frequencies), the spacecraft in cruise and as operationally quiet as possible (far from perturbing masses and with minimal unmodeled motion of the spacecraft), high radio frequency (or multiple links for tropospheric and charged particle calibration), and of course a highly stable ground system driven by a high-stability frequency standard.

III. Principal Noise Sources

Attempts to detect GWs must deal with other variations in the Doppler record. After all "deterministic effects" (such as orbital signature) have been removed, the principal sources of variability in the Doppler record are: frequency and timing system (FTS) noise; propagation noise (an extended medium--the solar wind--and a medium that is localized very close to the antenna--ionosphere and solar wind); thermal noise; spacecraft antenna unmodeled motion; ground antenna unmodeled motion; ground electronics and FTS distribution; spacecraft transponder noise; spacecraft buffeting; gravitational radiation; and systematic errors. These noises enter the observable via transfer functions (Estabrook and Wahlquist, 1975; Armstrong 1987) which are in general different from the three-pulse response function of the system to gravitation radiation. If "*" indicates convolution, the time series can be modeled as:

$$
\begin{aligned}
y(t) = \Delta f(t)/f_0 = \; & \text{solar wind plasma}(t) * [\delta(t) + \delta(t - T_2 + 2x/c)] + \\
& \text{ionospheric plasma}(t) * [\delta(t) + \delta(t - T_2)] + \\
& \text{troposphere}(t) * [\delta(t) + \delta(t - T_2)] + \\
& \text{ground station buffeting}(t) * [\delta(t) + \delta(t - T_2)] + \\
& \text{FTS}(t) * [\delta(t) - \delta(t - T_2)] + \\
& 2 \text{ x spacecraft antenna residuals}(t) * \delta(t - T_1) + \\
& \text{ground electronics}(t) + \text{FTS distribution}(t) + \\
& \text{thermal}(t) + \\
& \text{spacecraft transponder}(t) * \delta(t - T_1) + \\
& \text{spacecraft buffeting}(t) * \delta(t - T_1) +
\end{aligned}
$$

gravity waves(t) * $[(\mu-1)/2]\ \delta(t) - \mu\ \delta[t-(1/2)(1+\mu)T_2] + [(1+\mu)/2]\ \delta(t - T_2)$ +
systematic errors(t)

where T_1 and T_2 are the one- and two-way light times to the spacecraft, x is the distance of a solar wind plasma cloud from the earth, and μ is the cosine of the angle between the GW wavevector and a vector with its foot at the earth and its arrow at the spacecraft. A treatment of the noise sources and their spectra for a Ka-band observation is given by Riley et al. (1990); refinements of that model, based on the X-band Mars Observer observations in 1993, are given by Armstrong (1996) and Armstrong et al. (1997). For a properly-designed experiment at Ka-band, the leading noise sources will be the FTS and the residual troposphere after calibration.

Figure 1 shows the spectra of the principal noises for observations at S-, X-, and Ka-band. The power spectrum of $y = \Delta f/f_0$, $S_y(f)$, is plotted versus Fourier frequency on log-log scales. In some cases, measurements expressed as Allan deviation versus integration time have been converted to S_y under the assumption that S_y is powerlaw and smooth. References for the data are: Vessot and Levine (1978) for the GP-A raw S-band data; Armstrong, Woo, and Estabrook (1979) for the charged particles at elongations of 90 and 180 degrees, Keihm (1995) for the uncalibrated Goldstone winter troposphere, Armstrong and Sramek (1982) for the VLA troposphere; Cassini Project Document PD 699-501 (1995) for the stability of ground station plus residual uncalibrated troposphere for the gravity wave experiment. The Mars Observer X-band spectrum is unpublished, and the line marked 'frequency standard' is for a frequency standard with stability of 7×10^{-16} at 10^4 seconds and degrading like $\tau^{-1/2}$ for shorter integration times. It is clear that the tropospheric calibration is crucial for Ka-band observations.

IV A Specific Configuration Using Ka-Band Tracking Links

Any low-frequency gravity wave experiment added to a mission principally intended for redshift observations will necessarily increase the cost, if only because of the necessity to transfer the spacecraft to interplanetary orbit. Thus it is essential that the incremental cost be both low and credible (especially if the Clock Mission were to be proposed to a SMEX announcement of opportunity). Is there a way to get most or all of the sensitivity of a symmetrical multi-link system by building on a capability already in place?

