SYNCHRONIZED FORMATION ROTATION AND ATTITUDE CONTROL OF MULTIPLE FREE-FLYING SPACECRAFT

P.K.C. Wang *
University of California
Los Angeles, CA 90095-1594

F.Y. Hadaegh † and K. Lau ‡
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, CA 91109-8099

ABSTRACT

In the observation slewing of long base-line interferometers formed by multiple free-flying spacecraft in formation, it is required to rotate the entire formation about a given axis, and to synchronize individual spacecraft rotation with formation rotation. Using a particle model for spacecraft formation dynamics, and a rigid-body model for spacecraft attitude dynamics, attitude control laws are derived for this mode of operation in the absence of gravitational field and disturbances. A simplified control law suitable for implementation is also obtained. It is shown that under mild conditions, the formation alignment error decays to zero exponentially with time. Computer simulation studies are made for a free-flying spacecraft triad in a triangular formation. The results show that the developed control laws are effective in synchronized formation rotation.

INTRODUCTION

Recent interest in developing long base-line interferometers using multiple spacecraft

* Professor, Department of Electrical Engineering, AIAA Member.
† Senior Research Scientist and Technical Supervisor, Automation and Control Section, Associate Fellow of AIAA.
‡ Senior Engineer, Guidance and Control Analysis Group.

led to the study of various problems in the coordination and control of multiple spacecraft in formation. The use of multiple spacecraft for Michelson stellar interferometry was first proposed by Stachnik, Ashin, and Hamilton[1,2]. The feasibility of this approach for spacecraft in geosynchronous and low-earth orbits was studied by Johnson and Nock [3], and DeCue [4]. The operation of such interferometers calls for observation slewing which requires the rotation of the entire formation about a given axis. Moreover, each spacecraft must rotate in synchronism with formation rotation. In this paper, we consider multiple free-flying spacecraft in formation which are of basic importance in long base-line deep space interferometers[5]. Attention is focused on developing control laws for formation rotation and attitude synchronization.

We begin with the basic dynamic model of the multiple spacecraft to be used in the development of control laws for formation flying. This is followed by a discussion of the formation rotation and attitude synchronization problem. Then, control laws for performing the required task are derived. The paper concludes with typical results of a computer simulation study for a free-flying spacecraft triad.

DYNAMIC MODEL FOR FORMATION FLYING

We assume there are N spacecraft to be flown in formation in the three dimensional Euclidean space R^3, and each spacecraft is a rigid body with fixed center of mass. Let $\mathcal{F}_o$ denote the inertial frame with origin $O \in R^3$, and $\mathcal{F}_i$ a moving body frame whose origin O_i is at the mass center of the i-th spacecraft (see Fig.1). For a given orthonormal basis $\mathcal{B}_i$ for $\mathcal{F}_i$, the representation of a vector $\mathbf{a} \in R^3$ with respect to $\mathcal{B}_i$ will be denoted by $[\mathbf{a}]^i$. Let $\mathbf{r}_i(t)$ denote the position of the mass center of the i-th spacecraft at time t in R^3 relative to $\mathcal{F}_o$. In the absence of gravitational field and disturbances, the evolution of $\mathbf{r}_i(t)$ with time is governed by

$$\mathcal{D}_i^2(\mathbf{r}_i) = \mathbf{f}_{ci}/M_i, \tag{1}$$

where $\mathbf{f}_{ci}$ and M_i denote the control thrust and mass of the i-th spacecraft respectively; $\mathcal{D}_i^2$ is the time derivative operator defined by

$$\mathcal{D}_i^2 \triangleq \frac{{}^i d^2(\cdot)}{dt^2} + \frac{{}^i d\omega_i}{dt} \times (\cdot) + 2\omega_i \times \frac{{}^i d(\cdot)}{dt} + \omega_i \times (\omega_i \times \cdot), \tag{2}$$

where ${}^i d/dt$ and ${}^i(d^2/dt^2)$ denote the time derivative operators with respect to the body frame $\mathcal{F}_i$, and ω_i the angular velocity of $\mathcal{F}_i$ with respect to the inertial frame $\mathcal{F}_o$.

In formation flying, it is of importance to consider the relative motion between any pair of spacecraft. Let $\rho_{ji} = \mathbf{r}_j - \mathbf{r}_i$ denote the position of the j-th spacecraft relative to $\mathcal{F}_i$. Using (1) and (2), it can be verified that the evolution of $\rho_{ji}(t)$ with time is governed by

$$\mathcal{D}_i^2(\rho_{ji}) = \mathbf{f}_{cj}/M_j - \mathbf{f}_{ci}/M_i. \tag{3}$$

The attitude and angular velocity of the i-th spacecraft with respect to the inertial frame $\mathcal{F}_o$ can be described by the following quaternion equations[6]:

$$\frac{{}^o d\hat{\mathbf{q}}_i}{dt} = (q_{i4}\omega_i - \omega_i \times \hat{\mathbf{q}}_i)/2,$$

$$\frac{dq_{i4}}{dt} = -(\omega_i \cdot \hat{\mathbf{q}}_i)/2. \tag{4}$$

and the Euler's equation:

$$\frac{{}^o d(\mathbf{I}_i\omega_i)}{dt} = \mathbf{I}_i \frac{{}^i d\omega_i}{dt} + \omega_i \times (\mathbf{I}_i\omega_i) = \tau_{ci}, \tag{5}$$

where the centered dot denotes scalar product in R^3; $\mathbf{q}_i = (\hat{\mathbf{q}}_i^T, q_{i4})^T$ denotes the unit quaternion with q_{ij} being the Euler symmetric parameters; and $\hat{\mathbf{q}}_i = (q_{i1}, q_{i2}, q_{i3})^T$; $\mathbf{I}_i$ and

τ_{ci} are the tensor of inertia in the body frame $\mathcal{F}_i$ and control torque associated with the i-th spacecraft respectively.

FORMATION ROTATION AND ATTITUDE SYNCHRONIZATION

There are many ways for flying spacecraft in formation. In the approach of Wang and Hadaegh[7,8], the first spacecraft is taken as the reference spacecraft whose motion $\mathbf{r}_1 = \mathbf{r}_1(t), t \geq 0$ serves as the reference motion for the remaining $N-1$ spacecraft (see Fig.1). The desired motion for the i-th spacecraft, $i = 2, \ldots, N$, is specified by

$$\mathbf{d}_i(t) = \mathbf{r}_1(t) + \mathbf{h}_i(t), \quad t \geq 0, \tag{6}$$

where $\mathbf{h}_i(t) \in R^3$ is a specified nonzero *deviation vector* defined for all $t \geq 0$. The i-th spacecraft tries to track the motion of the reference spacecraft such that the norm of the tracking error

$$\mathbf{E}_i(t) \stackrel{\Delta}{=} \mathbf{d}_i(t) - \mathbf{r}_i(t) = \rho_{1i}(t) + \mathbf{h}_i(t) \tag{7}$$

is as small as possible. The *desired formation pattern* at any time $t \geq 0$ is specified by the point set $\mathcal{P}(t) = \{\mathbf{r}_1(t), \mathbf{r}_1(t) + \mathbf{h}_2(t), \ldots, \mathbf{r}_1(t) + \mathbf{h}_N(t)\}$.

Another approach is based on *nearest-neighbor tracking.* Here the motion of the $(i-1)$-th spacecraft serves as the reference motion for the i-th spacecraft, and the desired motion for the i-th spacecraft is specified by $\mathbf{d}_i(t) = \mathbf{r}_{i-1}(t) + \mathbf{h}_i(t), t \geq 0$. This approach could lead to undesirable formation pattern instability or oscillations. In what follows, we shall consider only the former case.

Now, consider the *formation body* $\mathcal{C}(t)$ defined by the convex hull of $\mathcal{P}(t)$ (i.e. the set of points formed by all convex combinations of the points in $\mathcal{P}(t)$). Let $\mathbf{R}(t)$ be a specified nonzero vector in R^3. It is required to rotate $\mathcal{C}(t)$ about an axis defined by the line

$\mathcal{L}(t) = \mathbf{a}(t) + span\{\mathbf{R}(t)\}$ with specified angular velocity $\Omega(t)$ as illustrated in Fig.2, where $\mathbf{a}(t)$ is a specified vector in R^3. Moreover, the reference spacecraft must rotate about a given body axis in *synchronism* with the formation body rotation, and the remaining spacecraft must track the reference spacecraft's attitude and angular velocity. This implies that the desired angular speed of the reference spacecraft is $\|\Omega(t) + \Omega_o(t)\|$, and the desired angular velocity $\omega_i^d(t)$ and unit quaternion $\mathbf{q}_i^d(t)$ for the i-th spacecraft, $i = 2, \ldots, N$, are $\omega_1(t)$ and $\mathbf{q}_1(t)$ respectively, where $\Omega_o(t)$ is the angular velocity of $\mathcal{L}(t)$ with respect to $\mathcal{F}_o$. Note that $\omega_i^d(t)$ and $\mathbf{q}_i^d(t)$ must be *compatible* with each other in the sense that they satisfy

$$\frac{{}^o d\hat{\mathbf{q}}_i^d}{dt} = (q_{i4}^d \omega_i^d - \omega_i^d \times \hat{\mathbf{q}}_i^d)/2, \qquad \frac{dq_{i4}^d}{dt} = -(\omega_i^d \cdot \hat{\mathbf{q}}_i^d)/2, \tag{8a}$$

and

$$\frac{{}^o d(\mathbf{I}_i \omega_i^d)}{dt} = \mathbf{I}_i \frac{{}^i d\omega_i^d}{dt} + \omega_i^d \times (\mathbf{I}_i \omega_i^d) \triangleq \tau_{ci}^d. \tag{8b}$$

where $\mathbf{q}_i^d = ((\hat{\mathbf{q}}_i^d)^T, q_{i4}^d)^T$.

To clarify the foregoing notion of formation rotation and attitude synchronization, we consider a spacecraft triad flying in a triangular formation as shown in Fig.3. Let $[\mathbf{r}]^o = (x, y, z)^T$ denote the representation of a point $\mathbf{r}$ with respect to an orthonormal basis $\mathcal{B}_o = \{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ in the inertial frame $\mathcal{F}_o$. We assume that the reference spacecraft moves along a circular orbit $\mathcal{O} = \{(x, y, z) : x^2 + y^2 = a_o^2, z = z_o\}$ with radius a_o and constant angular velocity $-\omega_o \mathbf{e}_z$, and the position of the reference spacecraft is given by

$$\mathbf{r}_1(t) = (a_o \cos(\omega_o t))\mathbf{e}_x - (a_o \sin(\omega_o t))\mathbf{e}_y + z_o \mathbf{e}_z. \tag{9}$$

To define the desired initial formation, we introduce constant deviation vectors $\mathbf{h}_2^o$ and $\mathbf{h}_3^o$:

$$\mathbf{h}_2^o = h_{2x}^o \mathbf{e}_x + h_{2y}^o \mathbf{e}_y + h_{2z}^o \mathbf{e}_z,$$

$$\mathbf{h}_3^o = -h_{2x}^o \mathbf{e}_x - h_{2y}^o \mathbf{e}_y + h_{2z}^o \mathbf{e}_z. \tag{10a}$$

Then the desired initial formation pattern at time t is specified by the point set

$$\mathcal{P}(t) = \{\mathbf{r}_1(t), \mathbf{r}_1(t) + \mathbf{h}_2^o, \mathbf{r}_1(t) + \mathbf{h}_3^o\} \tag{10b}$$

and the formation body $\mathcal{C}(t)$ is the plane domain bounded by the isosceles triangle with vertices given by $\mathcal{P}(t)$. For a space interferometer, the combiner and collectors correspond to the reference and the remaining spacecraft respectively[1−5]. We assume that the spacecraft body is a rigid cylinder with uniform mass density. Let $\mathcal{B}_i = \{\mathbf{e}_{x'}^i, \mathbf{e}_{y'}^i, \mathbf{e}_{z'}^i\}$ be an orthonormal basis for the body frame $\mathcal{F}_i$ for the i-th spacecraft such that $\mathbf{e}_{z'}^i$ is along the cylinder axis which is also aligned with $\mathbf{e}_z$. We consider two different cases: (a) the x'-axis is aligned with x-axis; and (b) the y'-axis passes through the center of the circular orbit $\mathcal{O}$ at all times, except during formation rotation (see Fig.3). Case (a) corresponds to the situation where it is required to have the instruments or solar panels onboard the spacecraft pointing to a fixed direction. Thus, the desired angular velocities ω_d^1 for Cases (a) and (b) are zero and $-\omega_o \mathbf{e}_z$ respectively.

Now, suppose that the objective is to rotate the triangular formation about the reference spacecraft's cylinderical body axis with constant angular velocity $\Omega = -\omega_R \mathbf{e}_z$. Thus, the rotation axis $\mathcal{L}(t)$ for $\mathcal{C}(t)$ is simply $span\{\mathbf{e}_z\}$, and the deviation vectors $\mathbf{h}_2(t)$ and $\mathbf{h}_3(t)$ take on the form:

$$\mathbf{h}_2(t) = (h_{2x}^o \cos(\omega_R t) - h_{2y}^o \sin(\omega_R t))\mathbf{e}_x + (h_{2x}^o \sin(\omega_R t) + h_{2y}^o \cos(\omega_R t))\mathbf{e}_y + h_{2z}^o \mathbf{e}_z,$$

$$\mathbf{h}_3(t) = (-h_{2x}^o \cos(\omega_R t) + h_{2y}^o \sin(\omega_R t))\mathbf{e}_x + (-h_{2x}^o \sin(\omega_R t) - h_{2y}^o \cos(\omega_R t))\mathbf{e}_y + h_{2z}^o \mathbf{e}_z. \tag{11}$$

During formation rotation, the desired angular velocity ω_1^d for the reference spacecraft relative to the inertial frame $\mathcal{F}_o$ is $-\omega_R \mathbf{e}_z$ for Case (a), and $-(\omega_o + \omega_R)\mathbf{e}_z$ for Case (b). The corresponding desired quaternion for the reference spacecraft can be determined from (8a) which can be written explicitly as

$$\frac{{}^{o}d}{dt}\begin{bmatrix} q_{11}^{d} \\ q_{12}^{d} \\ q_{13}^{d} \\ q_{14}^{d} \end{bmatrix} = \begin{bmatrix} 0 & \omega_T & 0 & 0 \\ -\omega_T & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_T \\ 0 & 0 & -\omega_T & 0 \end{bmatrix} \begin{bmatrix} q_{11}^{d} \\ q_{12}^{d} \\ q_{13}^{d} \\ q_{14}^{d} \end{bmatrix}, \tag{12}$$

where $\omega_T = \omega_R/2$ for Case (a), and $\omega_T = (\omega_o + \omega_R)/2$ for Case (b).

The solution to (12) with initial condition $(\hat{\mathbf{q}}_1(t_1)^T, q_{14}(t_1))^T$ at time t_1 (the starting time for formation rotation) is given by

$$\begin{bmatrix} q_{11}^{d}(t) \\ q_{12}^{d}(t) \end{bmatrix} = \Phi(t - t_1) \begin{bmatrix} q_{11}(t_1) \\ q_{12}(t_1) \end{bmatrix}, \quad \begin{bmatrix} q_{13}^{d}(t) \\ q_{14}^{d}(t) \end{bmatrix} = \Phi(t - t_1) \begin{bmatrix} q_{13}(t_1) \\ q_{14}(t_1) \end{bmatrix}, \quad t \geq t_1, \tag{13}$$

where

$$\Phi(t - t_1) = \begin{bmatrix} \cos(\omega_T(t - t_1)) & \sin(\omega_T(t - t_1)) \\ -\sin(\omega_T(t - t_1)) & \cos(\omega_T(t - t_1)) \end{bmatrix}. \tag{14}$$

CONTROL LAWS

To derive a control law for formation acquisition and rotation, we consider the following equation for the tracking error $\mathbf{E}_i(t), i = 2, \ldots, N$ defined by (7):

$$\mathcal{D}_i^2(\mathbf{E}_i) \triangleq \ddot{\mathbf{E}}_i + \dot{\omega}_i \times \mathbf{E}_i + 2\omega_i \times \dot{\mathbf{E}}_i + \omega_i \times (\omega_i \times \mathbf{E}_i) = \mathcal{D}_i^2(\mathbf{h}_i) + \mathbf{f}_{c1}/M_1 - \mathbf{f}_{ci}/M_i, \tag{15}$$

and the following positive definite function of $(\mathbf{E}_i, \dot{\mathbf{E}}_i)$ defined on R^6:

$$V_{1i} = (K_{1i}\mathbf{E}_i \cdot \mathbf{E}_i + \dot{\mathbf{E}}_i \cdot \dot{\mathbf{E}}_i)/2, \quad K_{1i} > 0, \tag{16}$$

where $\dot{\mathbf{E}}_i \overset{\Delta}{=} {}^i d\mathbf{E}_i/dt$, $\ddot{\mathbf{E}}_i \overset{\Delta}{=} {}^i d^2\mathbf{E}_i/dt^2$, and $\dot{\omega}_i \overset{\Delta}{=} {}^i d\omega_i/dt$. The time rate-of-change of V_{1i} along any solution of (15) is given by

$$dV_{1i}/dt = \dot{\mathbf{E}}_i \cdot \{\mathcal{D}_i^2(\mathbf{h}_i) + \mathbf{f}_{c1}/M_1 - 2\omega_i \times \dot{\mathbf{E}}_i + \mathbf{I}_i^{-1}(\omega_i \times (\mathbf{I}_i\omega_i) - \tau_{ci}) \times \mathbf{E}_i$$

$$-\omega_i \times (\omega_i \times \mathbf{E}_i) + K_{1i}\mathbf{E}_i - \mathbf{f}_{ci}/M_i\} = \dot{\mathbf{E}}_i \cdot \{\mathbf{w}_i - \mathbf{f}_{ci}/M_i\}, \tag{17}$$

where

$$\mathbf{w}_i = \mathbf{f}_{c1}/M_1 + \mathbf{I}_i^{-1}(\omega_i \times (\mathbf{I}_i\omega_i) - \tau_{ci}) \times \mathbf{E}_i - (\omega_i \cdot \mathbf{E}_i)\omega_i + (K_{1i} + \|\omega_i\|^2)\mathbf{E}_i + \mathcal{D}_i^2(\mathbf{h}_i). \tag{18}$$

If we set

$$\mathbf{f}_{ci}/M_i = \mathbf{w}_i + K_{2i}\dot{\mathbf{E}}_i, \tag{19}$$

where K_{2i} is a positive constant, then $dV_{i1}/dt = -K_{2i}\|\dot{\mathbf{E}}_i\|^2 \leq 0$, and (15) reduces to

$$\ddot{\mathbf{E}}_i(t) + K_{2i}\dot{\mathbf{E}}_i(t) + 2\omega_i(t) \times \dot{\mathbf{E}}_i(t) + K_{1i}\mathbf{E}_i(t) = \mathbf{0}. \tag{20}$$

Since $\dot{\mathbf{E}}_i(t) \cdot (\omega_i(t) \times \dot{\mathbf{E}}_i(t)) = 0$ for all $\omega_i(t)$, it follows that all solutions $(\mathbf{E}_i(t), \dot{\mathbf{E}}_i(t))$ of (20) tend to $(\mathbf{0}, \mathbf{0}) \in R^6$ as $t \to \infty$ for any $K_{1i}, K_{2i} > 0, i = 2, \ldots, N$. Moreover, the zero state of (20) is totally stable or stable under persistent disturbances[6,8]. This property implies that asymptotic stability of the zero state of (20) is preserved in the presence of small state-dependent perturbations.