In the Cassini-era (> April 2001), DSS 25 at Goldstone will be instrumented for high sensitivity two-way Ka-band tracking in support of Cassini radio science investigations, including an elaborate tropospheric calibration system. What would the sensitivity be if we were able to take advantage of the investment that the Cassini Project and DSN will have already made? In particular, suppose the Clock Mission had a simplified-version of the Cassini-heritage radio system: Ka-band two-way, plus a one-way downlink Doppler referenced to the high-quality on-board frequency standard required for the redshift experiment.

It is clear from Figure 1 that the two way link would be limited by frequency standard or residual troposphere noise for observations made in the antisolar direction, since the high radio frequency has drastically reduced charged particle effects compared to these other noises. Figure 2 shows how one might employ the extra information from the one-way link. The top diagram in Figure 2 is a space-time plot showing the earth and the spacecraft with some of the microwave photon paths illustrated. The gravity wave signature for a wavevector 60 degrees from the earth-spacecraft line is shown for the two-way and one-way links. If the tropospheric calibration is as good as expected, then this "5-pulse" response could be used in the GW pattern recognition algorithm. Alternatively, one could use the one-way data themselves to correct for the troposphere. Illustrated in Figure 2 are the tropospheric response functions for the two- and one-way links. One can cancel the troposphere to the level of the clocks by forming the observable $D_2(t) - (D_1(t) + D_1(t + T_2))$, shown in the bottom panel. This might be operationally preferable to the tropospheric calibration scheme and might be the most effective use of the one-way link. With this tropospheric cancellation or with calibration using water vapor radiometers, GW sensitivity at the Cassini-level would be possible for observations with elongation > 150 degrees; slightly degraded sensitivity would be possible to elongation ≈90 degrees (i.e., an observing duty cycle of ~1/2 year per year can be contemplated).

As a variation on this theme, one can also consider a configuration where one-way Doppler is additionally read out at the spacecraft and telemetered to the ground. This would require additional spacecraft instrumentation (the Doppler readout system). This provides the two-way and one-way signals described above, and another one-way signal received on the spacecraft. As before, this configuration gives the opportunity to use the data themselves to correct for the troposphere, and now there is the potential for a "7-pulse response" to gravitational waves: three pulses on the two-way link and two pulses on each of the one-way links. This offers a very powerful signature for rejection of false alarms.

V Some practical considerations

Adding a low-frequency GW experiment to a predominately earth-orbiting redshift experiment, although highly-desirable from a science viewpoint, does require additional cost and complexity to transfer the spacecraft to an interplanetary orbit. Also there is a possible power consideration: the maximum two-way light time might be set by distance over which solar power is practical, thus limiting Fourier bandpass to $f > 0.001$ Hz (or so). (This is not as bad as it might seem--as noted above, the transfer function for waves of duration $\tau >$ two-way light time is geometry-dependent and only falls off like two-way light time/τ, asymptotically, so sensitivity degrades gracefully at the low-frequency band "edge".) Finally, there is probably additional complexity in the attitude control system--the spacecraft would have to point a Ka-band high-gain antenna accurately enough for these observations.

VI Conclusions

Low-frequency gravity wave observations are a natural science addition to a redshift experiment. A Clock Mission with Ka-band radio links could reach gravitational wave sensitivity set by clock stability, not charged particles, at sun-earth-probe angles greater than 150 degrees. With minimally degraded sensitivity, observations could be made for solar elongations as small as 90 degrees (i.e., with a duty cycle of ~1/2 year of observations/year.) The use of a Ka-band radio system builds on ground-system investment for Cassini (station stability and tropospheric calibration); one would then get a Cassini-sensitivity experiment (at least for high-frequencies: $f > 0.001$ Hz) for a small incremental cost. As proposed here, "5- or 7-pulse response" allows tropospheric calibration plus some sensitivity improvement in signal processing (the efficacy of which will depend on waveform). The real utility of a 5- or 7-pulse-response, however, is tropospheric calibration and removal of systematic effects and false alarms, not SNR improvement. Finally, adding a gravity wave search to the Clock Mission fits in with the Zeitgeist--testing fundamental effects at the "right price".

Acknowledgment

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Figure Captions

Figure 1. Power spectrum of $y = \Delta f/f_0$, $S_y(f)$, versus Fourier frequency, for major noise sources. In some cases, measurements expressed as Allan deviation versus integration time have been converted to S_y under the assumption that S_y is powerlaw and smooth. See text for references.