In the case where the components of the control thrusts $\mathbf{f}_{ci}$ are amplitude limited, i.e. $[\mathbf{f}_{ci}]^i = (f_{ci1}, f_{ci2}, f_{ci3})^T$ satisfies $|f_{cij}| \leq F_{ci}, j = 1, 2, 3$, where $F_{ci}, i = 2, \ldots, N$ are given positive constants, we consider again (17) rewritten in reference to a given orthonormal basis $\mathcal{B}_i$ for $\mathcal{F}_i$:

$$dV_{1i}/dt = \sum_{j=1}^{3}\{[\dot{\mathbf{E}}_i]_j^i([\mathbf{w}_i]_j^i - f_{cij}/M_i)\}, \tag{17'}$$

where $[\mathbf{w}_i]_j^i$ denotes the j-th component of $[\mathbf{w}_i]^i$. If we set

$$f_{cij} = F_{ci}\text{sat}(g_{ij}(\omega_i, [\mathbf{E}_i]_j^i, [\dot{\mathbf{E}}_i]_j^i, \mathbf{f}_{c1}, \mathbf{h}_i)), \qquad j = 1, 2, 3, \tag{21}$$

where sat$(\cdot)$ denotes the saturation function defined by sat(a) = sign(a) if $|a| \geq 1$ and sat(a) $= a$ if $|a| \leq 1$, and

$$g_{ij}(\omega_i, [\mathbf{E}_i]^i_j, [\dot{\mathbf{E}}_i]^i_j, \mathbf{f}_{c1}, \mathbf{h}_i) \triangleq \frac{M_i}{F_{ci}}\left([\mathbf{w}_i]^i_j + K_{2i}[\dot{\mathbf{E}}_i]^i_j\right), \qquad j = 1, 2, 3, \tag{22}$$

then

$$dV_{1i}/dt = \sum_{j=1}^{3}\{[\dot{\mathbf{E}}_i]^i_j([\mathbf{w}_i]^i_j - \frac{F_{ci}}{M_i}\text{sat}(g_{ij}(\omega_i, [\mathbf{E}_i]^i_j, [\dot{\mathbf{E}}_i]^i_j, \mathbf{f}_{c1}, \mathbf{h}_i))\}, \tag{23}$$

For control law (21), (15) has the following representation with respect to basis $\mathcal{B}_i$:

$$[\mathcal{D}_i^2(\mathbf{E}_i)]^i_j = [\mathcal{D}_i^2(\mathbf{h}_i) + \mathbf{f}_{c1}/M_1]^i_j - \frac{F_{ci}}{M_i}\text{sat}(g_{ij}(\omega_i, [\mathbf{E}_i]^i_j, [\dot{\mathbf{E}}_i]^i_j, \mathbf{f}_{c1}, \mathbf{h}_i)), \qquad j = 1, 2, 3. \tag{24}$$

It can be readily verified that if $\|\omega_i(t)\|, \|\mathbf{f}_{c1}(t)\|, \|\mathcal{D}_i^2(\mathbf{h}_i)(t)\|$, and $\|\tau_{ci}(t)\|$ are uniformly bounded for all $t \geq 0$, then the set $\{([\mathbf{E}_i]^i, [\dot{\mathbf{E}}_i]^i) \in R^6 : |g_{ij}(\omega_i, [\mathbf{E}_i]^i_j, [\dot{\mathbf{E}}_i]^i_j, \mathbf{f}_{c1}, \mathbf{h}_i)| \leq 1, j = 1, 2, 3\}$ contains a neighborhood of the zero state $(\mathbf{0}, \mathbf{0}) \in R^6$. Consequently, any solution $[(\mathbf{E}_i(t), \dot{\mathbf{E}}_i(t))]^i$ of (24) tends to the zero state as $t \to \infty$ for any feedback gains K_{1i} and $K_{2i} > 0$, and sufficiently small $\|([\mathbf{E}_i(0)]^i, [\dot{\mathbf{E}}_i(0)]^i)\|$.

To derive control laws for the reference and remaining spacecraft to achieve attitude synchronization, we introduce

$$\delta\mathbf{q}_i \triangleq \mathbf{q}_i^d - \mathbf{q}_i = ((\hat{\mathbf{q}}_i^d - \hat{\mathbf{q}}_i)^T, q_{i4}^d - q_{i4})^T, \quad \delta\omega_i \triangleq \omega_i^d - \omega_i. \tag{25}$$

where $\mathbf{q}_i^d = \mathbf{q}_1$, and $\omega_i^d = \omega_1$, for $i = 2, \ldots, N$. It follows from (4),(5), and (8) that $\delta\mathbf{q}_i$ and $\delta\omega_i$ satisfy

$$\frac{{}^{o}d\delta\hat{\mathbf{q}}_i}{dt} = (q_{i4}^d\omega_i^d - q_{i4}\omega_i - \omega_i^d \times \mathbf{q}_i^d + \omega_i \times \hat{\mathbf{q}}_i)/2,$$

$$\frac{d\delta q_{14}}{dt} = -(\omega_i^d \cdot \hat{\mathbf{q}}_i^d - \omega_i \cdot \hat{\mathbf{q}}_i)/2. \tag{26}$$

and

$$\frac{{}^{o}d(\mathbf{I}_i\delta\omega_i)}{dt} = \mathbf{I}_i\frac{{}^{i}d\delta\omega_i}{dt} + \delta\omega_i \times \mathbf{I}_i\omega_i^d + \omega_i^d \times \mathbf{I}_i\delta\omega_i - \delta\omega_i \times \mathbf{I}_i\delta\omega_i = \tau_{ci}^d - \tau_{ci}. \tag{27}$$

By requiring the time rate-of-change of the following positive definite function along any solution of (26) and (27) to be nonpositive for all $t \geq 0$:

$$V_{2i} = K_{qi}(\delta q_{i4}^2 + \delta\hat{\mathbf{q}}_i \cdot \delta\hat{\mathbf{q}}_i) + (\delta\omega_i \cdot \mathbf{I}_i\delta\omega_i)/2, \tag{28}$$

we obtain the following attitude control law:

$$\tau_{ci} = K_{qi}\mathbf{s} + K_{\omega i}\mathbf{I}_i\delta\omega_i + \tau_{ci}^d - \omega_i^d \times (\mathbf{I}_i\delta\omega_i)/2, \tag{29}$$

where K_{qi} and $K_{\omega i}$ are positive constant feedback gains, and

$$\mathbf{s} = q_{i4}^d\delta\hat{\mathbf{q}}_i - \delta q_{i4}\hat{\mathbf{q}}_i^d - \hat{\mathbf{q}}_i^d \times \delta\hat{\mathbf{q}}_i. \tag{30}$$

It can be verified[7] that under the action of control law (29), any solution $(\delta\hat{\mathbf{q}}_i(t), \delta q_{14}(t), \delta\omega_i(t))$ of (26) and (27) tends to $(0,0,0) \in R^7$ as $t \to \infty$.

In the case where the control torques τ_{ci} are amplitude limited (i.e. $[\tau_{ci}]^i = (\tau_{ci1}, \tau_{ci2}, \tau_{ci3})^T$ satisfies $|\tau_{cij}| \leq T_{ci}, j = 1,2,3$), where $T_{ci}, i = 1,\ldots,N$ are given positive constants, we set

$$[\tau_{ci}]_j^i = T_{ci}\mathrm{sat}\{[K_{qi}\mathbf{s} + K_{\omega i}\mathbf{I}_i\delta\omega_i + \tau_{ci}^d - \omega_i^d \times (\mathbf{I}_i\delta\omega_i)/2]_j^i/T_{ci}\}, \quad j = 1,2,3. \tag{31}$$

Here, since the control torques are amplitude limited, loss of synchronization can occur when the angular speed of formation rotation or the reference spacecraft is so high that the remaining spacecraft are unable to track the formation rotation. The possible occurrence of this phenomenon can be deduced from the following inequality obtained by first integrating

(27) from time t_o to t, and making use of the triangle inequality $\|\mathbf{a}-\mathbf{b}\| \geq \|\mathbf{a}\| - \|\mathbf{b}\|$ for norms of vectors $\mathbf{a}$ and $\mathbf{b}$:

$$\|\mathbf{I}_i(\delta\omega_i(t) - \delta\omega_i(t_o))\|_\infty = \left\| \int_{t_o}^{t} (\tau_{ci}^d(s) - \tau_{ci}(s))ds \right\|_\infty$$

$$\geq \left\| \int_{t_o}^{t} \tau_{ci}^d(s)ds \right\|_\infty - \left\| \int_{t_o}^{t} \tau_{ci}(s)ds \right\|_\infty \geq \left\| \int_{t_o}^{t} \tau_{ci}^d(s)ds \right\|_\infty - \int_{t_o}^{t} \|\tau_{ci}(s)\|_\infty ds$$

$$\geq \left\| \int_{t_o}^{t} \tau_{ci}^d(s)ds \right\|_\infty - T_{ci}(t-t_o), \qquad t \geq t_o, \tag{32}$$

where $\|\mathbf{a}\|_\infty \stackrel{\Delta}{=} \max\{|a_i|; i = 1,2,3\}$ and $\mathbf{a} = (a_1, a_2, a_3)$. Now, suppose that the desired angular speed is such that the corresponding τ_{ci}^d's satisfy $\| \int_{t_o}^{t} \tau_{ci}^d(s)ds\|_\infty - T_{ci}(t-t_o) \geq \eta_i(t-t_o)$ for all t in some time interval $I_T = [t_o, T]$, and some positive constant η_i. Then the norm of the deviation in the angular momentum of the i-th spacecraft from its desired value grows with time over I_T. Consequently, loss of synchronization results.

From (18) and (19), it is evident that the formation control law consists of feedback terms involving $\mathbf{E}_i(t)$, $\dot{\mathbf{E}}_i(t), \omega_i(t)$ and $\tau_{ci}(t)$; and feedforward terms involving $\mathcal{D}_i^2(\mathbf{h}_i)(t)$ and the control thrust $\mathbf{f}_{c1}(t)$ of the reference spacecraft. Figure 4 shows the structure of the overall control system. The reference-path parameters are fed into the reference-path command generator which produces the necessary data for generating the controls for the reference spacecraft. Similarly, the formation-pattern parameters are fed into the formation-command generator for the i-th spacecraft, whose output along with the displacement, velocity, quaternion, and angular velocity of the i-th spacecraft relative to the reference spacecraft are used in generating the controls for the i-th spacecraft.

SIMPLIFIED CONTROL LAWS

Control laws (19) and (21) for formation acquisition and rotation have complex forms which cannot be readily implemented. In what follows, we shall consider a simplified version of control (19) given by

$$\mathbf{f}_{ci}/M_i = \mathbf{K}_{1i}\mathbf{E}_i + \mathbf{K}_{2i}\dot{\mathbf{E}}_i + \tilde{\mathbf{w}}_i, \tag{33}$$

where $\mathbf{K}_{1i}$ and $\mathbf{K}_{2i}$ are positive self-adjoint linear transformations on R^3 onto R^3, and

$$\tilde{\mathbf{w}}_i = \mathbf{f}_{c1}/M_1 + \mathcal{D}_i^2(\mathbf{h}_i). \tag{34}$$

Substituting (33) into (15) leads to the following linear time-varying differential equation for the tracking error:

$$\ddot{\mathbf{E}}_i(t) + \mathbf{R}_i(t)\dot{\mathbf{E}}_i(t) + \mathbf{S}_i(t)\mathbf{E}_i(t) = \mathbf{0}, \tag{35}$$

where $\mathbf{R}_i(t)$ and $\mathbf{S}_i(t)$ are linear transformations on R^3 into R^3 defined by

$$\mathbf{R}_i(t) = \mathbf{K}_{2i} + 2\omega_i(t) \times (\cdot), \tag{36}$$

and

$$\mathbf{S}_i(t) = \dot{\omega}_i(t) \times (\cdot) + \omega_i(t)(\omega_i(t) \cdot) + (\mathbf{K}_{1i} - \|\omega_i(t)\|^2\mathbf{I}). \tag{37}$$

Consider the following function defined on R^6:

$$\tilde{V}_i = (\mathbf{E}_i \cdot \mathbf{K}_{1i}\mathbf{E}_i + \dot{\mathbf{E}}_i \cdot \dot{\mathbf{E}}_i + 2\gamma\dot{\mathbf{E}}_i \cdot \mathbf{E}_i)/2, \tag{38}$$

where γ is a positive constant. Since $\mathbf{K}_{1i}$ is a positive self-adjoint linear transformation, hence

$$\mathbf{E}_i \cdot \mathbf{K}_{1i}\mathbf{E}_i \geq \lambda_{min}(\mathbf{K}_{1i})\|\mathbf{E}_i\|^2, \tag{39}$$

where $\lambda_{min}(\mathbf{K}_{1i})$ denotes the minimum eigenvalue of $\mathbf{K}_{1i}$. From (39) and the inequality $\dot{\mathbf{E}}_i \cdot \mathbf{E}_i \geq -\|\dot{\mathbf{E}}_i\|\|\mathbf{E}_i\|$, it follows that $\tilde{V}_i$ satisfies the lower bound:

$$\tilde{V}_i \geq \frac{1}{2}\begin{bmatrix} \|\mathbf{E}_i\| \\ \|\dot{\mathbf{E}}_i\| \end{bmatrix}^T \mathbf{P} \begin{bmatrix} \|\mathbf{E}_i\| \\ \|\dot{\mathbf{E}}_i\| \end{bmatrix}$$

$$\geq \frac{1}{2}\lambda_{min}(\mathbf{P})\|(\mathbf{E}_i, \dot{\mathbf{E}}_i)\|^2, \tag{40}$$

where $\|(\mathbf{E}_i, \dot{\mathbf{E}}_i)\|^2 = \|\mathbf{E}_i\|^2 + \|\dot{\mathbf{E}}_i\|^2$;

$$\mathbf{P} = \begin{bmatrix} \lambda_{min}(\mathbf{K}_{1i}) & -\gamma \\ -\gamma & 1 \end{bmatrix}. \tag{41}$$

Thus, if

$$\lambda_{min}(\mathbf{K}_{1i}) \geq \gamma^2, \tag{42}$$

then $\tilde{V}_i$ is positive definite on R^6. We shall show that under certain mild conditions, the zero state of (35) is exponentially stable.

Consider the time-rate-of-change of $\tilde{V}_i$ given by

$$d\tilde{V}_i/dt = \gamma\mathbf{E}_i \cdot (\|\omega_i\|^2\mathbf{I} - \mathbf{K}_{1i})\mathbf{E}_i + \dot{\mathbf{E}}_i \cdot (\gamma\mathbf{I} - \mathbf{K}_{2i})\dot{\mathbf{E}}_i - \gamma(\omega_i \cdot \mathbf{E}_i)^2$$

$$-\dot{\mathbf{E}}_i \cdot (\dot{\omega}_i \times \mathbf{E}_i) - (\omega_i \cdot \mathbf{E}_i)(\dot{\mathbf{E}}_i \cdot \omega_i) + \mathbf{E}_i \cdot (\|\omega_i\|^2\mathbf{I} - \gamma\mathbf{K}_{2i})\dot{\mathbf{E}}_i - 2\gamma\mathbf{E}_i \cdot (\omega_i \times \dot{\mathbf{E}}_i). \tag{43}$$

From (5), it is evident that for bounded control torque τ_{ci} (i.e. $\|\tau_{ci}(t)\| \leq \hat{\tau}_{ci} < \infty$ for all $t \geq 0$), there exist positive constants α_i and β_i such that

$$\sup\{\|\omega_i(t)\|; t \geq 0\} \leq \alpha_i < \infty, \qquad \sup\{\|\dot{\omega}_i(t)\|; t \geq 0\} \leq \beta_i < \infty. \tag{44}$$

Then $d\tilde{V}_i/dt$ satisfies the following estimate:

$$d\tilde{V}_i/dt \leq -\begin{bmatrix} \|\mathbf{E}_i\| \\ \|\dot{\mathbf{E}}_i\| \end{bmatrix}^T \mathbf{Q} \begin{bmatrix} \|\mathbf{E}_i\| \\ \|\dot{\mathbf{E}}_i\| \end{bmatrix} \tag{45}$$

where

$$\mathbf{Q} = \begin{bmatrix} \gamma(\lambda_{min}(\mathbf{K}_{1i}) - \alpha_i^2) & \delta_{1i} \\ \delta_{1i} & \lambda_{min}(\mathbf{K}_{2i}) - \gamma \end{bmatrix}, \tag{46}$$

$\delta_{1i} = (\beta_i - 2\gamma\alpha_i + \alpha_i^2 + \gamma\|\mathbf{K}_{2i}\|)/2$. Now, if

$$\lambda_{min}(\mathbf{K}_{1i}) > \alpha_i^2, \quad \lambda_{min}(\mathbf{K}_{2i}) > \gamma,$$

$$\gamma(\lambda_{min}(\mathbf{K}_{1i}) - \alpha_i^2)(\lambda_{min}(\mathbf{K}_{2i}) - \gamma) > \delta_{1i}^2, \tag{47}$$

then $\mathbf{Q}$ is positive definite. Thus, under conditions (42) and (47), we have

$$d\tilde{V}_i/dt \leq -\lambda_{min}(\mathbf{Q})(\|\mathbf{E}_i\|^2 + \|\dot{\mathbf{E}}_i\|^2)$$

$$\leq -2\{\lambda_{min}(\mathbf{Q})/\lambda_{min}(\mathbf{P})\}\tilde{V}_i. \tag{48}$$

It follows that

$$\tilde{V}_i(t) \leq \tilde{V}_i(0)\exp\{-2(\lambda_{min}(\mathbf{Q})/\lambda_{min}(\mathbf{P}))t\} \tag{49}$$

for all $t \geq 0$, which implies exponential stability of the zero state of (35).

Remark: The inclusion of the term $\tilde{\mathbf{w}}_i$ in control law (33) is essential in achieving exponential stability. Since $\tilde{\mathbf{w}}_i$ given by (34) involves the deviation vector $\mathbf{h}_i(t)$ and control law $\mathbf{f}_{c1}$ associated with the reference spacecraft, these data must be transmitted to the i-th spacecraft. If the i-th spacecraft receives the deviation vector $\mathbf{h}_i(t)$ and its velocity $\dot{\mathbf{h}}_i(t)$ from the reference spacecraft, and has onboard sensors to measure the relative position $\rho_{i1}(t)$ and velocity $\dot{\rho}_{i1}(t)$, then the positional error and error rate can be determined by $\mathbf{E}_i(t) = \mathbf{h}_i(t) - \rho_{i1}(t)$ and $\dot{\mathbf{E}}_i(t) = \dot{\mathbf{h}}_i(t) - \dot{\rho}_{i1}(t)$ respectively. These data are required for the implementation of the simplified control law (33).

SIMULATION STUDY

To determine the effectiveness of the proposed control laws, we consider a spacecraft triad discussed earlier. The spacecraft parameter values are given in the Appendix. We only present typical simulation results for Case (a) where the desired angular velocity for the reference spacecraft is zero before and after formation rotation.

Consider the scenario where the first spacecraft generates a circular reference orbit for the remaining two spacecraft given by (9) with $\omega_o = 0.071$ rad/sec. The spacecraft are initially clustered and expand to a triangular formation pattern specified by $\mathcal{P}(t)$ given by (10b) with

$$\mathbf{h}_2^o = 4\mathbf{e}_y - 6\mathbf{e}_z, \quad \mathbf{h}_3^o = -4\mathbf{e}_y - 6\mathbf{e}_z. \tag{50}$$

At $t = 50$ sec, the formation begins rotation for half a revolution about the vertical axis with desired constant angular velocity $\Omega = -0.25\mathbf{e}_z$ rad/sec. It is required that the reference spacecraft also rotates about the vertical axis in synchronism with the formation rotation, and the remaining two spacecraft track the attitude of the reference spacecraft. At $t = 100$ sec, the spacecraft triad expands to a larger triangular formation with $\mathcal{P}(t)$ given by (10b) with

$$\mathbf{h}_2^o = -8\mathbf{e}_y - 12\mathbf{e}_z, \quad \mathbf{h}_3^o = 8\mathbf{e}_y - 12\mathbf{e}_z. \tag{51}$$

Finally, at $t = 150$, the formation rotation is repeated. Typical simulation results for control law (19) are shown in Fig.5 depicting the trajectories of the spacecraft triad in 3-dimensional space. The time records for the quaternions and angular velocities are shown in Figs.6a and 6b respectively. The time records for the attitude errors and the norms of the position tracking errors for spacecraft 2 and 3 relative to the reference spacecraft are shown in Figs.7a and 7b respectively. The time records for the distances between spacecraft 2,3 and the reference spacecraft are shown in Fig.8. Here, the feedback gains K_{1i}, K_{2i}, K_{qi} and $K_{\omega i}$ have been tuned to achieve rapid response with acceptable overshoot. Their values are given in the Appendix. Simulation studies for the case with simplified control law (33) with $\mathbf{K}_{1i} = K_{1i}\mathbf{I}$ and $\mathbf{K}_{2i} = K_{2i}\mathbf{I}$ have also been obtained. The results do not differ appreciably from those given here for control law (19). A video depicting the simulation results for various scenarios in synchronized formation rotation is available[10].