Figure 2. Impulse responses in two- and one-way Doppler links, illustrating how the one-way Doppler link might be used to correct the two-way link for tropospheric scintillation to the level of the clock noise. See text for discussion.

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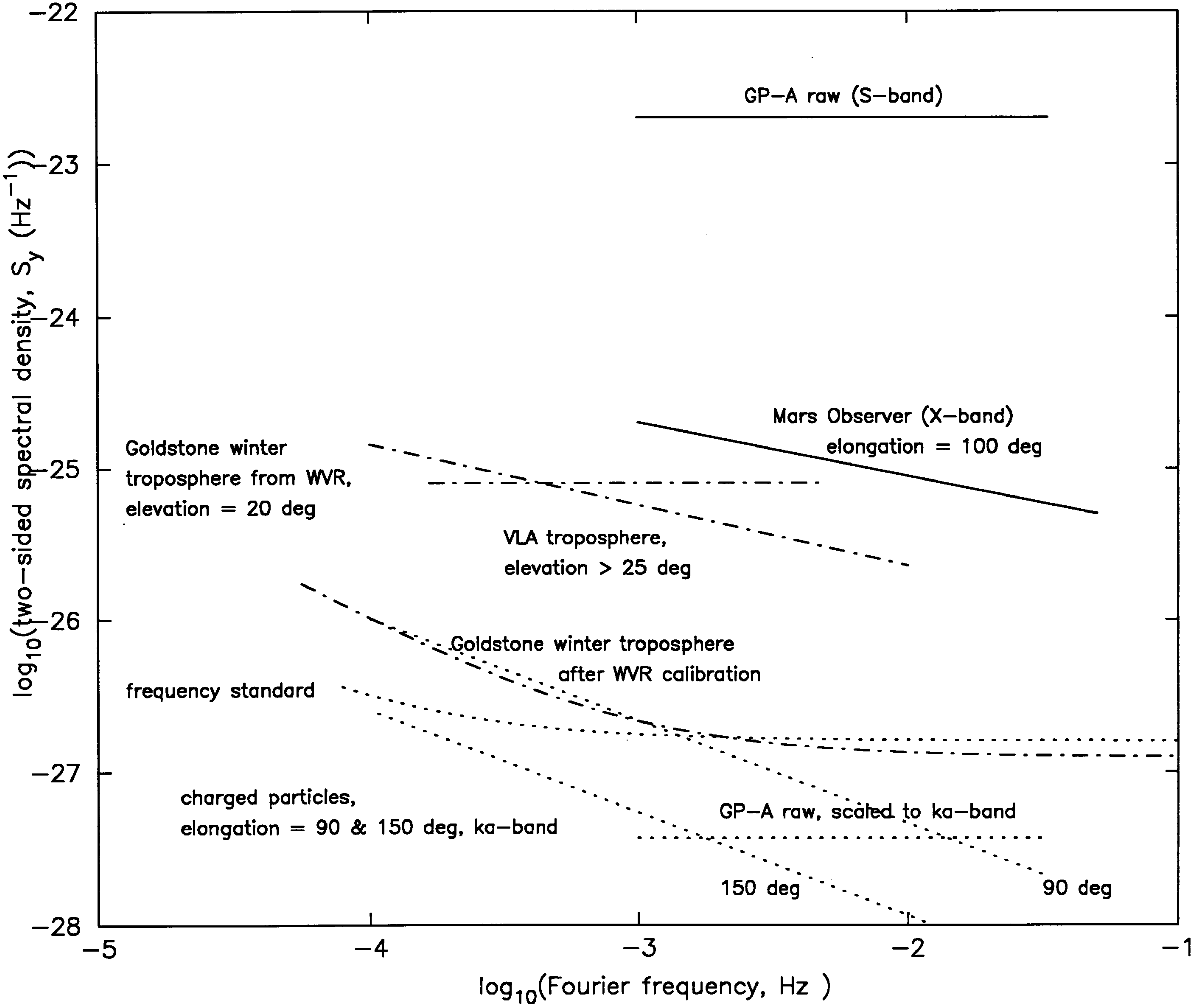
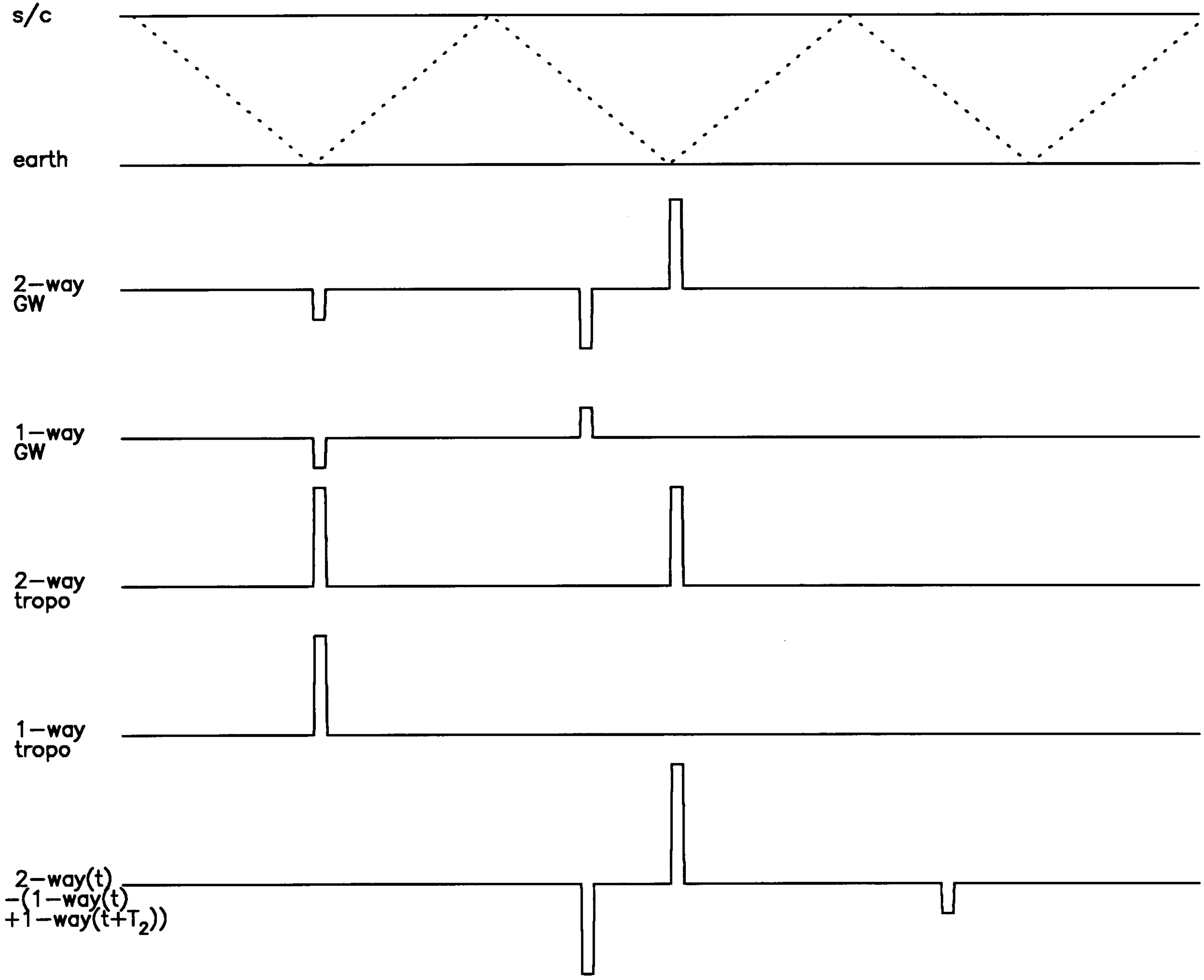

GP–A raw (S–band)
Mars Observer (X–band)
elongation = 100 deg
Goldstone winter
troposphere from WVR,
elevation = 20 deg
VLA troposphere,
elevation > 25 deg
Goldstone winter troposphere
after WVR calibration
frequency standard
charged particles,
elongation = 90 & 150 deg, ka–band
GP–A raw, scaled to ka–band
150 deg
90 deg
−22
−23
−24
−25
−26
−27
−28
−5
−4
−3
−2
−1
$\log_{10}$(two–sided spectral density, S_y (Hz^{-1}))
$\log_{10}$(Fourier frequency, Hz)


s/c
earth
2—way GW
1—way GW
2—way tropo
1—way tropo
2—way(t) −(1—way(t) +1—way(t+T_2))