CONCLUSION

In this paper, control laws for formation rotation of multiple free-flying spacecraft about a given axis, and synchronization of individual spacecraft rotation with formation rotation are derived using a simplified dynamic model in the absence of gravitational field and other disturbances. Simulation studies based on a generic spacecraft model showed that the derived control laws are effective for this mode of operation. In the presence of amplitude limited control thrusts, the derived control laws are effective provided that the formation rotation speed is sufficiently low. Various problems associated with the implementation of the control laws for real spacecraft, and the effect of gravitational and environmental disturbances on the spacecraft motion are not considered here. They require further study.

Acknowledgment

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APPENDIX

Parameter Values for Simulation Study

Spacecraft mass (kg):

$$M_1 = 20;\ M_2 = M_3 = 10.$$

Moment of inertia about x_i-axis (kg m^2):

$$I_{x1} = 0.7290;\ I_{x2} = I_{x3} = 0.3645.$$

Moment of inertia about y_i-axis (kg m^2):

$$I_{y1} = 0.54675;\ I_{y2} = I_{y3} = 0.2734.$$

Moment of inertia about z_i-axis (kg m^2):

$$I_{z1} = 0.625;\ I_{z2} = I_{z3} = 0.3125.$$

Feedback gains:

$$K_{1i} = 1, \quad K_{2i} = 10, \quad i = 1, 2, 3;$$

$$K_{q1} = 1; \quad K_{q2} = K_{q3} = 0.335.$$

$$K_{\omega 1} = 5.5; \quad K_{\omega 2} = K_{\omega 3} = 10;$$

Figure Captions

Fig.1 Sketch of coordinate systems.

Fig.2 Formation rotation about the line $\mathcal{L}(t)$.

Fig.3 Rotation of spacecraft triad in a triangular formation.

Fig.4 Structure of the overall control system.

Fig.5 Trajectories of a spacecraft triad undergoing formation reconfiguration and rotation in 3-dimensional space.

Fig.6a Quaternion components of spacecraft triad versus time.

Fig.6b Angular velocity components of spacecraft triad versus time.

Fig.7a Time records for the attitude errors of spacecraft 2 and 3 relative to the reference spacecraft (A_{1i} - relative attitude angle between the i-th and the reference spacecraft).

Fig.7b Time records for the norm of position tracking errors of spacecraft 2 and 3 relative the reference spacecraft.

Fig.8 Time records for the distances between spacecraft 2,3 and the reference spacecraft (d_{i1}- distance between the i-th and the reference spacecraft).

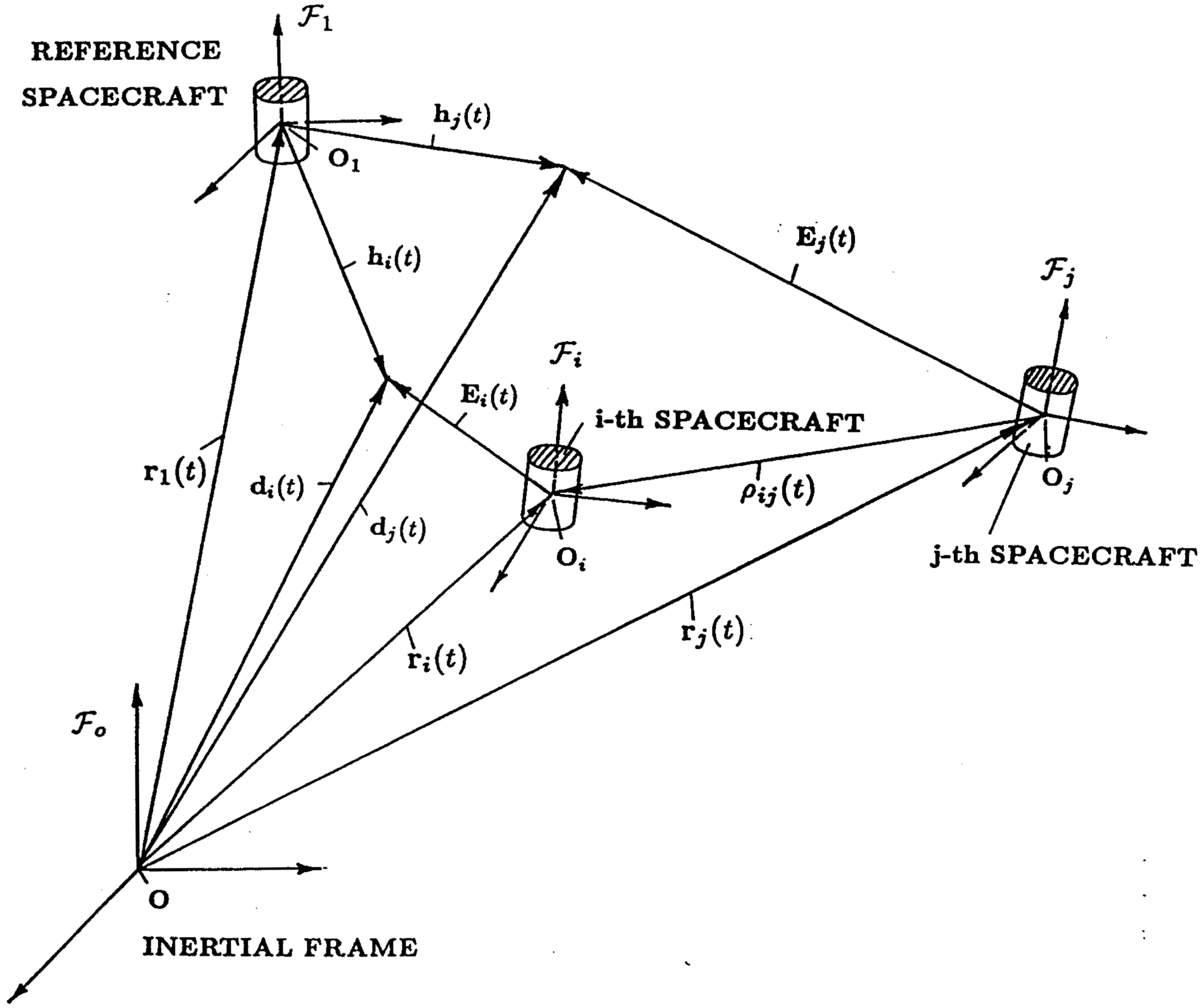

Fig. 1 Sketch of coordinate systems.

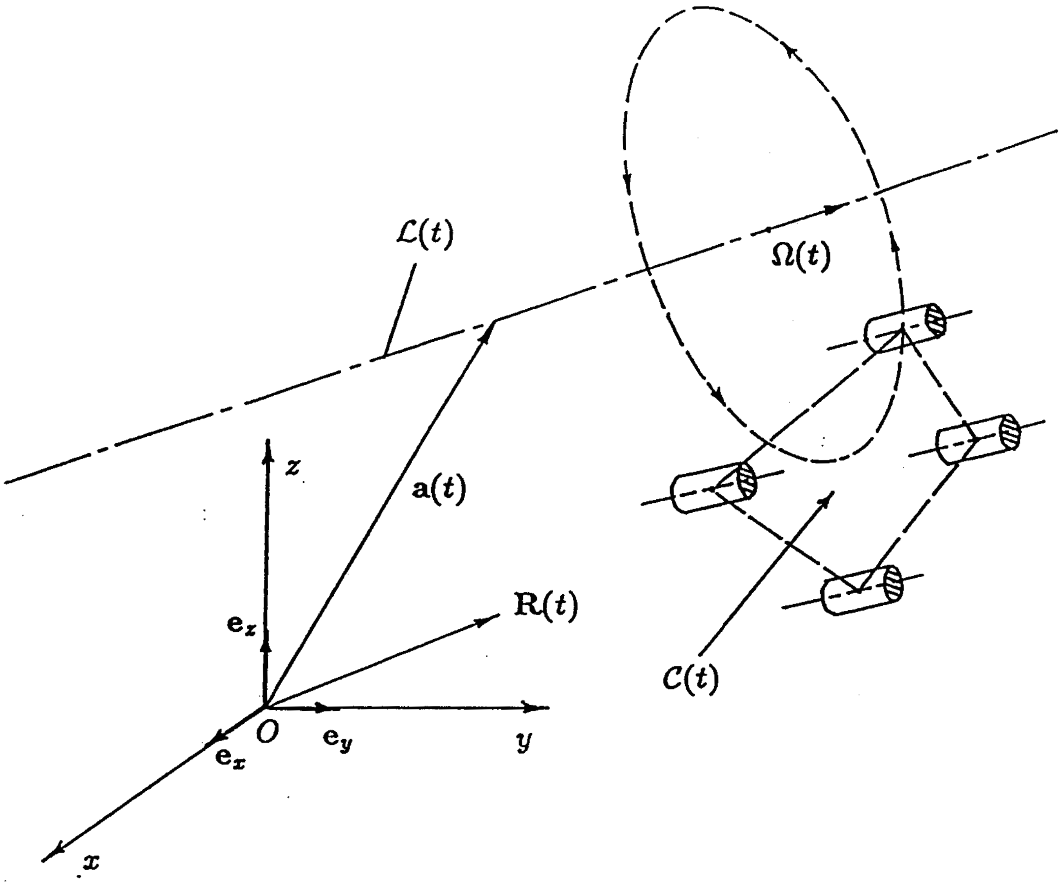

Fig.2 Formation rotation about the line $\mathcal{L}(t)$.

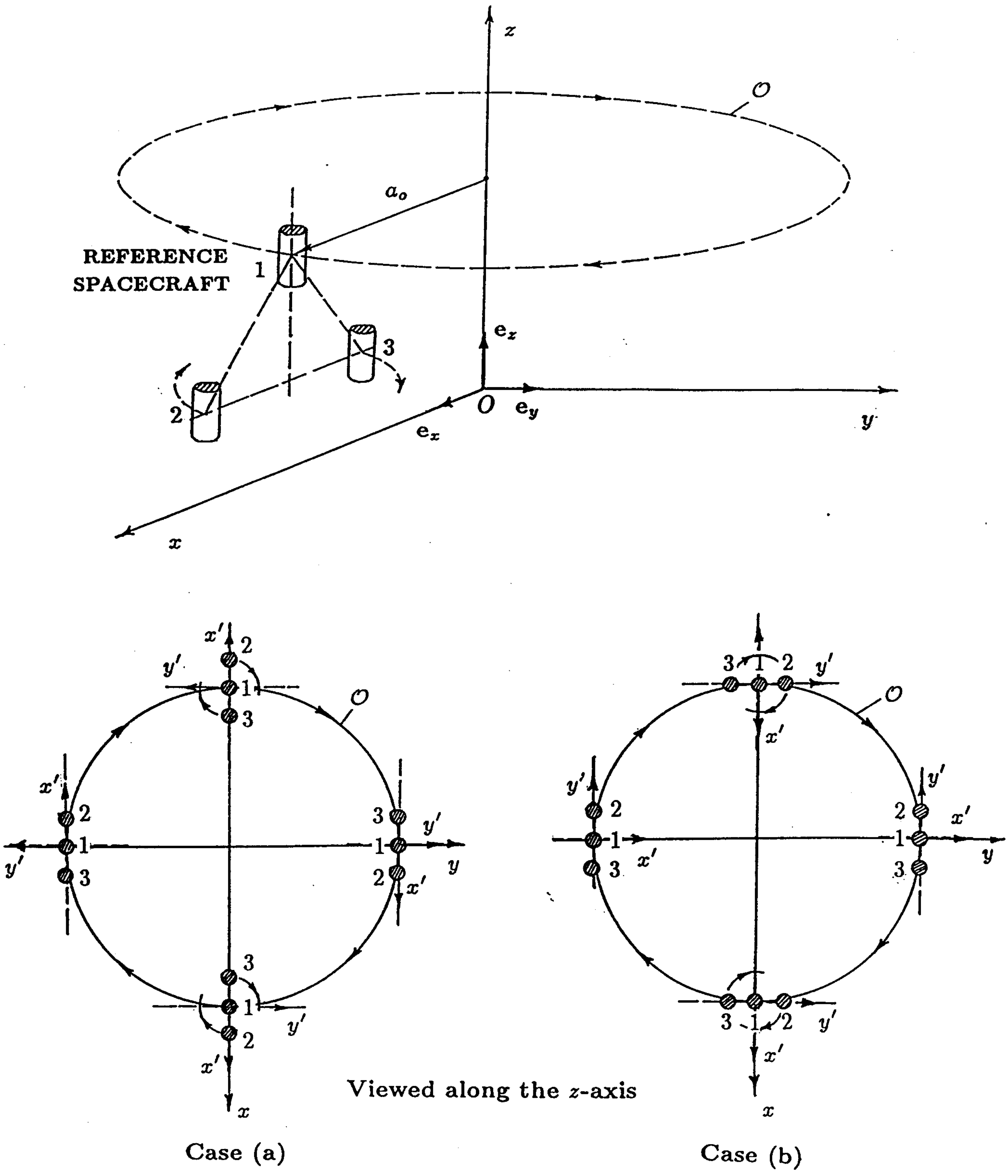

Fig.3 Rotation of spacecraft triad in a triangular formation.

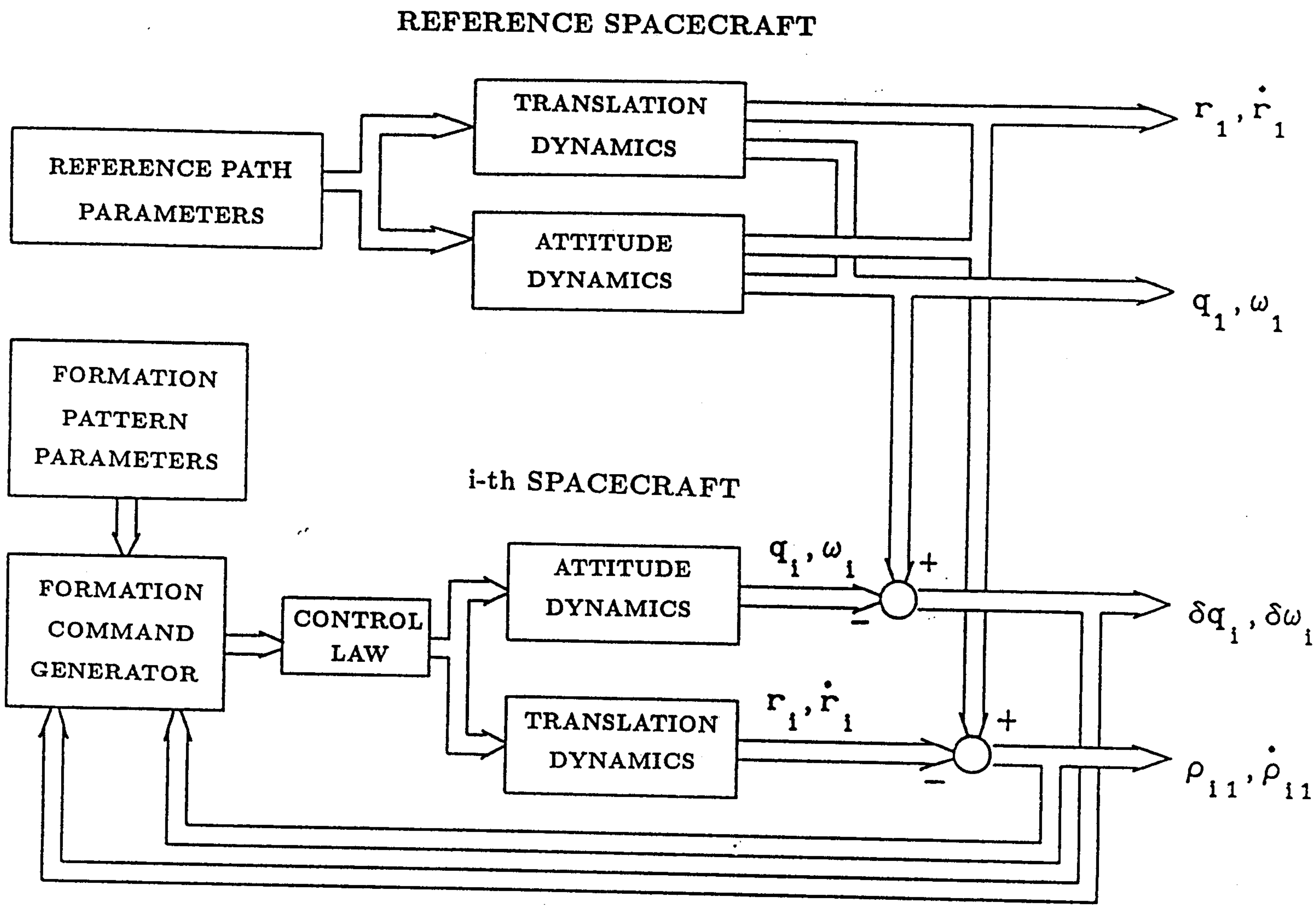

Fig.4 Structure of the overall control system.

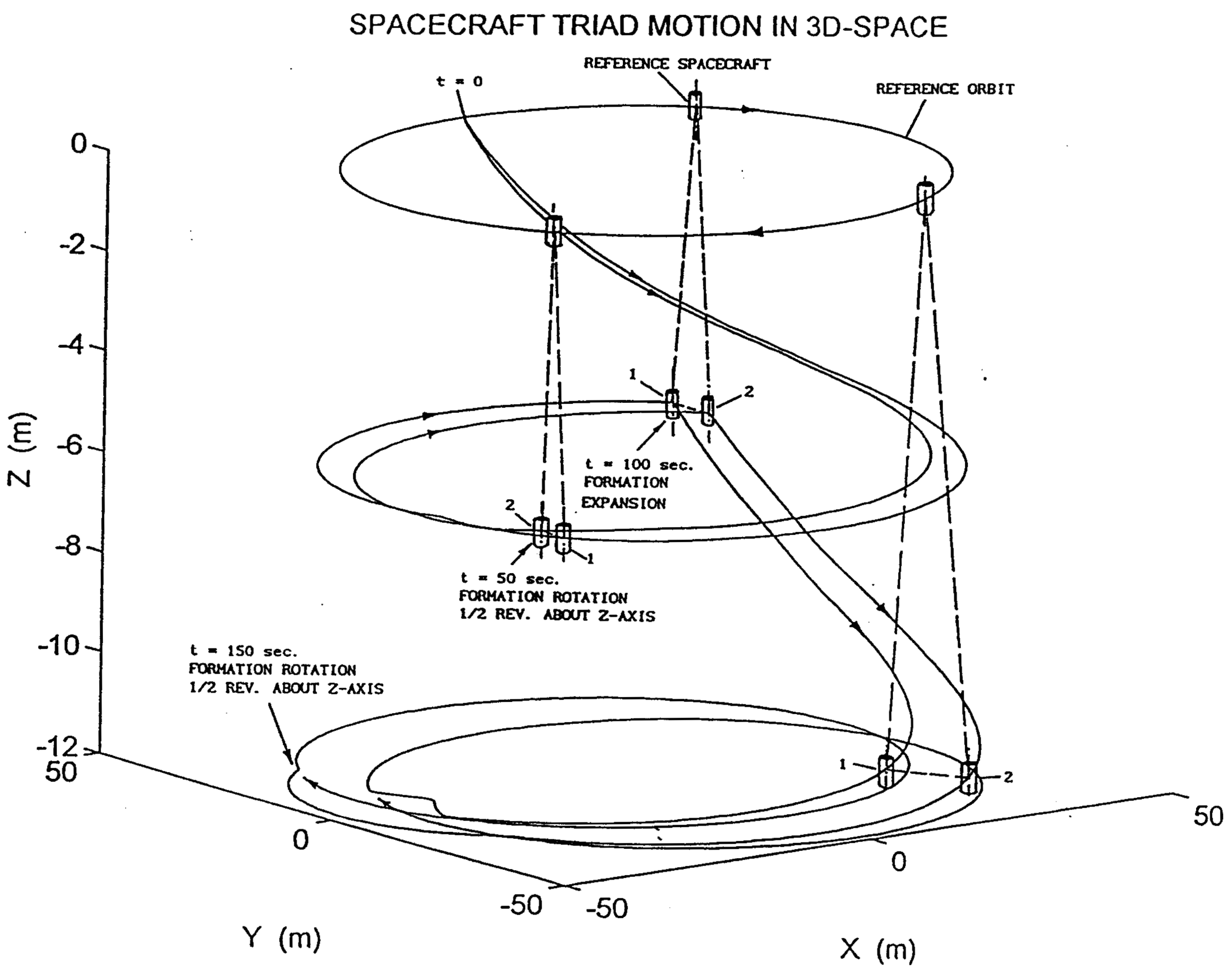

Fig.5 Trajectories of a spacecraft triad undergoing formation reconfiguration and rotation in 3-dimensional space.

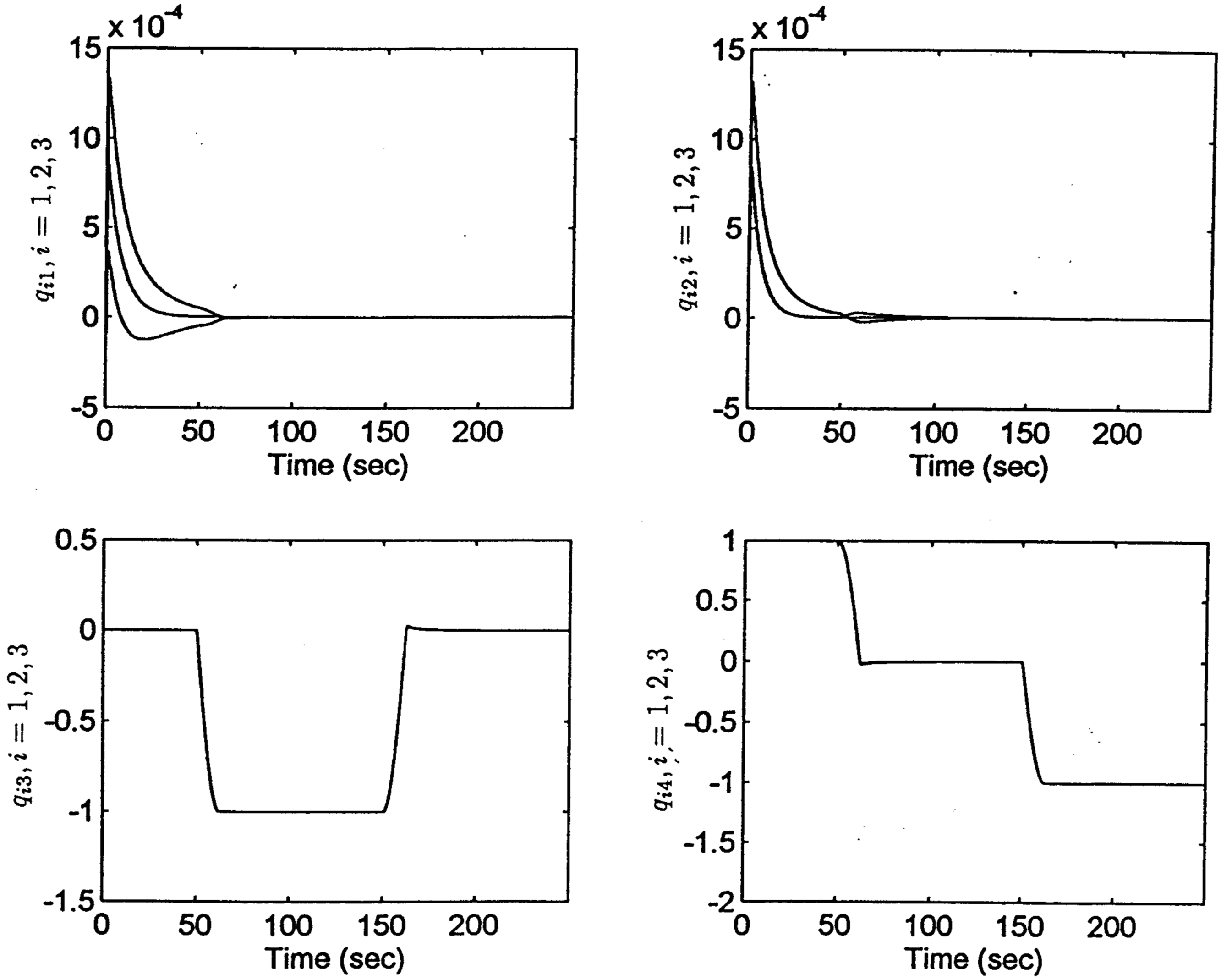

Fig.6a Quaternion components of spacecraft triad versus time.

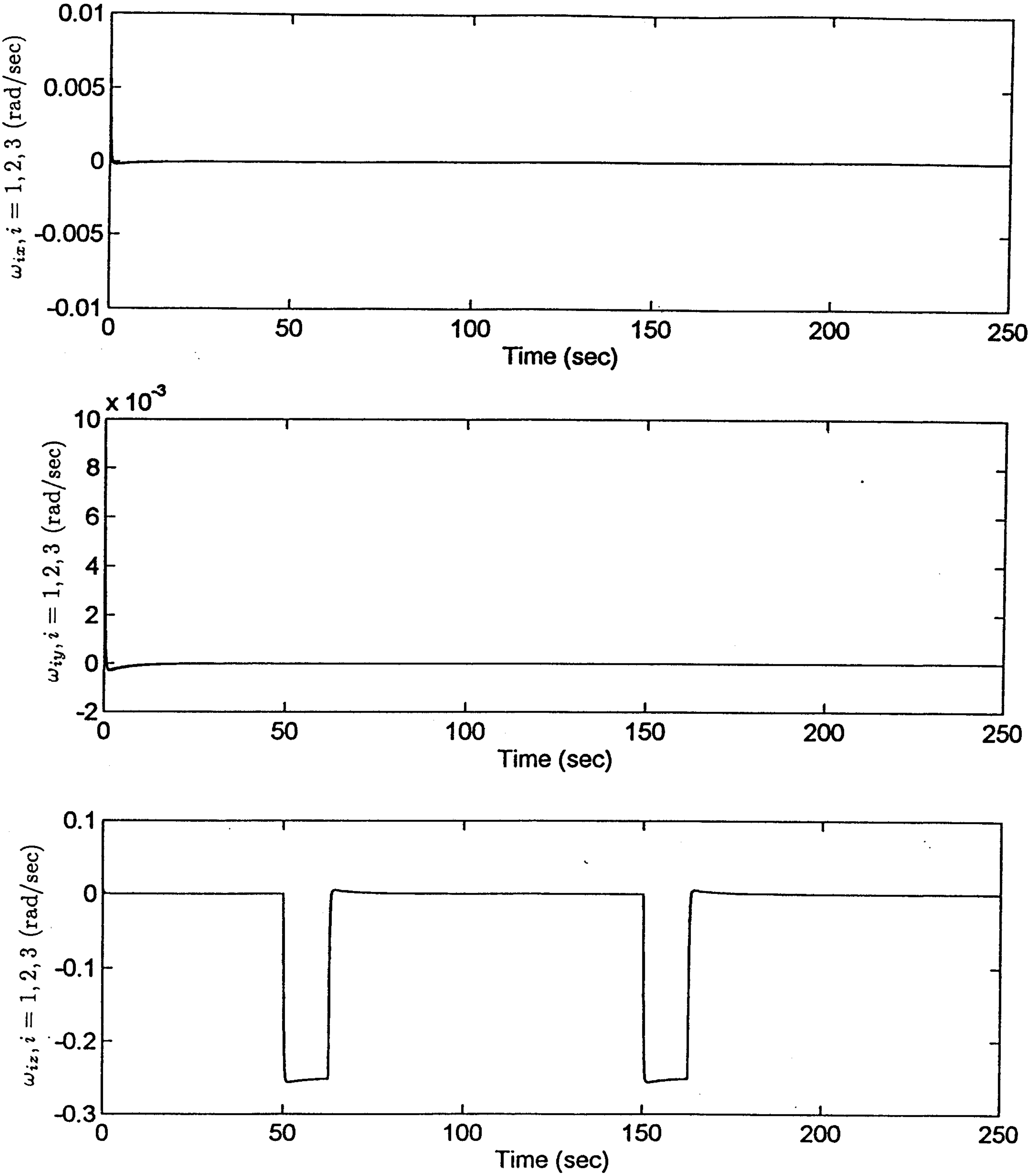

Fig.6b Angular velocity components of spacecraft triad versus time.

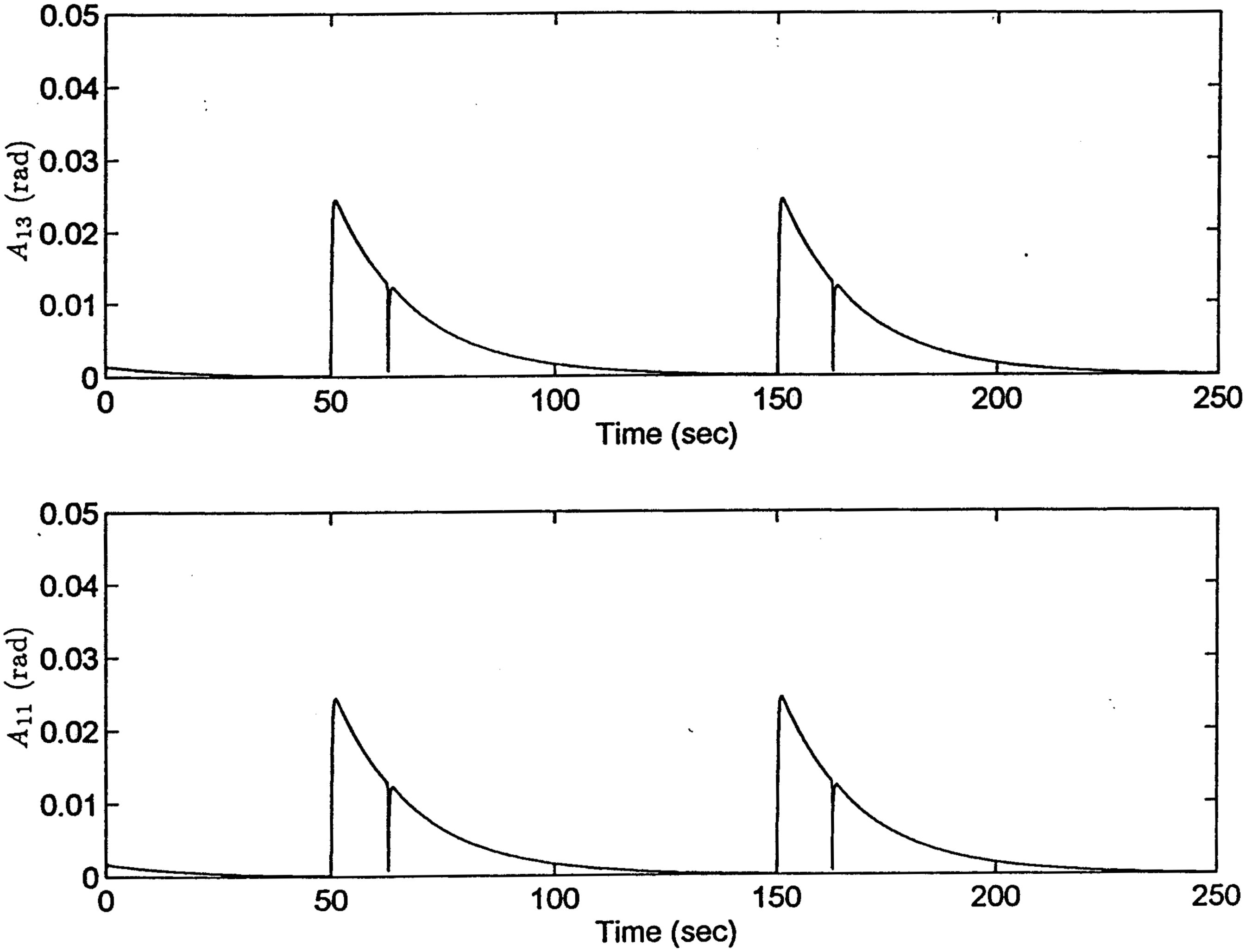

Fig.7a Time records for the attitude errors of spacecraft 2 and 3 relative to the reference spacecraft. (A_{1i} - the relative attitude angle between i-th spacecraft and the reference spacecraft)

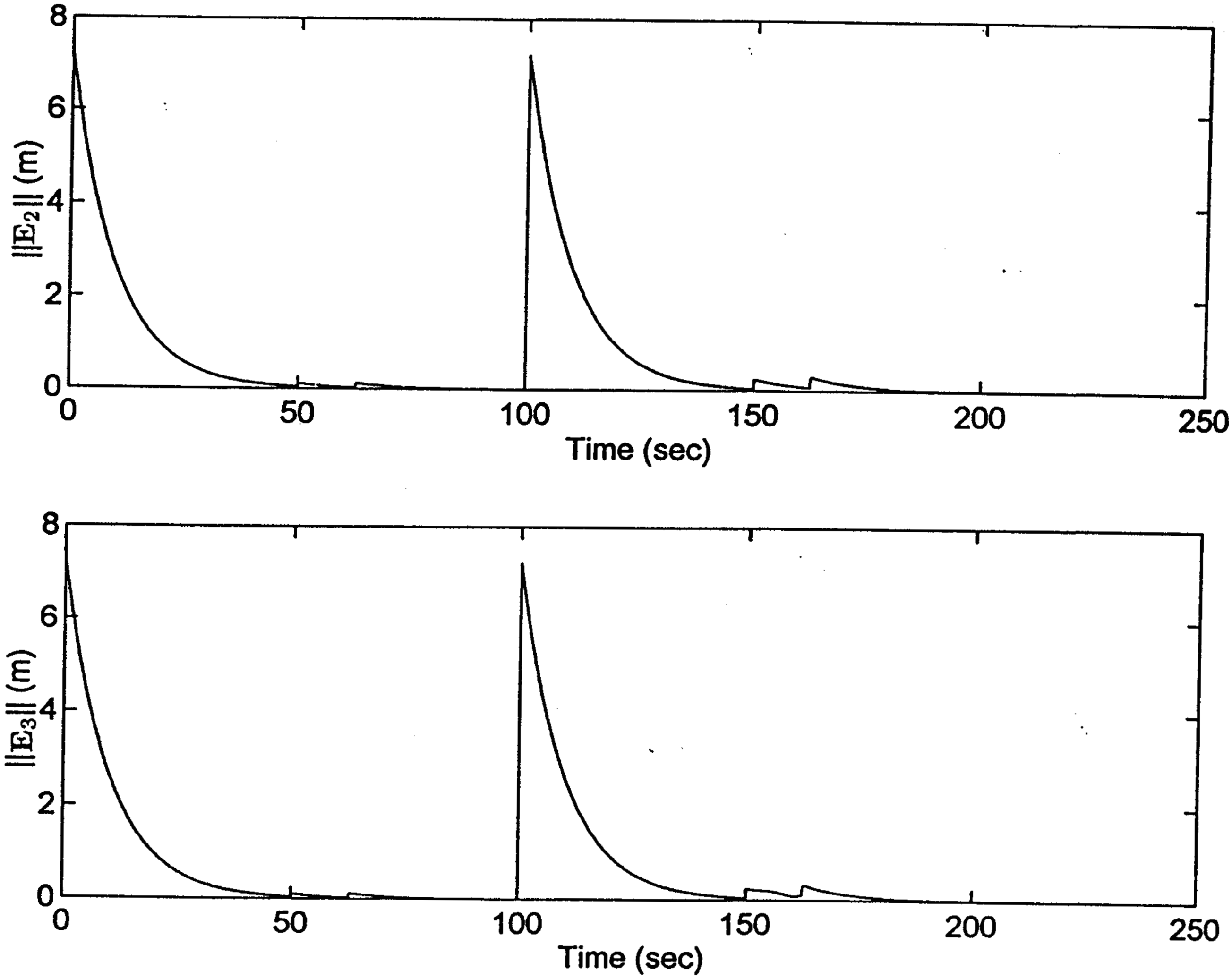

Fig.7b Time records for the norm of position tracking errors of spacecraft 2 and 3 relative to the reference spacecraft.

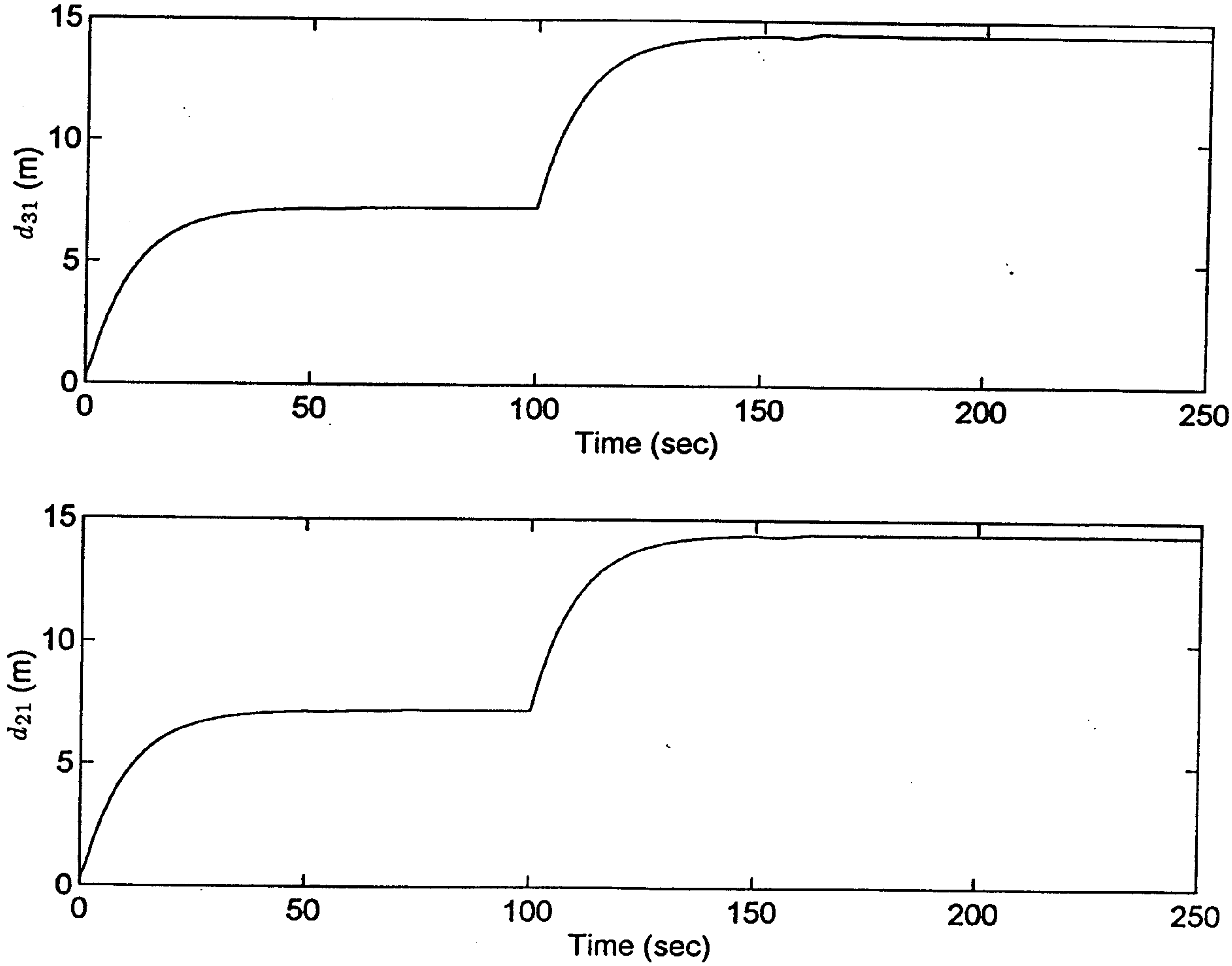

Fig.8 **Time records for the distances between spacecraft 2,3 and the reference spacecraft.**
(d_{i1} - distance between the i-th spacecraft and the reference spacecraft)